Cube Compound Puzzles

Supplement

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It is difficult to gain an understanding of how these puzzles go together by viewing the assembled puzzle or a single piece. In order to give the reader insight into their construction, we show here additional photos of the puzzles partially assembled. We also give further details regarding *Bakos' Puzzle*, and a few thoughts on making your own puzzles.

Visualization of the four puzzles in Figure 1

When the Figure 1 puzzles are assembled, the appearance is that of a Compound of *n* Cubes, C_n. To give the reader insight into their construction, Figures 1s and 2s show the assembled puzzles with one piece missing. The missing piece is displayed in front of the puzzle.

Figure 1s: *Kubusmix and the Compound of Three Cubes assembled with one piece removed. These puzzles are assembled by grouping half the pieces in each hand and then sliding the halves together.*

Recall that the pieces are either identical in shape, or present in mirror image pairs. Thus, the foreground piece gives an understanding of the other pieces. The coloring of the remaining pieces can be different, again this depends on the puzzle. *Kubusmix,* for example, has all identical pieces but half are red and half are green. With the *Compound of Three Cubes*, each piece is made using all three colors, but the pieces come in mirror image pairs.

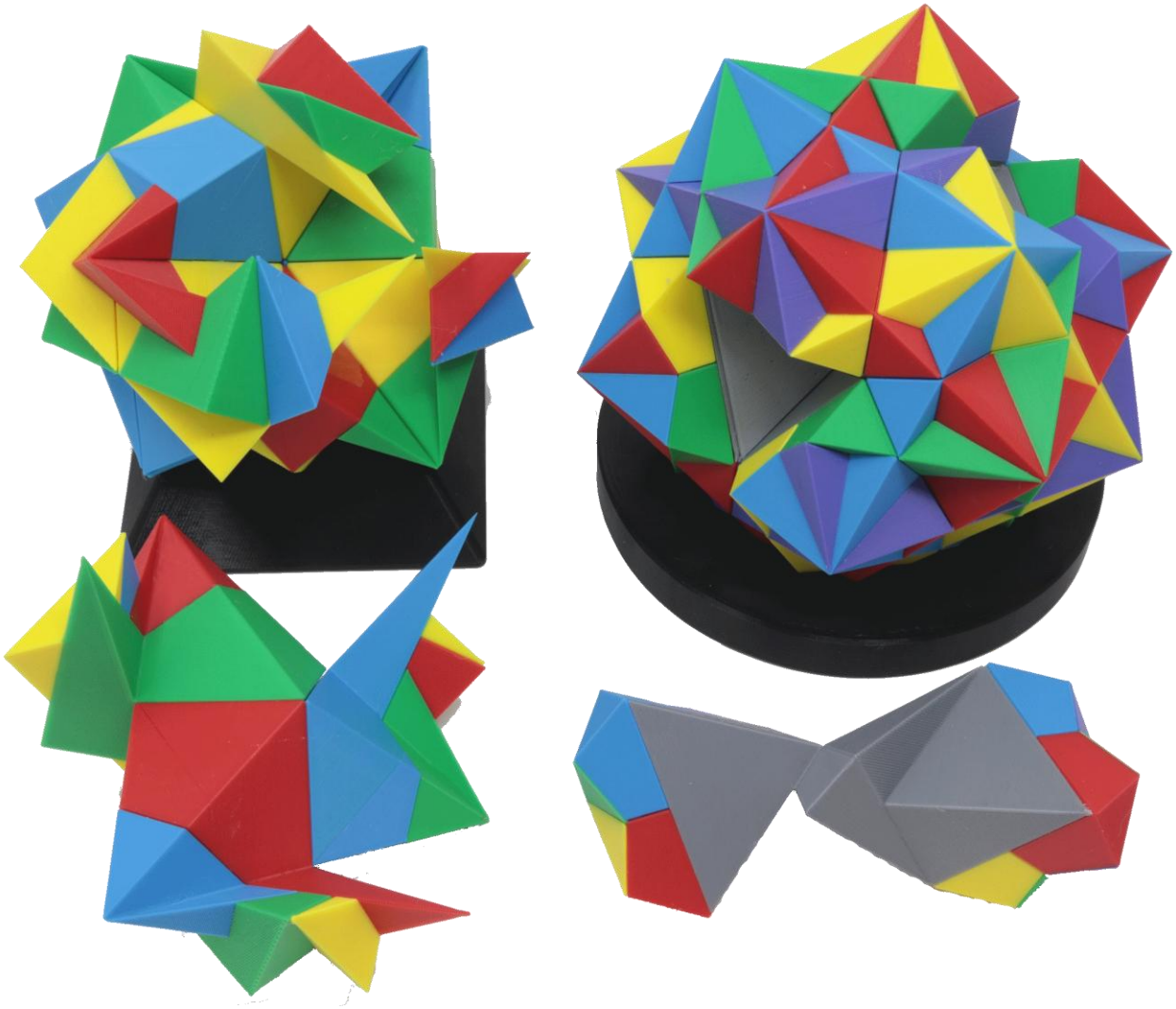


Figure 2s: *Bakos' Puzzle and the Compound of Five Cubes assembled with one piece removed. These puzzles are assembled by moving all pieces toward the center simultaneously, known as coordinate-motion [4].*

Figure 2s shows the construction of *Bako's Puzzle* and *The Compound of Five Cubes*. These puzzles are more difficult to assemble. In order to add the final piece, the pieces must be expanded until they nearly come apart. With some luck, the final piece can be added, and the entire assembly is then allowed to contract into the final shape.

Details in the construction of *Bakos' Puzzle*

Bakos Puzzle is constructed differently from the other three puzzles. The basic piece is obtained by performing an intersection with a puzzle piece for a rhombic dodecahedron. For this reason we now go over its construction in some detail.

First we show how to dissect a rhombic dodecahedron into four identical pieces. This is done using a face dissection by a tetrahedron, shown in Figure 3s. This results in four identical rhombohedra (6-sided polyhedra with six identical rhombic faces).

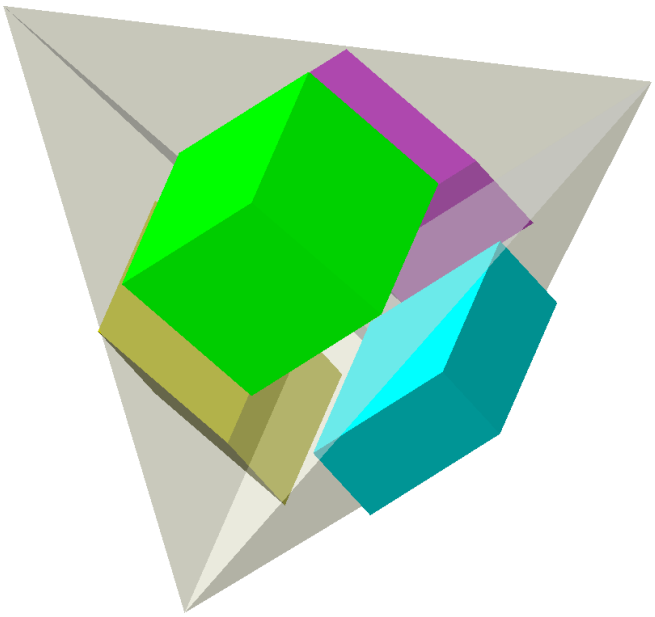
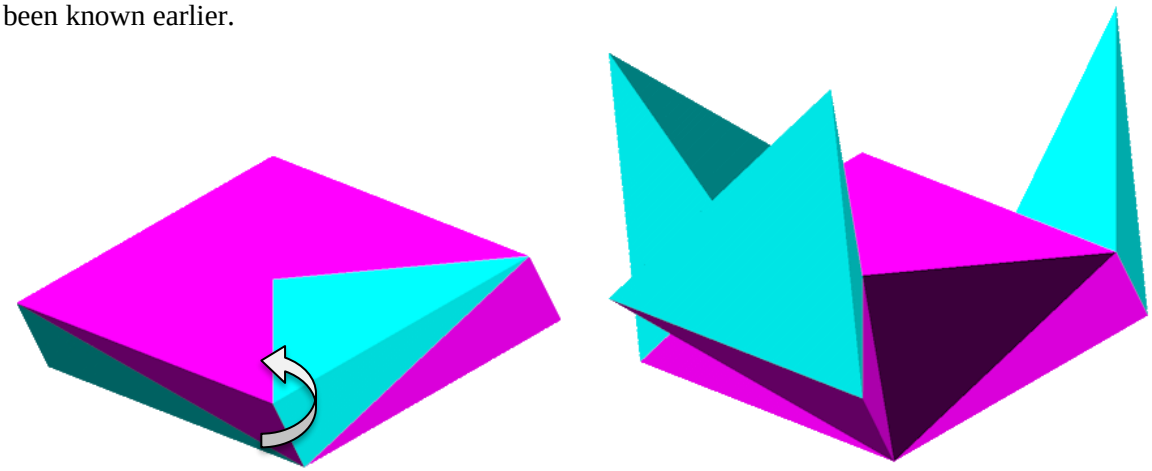


Figure 3s: *Face dissection of a rhombic dodecahedron using P=tetrahedron. The result is four identical rhombohedra.*

Next we make three (aqua) cuts into a rhombohedron and rotate each cut 90° around a hinge as shown in Figure 4s. Figure 4s(b) shows the finished puzzle piece, four of them assemble into a rhombic dodecahedron (RD) using coordinate-motion [4]. Stephen Chin discovered this puzzle in 2013, but it may have been known earlier.



(a) (b)

Figure 4s: *Cutting three aqua tetrahedra from a rhombohedron, and rotating 90° about a hinge to make the completed RD puzzle piece.*

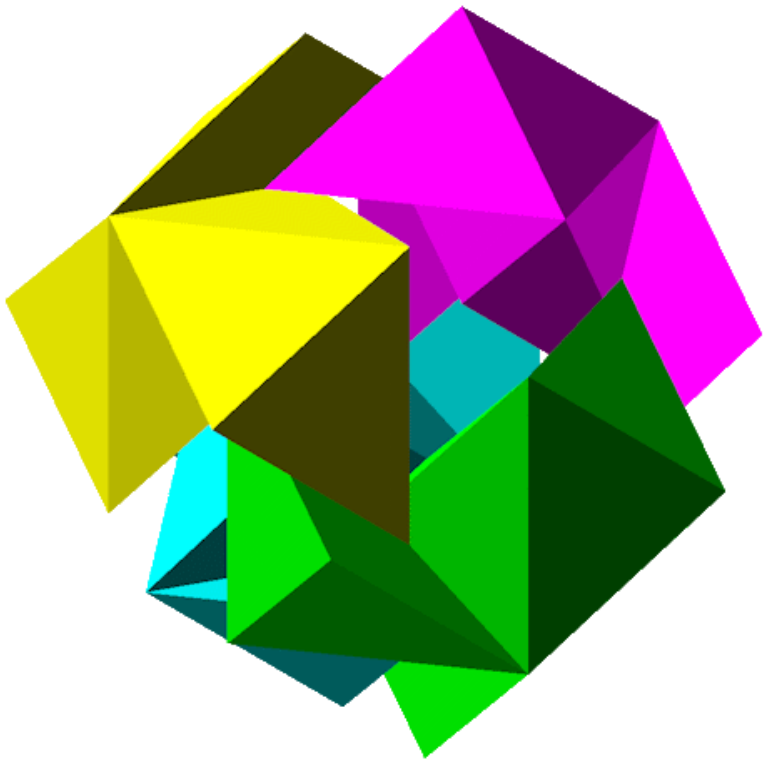


Figure 5s: *Assembly of four RD puzzle pieces into a rhombic dodecahedron.*

The final step is now to intersect the piece in Figure 4s(b) with C_4. To do this we align the axes of 3 and 4-fold symmetry between the rhombic dodecahedron and Bakos' solid. The resulting puzzle piece is shown in Figure 7 and Figure 2s. It is unfortunate that each piece contains three sharp points, which tend to break off.

Building your own puzzles

It can be quite rewarding to make these puzzles, either using 3D printing or woodworking. I have experience only with the former. You can download STL files for printing the first two puzzles at [3], I plan to sell plans for the other two puzzles for a small fee [5].

Kubusmix is the easiest to make, because each piece can be printed in a single color and no supports are needed. With the other puzzles, there are two options for making the pieces. The first is to print each component in a single color, and then connect all the components (using glue or some other technique) to create each piece. The second option is to print each piece in multiple colors (which requires a printer that can print in several colors simultaneously). I use the first option for *The Compound of Three Cubes* and *The Compound of Five Cubes*. For *Bakos' Puzzle* I use a hybrid technique where components are printed in multiple colors but then glued together to make each piece.