The Aesthetics of Mutual Support: Geoband Polyhedra Workshop

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Abstract

Geoband Polyhedra Workshop leverages the elastic bending of bands to form flexible, edge-based frameworks that enhance participants' spatial perception, problem-solving, and collaboration skills [1,5,7]. This workshop aims to construct a closed bending-active skeletal structure resembling a soccer ball, consisting of 12 pentagonal and 20 triangular regions, fostering spatial reasoning, collaboration, and creative exploration through hands-on manipulation of flexible Geoband strips. Participants explore mathematical concepts like structural stability and Euler's characteristic while engaging in creative design.

Introduction

In mathematics education and STEM fields, spatial reasoning and geometric understanding represent critical cognitive skills that support student development across multiple disciplines. Polyhedron building activities have long been recognized as effective pedagogical tools for developing these essential spatial abilities, allowing students to physically interact with three-dimensional concepts that might otherwise remain abstract. Such activities provide concrete experiences with mathematical structures, helping learners develop intuitive understanding of geometric principles through tactile engagement.

Traditional approaches to teaching polyhedra concepts frequently rely on rigid construction materials that limit experiential learning opportunities. While conventional manipulatives such as paper folding, plastic polydron pieces, and rod-based construction kits offer valuable learning experiences, they often constrain exploration to predetermined structures with fixed properties. These traditional tools, though beneficial, typically result in static final products that fail to demonstrate the flexible interrelationships between structural elements. This limitation, observed in tools like Polydron and Zometool [6], restricts students' ability to explore dynamic force interactions, hindering conceptual understanding of 3D geometry.

The concept of mutual support structures offers a compelling alternative approach to polyhedron construction. In these systems, individual components work together to create a structurally sound whole, with each element simultaneously supporting and being supported by adjacent parts. This architectural principle, evident in various historical designs from ancient wooden joinery to Leonardo da Vinci's self-supporting bridges, provides a powerful metaphor for interdependence that extends beyond mathematics into broader educational contexts. Geobands represent an innovative educational tool specifically designed to implement this principle, utilizing flexible bands of varying lengths that interconnect to form stable three-dimensional structures without centralized support. The workshop described in this paper introduces a systematically designed learning experience using Geobands to construct bending-active skeletal structures through mutual support principles. Unlike traditional polyhedron activities, this approach emphasizes the flexible relationships between components, allowing participants to observe and manipulate the elastic interactions of bands, which redistribute forces to maintain structural stability. By progressing from basic modules to complex three-dimensional forms, learners engage with essential geometric concepts while simultaneously developing collaborative problem-solving skills and spatial reasoning abilities. The unique properties of Geoband materials—flexibility, interconnectivity, and resilience—create learning opportunities that transcend conventional construction activities, fostering both mathematical understanding and appreciation for the aesthetic dimensions of geometric structures.

This workshop methodology represents an educational innovation that bridges mathematical theory with physical experience, providing a multifaceted learning environment that simultaneously addresses cognitive, social, and creative development [2, 6].

Motivation

Geometric principles of mutual support structures

A reciprocal structure relies on interdependent members to achieve self-sustaining stability. Each part shares the load of others, resulting in self-sustaining stability. Modern examples, such as reciprocal frames and nexorades, illustrate this principle's application in advanced structural design [1, 5, 7]. These structures are characterized by balanced force distribution, where the failure of one element can lead to the collapse of the whole. Educationally, this concept helps students develop intuition about the relationship between parts and wholes, the distribution of forces, and the philosophical implications of cooperation.

Improve mathematical structure and spatial awareness of polyhedra

In traditional geometry, a polyhedron is a three-dimensional solid bounded by flat polygonal faces, while a spherical polyhedron(e.g., a geodesic dome) approximates a sphere but still consists of flat faces. In contrast, a sphere is a perfectly round three-dimensional shape with no flat faces. The structures created in this workshop using Geobands do not fit these categories. Instead, they are bending-active skeletal structures, where flexible, curved bands form a network of edges rather than solid faces. These structures derive their form and stability from the elastic bending of the bands and their unique interconnections, including connectors at vertices of one face and midpoints of another, a feature that enhances their structural and aesthetic appeal [1, 4, 11]. For example, the workshop's target structure resembles a truncated icosahedron, with 12 pentagonal and 20 triangular regions, where each vertex typically has a degree of 3 (three edges meeting at a vertex). For the truncated icosahedron, Euler's characteristic (V-E+F=2) holds, with 60 vertices, 90 edges, and 32 faces (12 pentagons, 20 triangles) [10]. This configuration allows participants to explore face shapes (pentagons and triangles), edge relationships (curved bands), and vertex connectivity, connecting to concepts like Euler's characteristic, which describes the topological properties of such forms [1, 10]. At the same time, it requires spatial visualization skills to envision the process of folding two-dimensional shapes into three-dimensional solids [1, 10].

Existing studies report that such three-dimensional building activities contribute significantly to students' spatial cognition. Research suggests that experimental activities with modular models are an effective instructional approach for building spatial skills, and reviews of spatial abilities in mathematics education emphasize that action-oriented learning involving hands-on construction and manipulation of three-dimensional shapes can significantly improve students' spatial skills [1, 5]. Spatial skills are an important cognitive component not only in math learning, but also in science, engineering, and everyday life, and activities such as building bending-active skeletal structures are needed to complement current curricula that lack experience with three-dimensional objects, especially in geometry classes.

How Geobands are different from traditional polyhedron manipulation activities

Typical polyhedron manipulatives utilized in schools include cutting and folding polyhedral nets out of paper, connecting plastic polygonal pieces (e.g., Polydron brand), or building skeletal structures using rod-and-bolt kits such as Zometool or 4DFrame [6]. While these traditional teaching aids have the advantage of using structured parts (e.g., straight rods, structured polygonal faces) to construct an object with an expected shape, the finished structure is usually rigid and limits the experience of active interaction between elements. The concept of creating bending-active skeletal structures using curved bands has precedent in mathematical art, notably in the work of Yananose [12], who explored curved band origami polyhedra using paper strips. This workshop builds upon such approaches while introducing the educational dimension of mutual support structures and the use of specialized materials designed for classroom implementation. This versatility extends beyond regular polygonal forms, as other researchers have explored mathematical structures using different geometrical shapes, such as rhombuses [8], which also employ interconnected elements to create stable three-dimensional forms. Geobands, on the other hand, use flexible, curved bands to represent the edges of bending-active skeletal structures, and the flexible stability of the structure can be experienced as each piece pulls and pushes against the others (Figure 1). Each Geoband band is perforated with four holes, which are designed to provide points where bands can be cross-linked, allowing for mutually supportive connections.

Figure 1: *Geoband strips in various colors and lengths (Left: red 12.3 cm, white 12 cm, bBlack 10.9 cm, green 10.7 cm) and connector buttons (Right).*

The bands are joined at the eyelets by specialized plastic clips (buttons), and when joined, the rule of "end eyelet (1) is over middle eyelet (2)" must be observed (Figure 3, 4). Thanks to this unique joining rule and flexibility, three-dimensional structures made with Geobands are resiliently stable, allowing for slight deformation but not collapsing, unlike structures made with ordinary plastic building kits. For example, conventional polydrons can only create curved polyhedra with straight polygonal faces, but Geobands can create closed bending-active skeletal structures with curved surfaces using the flexibility of the bands. In this sense, the mutually supporting structures constructed with Geobands have new educational and aesthetic value compared to the results of conventional polydrons.

Workshop

Geobands create bending-active skeletal structures by connecting flexible bands at specific points, forming a network of geometric edges. This workshop guides participants in constructing these structures using flexible Geoband strips. Participants will explore force distribution by deforming structures and observing their elastic responses (see Module applicability). They engage in hands-on activities to create lightweight, mutually supportive frameworks, exploring geometric forms and structural stability. The process involves basic operations, module assembly, and freeform experimentation, fostering spatial reasoning and creative problem-solving.

Participants construct a closed structure resembling a soccer ball, comprising 12 pentagonal and 20 triangular regions, leveraging principles of structural reciprocity [5]. Unlike traditional polyhedra with rigid faces, these structures derive form and stability from the elastic bending of bands and their interconnections, including connectors at vertices and midpoints, allowing exploration of flexible force interplay. Geobands are available from suppliers like On Education Company (contact: mjeong10@naver.com), with international shipping for Europe and North America. Each group requires approximately 120 Geoband strips of various lengths and 200 connector buttons, as detailed below. This hands-on approach introduces structural reciprocity, foundational to free-form reciprocal structures [5]. An instruction sheet with visual guides is included as a supplement for reproducibility.

Materials and Preparation

Required materials (per group of 4-6 participants):
- 120 Geoband strips (60 black/10.9 cm, 30 white/12 cm, 30 red/12.3 cm, optional 10 green/10.7 cm, all 1 cm wide)
- 200 connector buttons for securing band intersections
- 1 instruction sheet with visual guides (included in the supplement)
- 1 workspace large enough for collaborative construction (minimum 1m²).

Preparation checklist:

- Sort bands by color and length
- Count and verify button quantities
- Test sample connections to ensure bands and buttons function properly
- Arrange work areas with adequate space between groups
- Prepare visual aids showing progression of construction
- Set up completed model examples (if available).

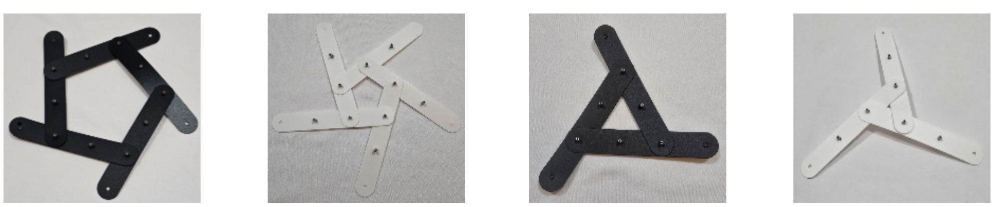
Basic operations

The key to joining bands is to overlap the band holes and insert the connector button. When cross-linking two bands, the button is inserted by aligning the end holes of one band with the middle holes of the other band. As mentioned earlier, it's important to keep the "end hole over the middle hole" orientation. While the opposite orientation (end eyelet under middle eyelet) is possible, the 'over' configuration is preferred for consistent structural integrity. This intersection pattern is crucial to the structural integrity and flexibility of the Geoband bending-active skeletal structures. When properly connected, these skeletal frameworks suggest the boundaries of faces without actually containing solid faces, allowing participants to visualize spatial relationships between vertices, edges, and implied faces. The resulting structures maintain topological closure while permitting physical deformation, combining stability with flexibility in a way that traditional rigid polyhedra cannot achieve. This unique combination of properties is what makes these structures particularly interesting from both mathematical and physical perspectives. Maintaining the correct orientation is crucial; otherwise, the bands will only overlap in one direction and the structure may warp or fail to stand stably [16]. See Figure 2. These operations encourage logical reasoning as participants deduce correct band placements through trial and error. You can also use one button to connect multiple (usually three or more) bands at the same time. For example, if you clip the ends of four bands together, you can create a bond that extends out in four directions from the center. In this way, multiple bands can meet at a single point by forming a node.

Assemble and extend modules

You can create a basic module and then expand on it to develop a whole structure. A basic module is a unit structure that will be used repeatedly in the workshop, and you will first complete this small structure and then work on extending it in the same pattern. The building process goes step by step as follows.

1) Basic module construction: Use 3–5 of the bands provided to create a closed curve-shaped module, as in Figure 2. For instance, three bands can form a triangular module, four bands a square module, or five bands a pentagonal module. Each band is connected to the two neighboring bands by a hole at each end, forming a ring with the bands staggered and supported by each other. When all the bands are connected in a ring, the structure becomes self-supporting due to mutual tension between the bands, creating a stable ring. This is the basic module. If you hold this small concave ring structure in your hand, you can observe how the balanced tension of the bands counteracts deformation, resulting in structural stability.



(a) (b) (c) (d)

Figure 2: *Preliminary structures. (a) and (b) : Pentagon modules (make 12) in two sizes. (c) and (d): Triangle modules (make 20) in two sizes.*

2) Expand the pattern of the module: Using each corner or face of the completed base module as a starting point, connect new bands. For example, add a new band along one side of a pentagonal module to extend the triangular side. See Figure 3, 4 for a close-up of triangular extensions. Create secondary modules by connecting new bands to each adjacent corner of the primary module, as in Figure 3, 4 [13].

Figure 3, 4 illustrates this progression using bands of varying lengths, showcasing the flexibility of the construction process. The new faces (e.g., triangular regions) created in this process wrap around the module, forming a dome centered on the initial pentagonal module.

(a) (b) (c)

Figure 3: *Sequential construction of a bending-active skeletal structure(Part1). Images (a-c) illustrate the progression from a single pentagon to a complete structure with 12 pentagonal and 20 triangular regions, using varying band lengths to demonstrate construction flexibility.*

(d) (e) (f)

Figure 4: *Sequential construction of a bending-active skeletal structure(Part2). Images (d-f) illustrate the progression from a single pentagon to a complete structure with 12 pentagonal and 20 triangular regions, using varying band lengths to demonstrate construction flexibility.*

3) Finalize and close the structure: As you expand the module, the structure begins flat, progresses through a half-closed, bowl-like shape, and finally transforms into a closed form. Start at the top, add bands to the sides, and finally fill in the empty space at the bottom. When all the bands are connected together to form a single closed surface, you have created a bending-active skeletal structure with curved edges that approximates a sphere (Figure 5, 6). In this final step, multiple participants work together to connect the bands in the remaining gaps, creating a sphere that is quite elastic as the structure closes completely. Participants finalize the structure by connecting the remaining modules to form a closed polyhedral shape, resembling a soccer ball with 12 pentagonal and 20 triangular regions. Additional buttons are added to strengthen the connections. Additional connector buttons are added at loose intersections, such as where multiple bands meet at a vertex, by inserting buttons through overlapping holes to secure the connection. The completed structure showcases mutual support and structural stability.

(a) (b) (c)

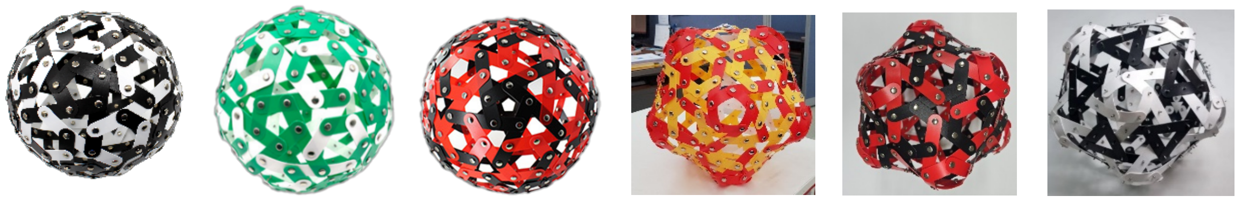
Figure 5: *Examples of completed bending-active skeletal structure(Part1): (a) Regular arrangement, (b) Dodecahedral configuration, (c) Mutually supportive structure deformed by twelve elements showing adaptive stability*

(d) (e) (f)

Figure 6: *Examples of completed bending-active skeletal structures(Part2): (d) Curved-surface hexahedral configuration demonstrating flexibility possibilities, (e) Alternative dodecahedral configuration, (f) Curved dodecahedral structure highlighting the flexibility of Geoband materials, akin to woven reciprocal systems [9].*

Module applicability

After completing the basic structure, spend some time exploring the beautiful bending-active skeletal structures that can be obtained by varying the geometry and connections of the Geobands as time allows. Most configurations, as shown in Figure 7, consistently use 120 Geoband strips in varying arrangements. Completed examples, including those shown in Figure 7, will be displayed at the workshop for participants to examine.



White/12cm/60p Black/10.9cm/60p | White/12cm/30p Green/10.7cn/30p | Red/12.3 cm/60p Black/10.9cm/60p | Red/12.3 cm/60p Yellow/12cm/60p | Red/12.3 cm/60p Black/10.9cm/60p | White/12cm/60p Black/10.9cm/60p

Figure 7: V*arious bending-active skeletal structures using different combinations of Geoband colors and lengths, demonstrating the versatility of the system.*

1) Change the shape of the modules: If you used pentagons as the base modules earlier, try using triangular or square modules to create a smaller structure. You can compare how changing the shape of the modules changes the curvature and shape of the final object. For example, you can observe that connecting only triangular modules results in a more pointed dome shape, while using square modules results in a structure that is closer to a cube.

2) Vary the band length: Try using Geobands of different lengths. You can experiment with mixing and matching two different lengths of bands to see what happens to the structure. You may find that using different lengths of bands breaks symmetry or creates new shapes (such as an ellipsoidal form rather than a sphere).

3) Deform a partial structure: You can experiment with elastic resilience by intentionally deforming (pushing or pulling) a part of the finished structure. By gently pushing or pulling the structure, participants can observe how forces redistribute across the bands, maintaining equilibrium or stabilizing new shapes. A mutually supportive structure made of bands can return to its original shape under some force or stabilize in a new shape without collapsing completely. Try deforming the structure with a small amount of force and observe whether it returns to its original shape when released or continues to take on a different shape, and discuss which parts are most vulnerable and how the force is distributed.

4) Freeform: You can also expand on this by designing a freeform structure using the leftover materials. It doesn't have to be a perfect sphere. Connect the bands to your heart's content to create an original abstract structure or piece of art (Figure 8). Notice whether you favor symmetry or asymmetry in your creation.

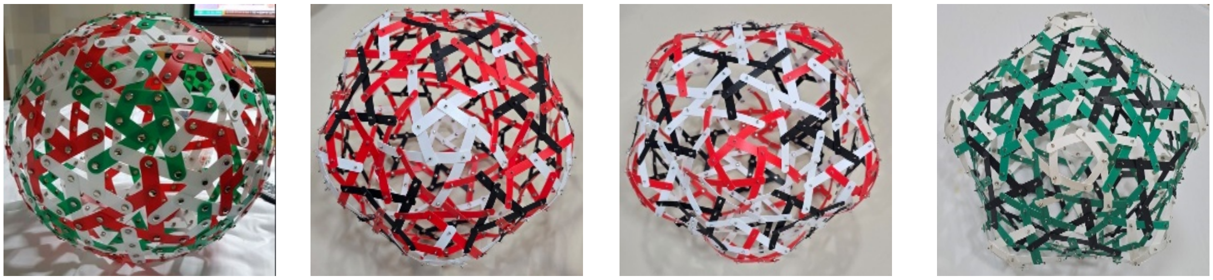


Figure 8: *Examples of freeform creations using Geobands, showing the creative possibilities beyond standard bending-active skeletal structures.*

By exploring these module applications, you can further explore the possibilities of mutually supportive structures and geobands, and use your experimental spirit to develop creative thinking about structure and form.

Summary

The Geoband Polyhedra Workshop offers participants a multifaceted learning experience that enhances spatial cognition, creative thinking, and collaborative problem-solving. By manipulating flexible Geoband strips, participants construct bending-active skeletal structures, exploring mathematical concepts such as Euler's characteristic and structural stability. Participants often encountered misconnections when linking bands, which they resolved through collaborative troubleshooting, enhancing their problem-solving skills and structural understanding. This hands-on approach fosters an intuitive understanding of geometric forms, bridging mathematics, art, and engineering. The workshop's engaging format encourages free experimentation, contributing to individual creativity and collective achievement.

Conclusion

This workshop introduces students to bending-active skeletal structures, a modern approach to structural design that emphasizes the role of flexibility and elasticity. By building these structures, participants gain hands-on experience with geometric principles, spatial reasoning, and the physics of mutual support, all while engaging in collaborative and creative problem-solving. This approach not only makes abstract concepts accessible but also connects to advanced structural design research, such as the development of free-form reciprocal structures [5, 7]. Future directions include longitudinal studies to measure long-term retention of spatial skills, development of differentiated instructions for various age groups, integration of digital modeling tools, and expansion into advanced topics like topology and non-Euclidean geometry.

This workshop represents a significant contribution to mathematics education by integrating geometric principles, spatial cognition, collaborative learning, and artistic appreciation. The unique properties of Geobands—flexibility, interconnectivity, and mutual support—create learning experiences that go beyond traditional polyhedron construction activities, enabling participants to develop intuitive understanding of complex structural relationships.

Through this workshop, participants not only gain mathematical knowledge but also experience the philosophical concept of interdependence, where the strength of the whole depends on the proper functioning of each component. This metaphor extends beyond mathematics to valuable life lessons about cooperation and community. Limitations of the current workshop design include time constraints that may prevent full exploration of all concepts for some participants and the need for careful facilitation to ensure productive collaboration. Additionally, formal assessment tools for measuring spatial cognition improvements specifically related to this activity are still under development.

As education increasingly emphasizes interdisciplinary approaches and 21st-century skills, this workshop offers a model for how mathematical learning can simultaneously develop content knowledge, spatial reasoning, collaborative skills, and creative thinking—all essential components for future success in STEM fields and beyond. The Geoband polyhedra workshop represents a significant innovation in mathematics education by transforming abstract geometric concepts into tangible, exploratory experiences that engage learners on multiple levels—cognitive, social, and aesthetic. By incorporating principles from architecture, engineering, and art into mathematical learning, this approach exemplifies how cross-disciplinary integration can enhance both educational outcomes and student engagement.

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