Supplementary Material for
How N. G. (Dick) de Bruijn Connected Mathematics and the Arts

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Abstract

More background information, references (also of what others wrote about de Bruijn), pictures, etc.

Introduction

I have consulted many sources and spoken with numerous people. Here, I collect some additional information on Dick de Bruijn.

Sources of Personal Information about Dick de Bruijn

I studied Applied Mathematics at TU Eindhoven from 1976 until 1985 (during my master phase, I also worked in industry for a couple of years, doing software development for a parallel computer). In 1985, I got a PhD position with Computer Science, and in 1987 this was converted to a tenured Assistant Professorship CS. While studying, I took several courses taught by de Bruijn:

- Measure Theory and Lebesgue Integration (2.220)
- Combinatorics (2.950)
- Mathematical Languages (2.960); included Automath
- Language and Structure of Mathematics (2.965)

These were all special in their own way, both in the choice of topics and in his approach to teaching the material. All these courses were quite challenging, and I liked them very much. I also attended various colloquia that de Bruijn gave at TU Eindhoven.

Other sources:

- Unpublished autobiographic material (in Dutch), 4+16 typewritten pages, covering 1918–1924 and 1924–1944; obtained from Henriëtte de Brouwer.
- [26] (in Dutch), where he mentions that he worked on Riemann's zeta-function, and that he tried to prove the *Riemann Hypothesis*, on and off for many years. Here he also mentioned that before coming up with the proof for the number of de Bruijn sequences, he had experimented with thousands of examples before he got the key insight.
- [14], where Dick mentions *The Epsilon Boys*; also contains the full text of Dick's song for Kloosterman.
- [15], where Dick mentions again the singing group *The Epsilon Boys*, explaining: "It was called The Epsilon Boys, epsilon being for us the true symbol for mathematics. We were always in tune, within an epsilon."
- [23, 24], obituary/in memoriam
- [29], personal correspondences between the author and Dick (especially Chapter 28).

IMO Problems by de Bruijn

De Bruijn had two problems accepted at the International Mathematical Olympiad (IMO).

1. Problem 6 at IMO 1993 (Turkey):

   There are n lamps $L_0, \ldots, L_{n-1}$ in a circle ($n > 1$), where we denote $L_{n+k} = L_k$. (A lamp at all times is either on or off.) Perform steps $s_0, s_1, \ldots$ as follows: at step s_i, if L_{i-1} is lit, switch L_i from on to off or vice versa, otherwise do nothing. Initially all lamps are on. Show that:

   (a) There is a positive integer $M(n)$ such that after $M(n)$ steps all the lamps are on again;

   (b) If $n = 2k$, we can take $M(n) = n^2 - 1$;

   (c) If $n = 2k + 1$, we can take $M(n) = n^2 - n + 1$.

   Also see [31] for an interactive demonstration.

2. Problem 2 at IMO 2005 (Mexico):

   Let $a_1, a_2, \ldots$ be a sequence of integers with infinitely many positive and negative terms. Suppose that for every positive integer n the numbers $a_1, a_2, \ldots, a_n$ leave n different remainders upon division by n. Prove that every integer occurs exactly once in the sequence $a_1, a_2, \ldots$.

Other Playful and Art-Relevant Publications by de Bruijn

- [3] (in Dutch): "Puzzle and game elucidated from a higher perspective"
- [4] (in Dutch): "The symmetry of 1961", newspaper article about half-turn symmetries
- [5], about peg solitaire
- [6] (in Dutch): "The programming of a pentomino puzzle"; note the year of publication, long before personal computers were available.
- [8], about an interesting solitaire card game.
- [11], generalizes the problem "Show that there exists a polygon with 1990 equal angles, of which the edges have length $1^2, 2^2, \ldots, 1990^2$ (in some order)."
- [12], concerns a famous problem that is easy state and hard to prove formally, involving the tiling of a hexagon with rhombs consisting of two equilateral triangles (such rhombs are know as *calissons*).

De Bruijn on Aperiodic Tilings

De Bruijn's key contribution to aperiodic tilings is no doubt his article [7], which hit the world like a bomb shell.

While implementing the ideas in [7] to help me reconstruct *The Wieringa Roof* and *The 'Knights' Tiling*, I ran into some typos:

- p.46, line -5: change '$180°$' to '$108°$'
- p.54, line 6: change 'ζ' to 'ξ'
- p.54, (9.7): change '$(u + v, u, 0, 0, v)$' to '$(u + v, -u, 0, 0, -v)$'
- p.56, line 19: change 'Let is call it' to 'Let us call it'
- p.57, line 4: change 'two-line' to '2-line'
- Also see [9] for some other notes.

I also wondered what the parameters for [7, Figure 2] were. When you look carefully, you see that it derives from the exceptionally singular pentagrid with $\xi = 0$ (the cartwheel): it is centered on the dodecagon just to the right of the center of [7, Figure 2].

In [7, §13], Dick investigates the symmetries that pentagrids can have.

Summarizing, we have the following essentially different cases of symmetry:

$$\xi = 0, \quad \xi = 1, \quad \xi = 2, \quad \xi = 5/2, \quad \xi = \tfrac{1}{2}(\zeta^2 - \zeta^3),$$

$$\xi = \tfrac{1}{2}(\zeta - \zeta^4), \quad \xi \in \mathbb{R}, \quad \xi \in i\mathbb{R},$$

apart from the fact that the latter two can be equivalent to one of the others in exceptional cases.

Figure 1: *Pentagrid parameters with non-trivial symmetries, from [7, §13].*

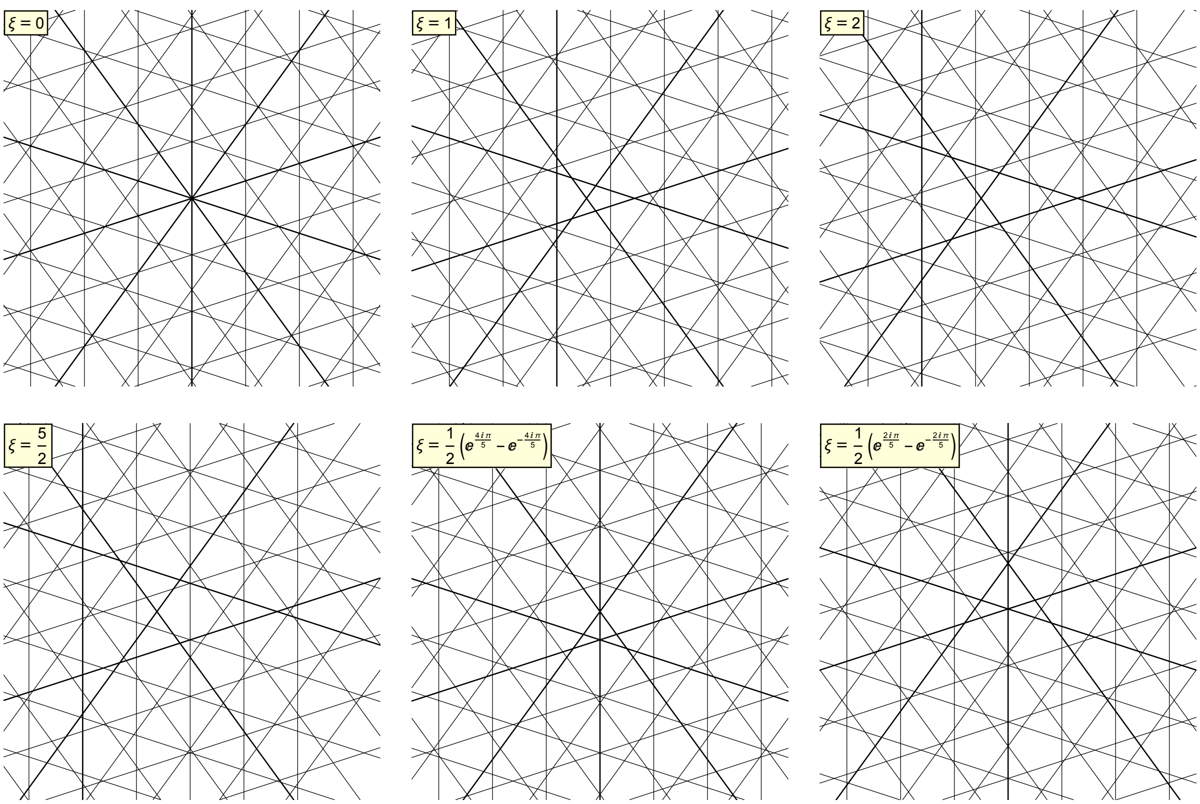

Figure 2: *Symmetric pentagrids*

Here is an explicit overview of the symmetries (in parentheses are approximate γ_i values; also see Figure 2 and 3 below):

- $\xi = 0$: $(0, 0, 0, 0, 0)$, a.k.a. *cartwheel*; exceptionally singular, order-10 pseudo-rotational symmetry, and pseudo-reflection symmetry (in the Conway worms); gives rise to 10 different rhombus tilings; note that the asymmetric Conway worms break the symmetry near the origin
- $\xi = 1$: $(-0.552786, 0.276393, 0, 0, 0.276393)$, a.k.a. *infinite star*; regular, order-5 rotational symmetry, reflection symmetry (e.g., in horizontal line)

- $\xi = 2$: $(-1.10557, 0.552786, 0, 0, 0.552786)$, a.k.a. *infinite sun*; regular, order-5 rotational symmetry, reflection symmetry (e.g., in horizontal line)
- $\xi = \frac{5}{2}$: $(-1.38197, 0.690983, 0, 0, 0.690983)$; singular, reflection symmetry in horizontal line, pseudo-reflection symmetry in vertical Conway worm; gives rise to 2 different rhombus tilings
- $\xi = \frac{1}{2}\left(e^{\frac{4\pi}{5}} - e^{-\frac{4\pi}{5}}\right)$: $(0, -0.5, 0, 0, 0.5)$; singular, reflection symmetry in horizontal line, pseudo-reflection symmetry in vertical Conway worm; gives rise to 2 different rhombus tilings
- $\xi = \frac{1}{2}\left(e^{\frac{2\pi}{5}} - e^{-\frac{2\pi}{5}}\right)$: $(0, -0.809017, 0, 0, 0.809017)$; singular, reflection symmetry in horizontal line, pseudo-reflection symmetry in vertical Conway worm; gives rise to 2 different rhombus tilings
- $\xi \in \mathbb{R}$: $u = v$; regular, reflection symmetry in horizontal line
- $\xi \in i\mathbb{R}$: $u = -v$; singular, pseudo-reflection symmetry in vertical Conway worm; gives rise to 2 different rhombus tilings

Note that Wieringa roofs live in three dimensions and therefore have slightly different symmetries.

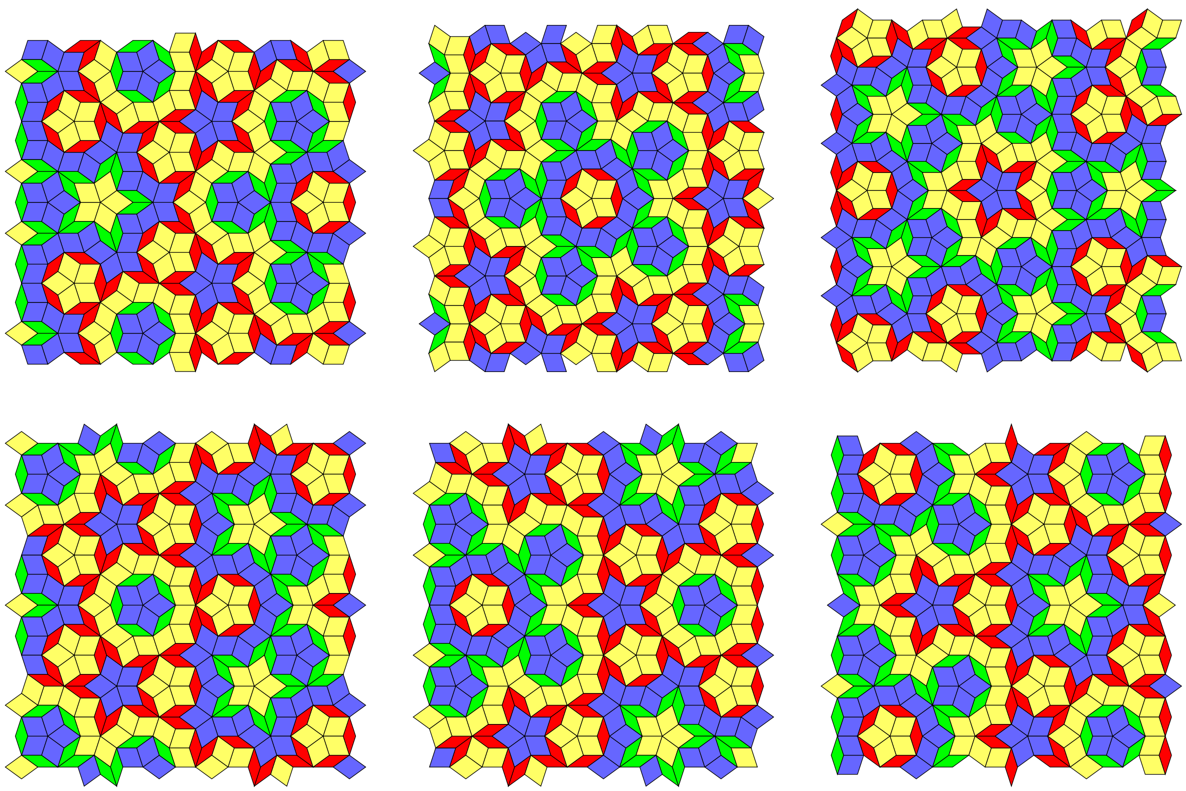

Figure 3: *Symmetric rhombus tilings corresponding to Figure 2*

The 'knight' tile also appears in [10, Figure 3] as the deflation of a dart.

In my article, I mention Conway's theorem on how far you need to look in an arbitrary aperiodic Penrose rhombus tiling to find a particular fragment of diameter d. There, I took the liberty of using the upper bound πd. But Conway actually proved a slightly tighter bound: $\left(1 + \frac{1}{2}\Phi^3\right) d \approx 3.11803d$, where Φ is the golden ratio [20, Ch.1, p.10].

De Bruijn Fequences for fountains

For comparison, the supplementary material includes six animated GIFs:

- `BinaryCountingFountain{4,6}.gif` using binary counting
- `GrayCodeFountain{4,6}.gif` using the reflected Gray code
- `deBruijnSequenceFountain{4,6}.gif` using a binary de Bruijn sequence

Others about the Mathematics of de Bruijn

[1, 2, 16, 18, 19, 21, 22, 25, 27, 30]

More Pictures of de Bruijn

Figure 4 shows Dick de Bruijn at the Math Department Party in 1979, singing a song that he wrote about Mrs. Geerts, who ran the department's canteen. I obtained the song text (in Dutch) from Henriëtte de Brouwer, long-time librarian of the department's library.

Figure 4: *Dick de Bruijn at the Math Department Party, September 1979: with Bert van Benthem Jutting behind the piano (left); by himself (middle); with Mrs. Geerts (right); image credits: Henk van Tilborg.*

Figure 5: *De Bruijn with Penrose tiling superimposed (1996, symposium 50 year professor).*

Dick briefly features in [28], with a picture from 1987. A special booklet was published 1996 when de Bruijn celebrated that he was first appointed as professor 50 years ago (Figure 5).

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