# Using the Golden Ratio to Construct Poems

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## Abstract

The ancient irrational number known as  $\phi$ , the *golden ratio*, has been used as a pattern in the arts since the beginning of the 20<sup>th</sup> century. Its geometrical and numerical properties lend content or structure to works of visual art, architecture, music, and poetry. This workshop explores three golden ratio poetry writing techniques, which were developed by the authors in previous papers in 2024 and 2025. Particularly, the workshop focuses on writing poems using the decimal expansion of  $\phi$ , the connection between  $\phi$  and the Fibonacci sequence, and the  $\phi$ -driven Le Modulor system in architecture.

#### Introduction

The most ancient text in which a definition of the golden ratio has been found is Euclid's (323–285 BCE) *Elements* [4], where it is called "extreme and mean ratio," and appears as the ratio of line segments:  $\frac{a+b}{a} = \frac{a}{b}$  (see Figure 1). In other • words, the total length is to the longer segment as the longer segment is to the shorter segment. This common ratio is the golden ratio,  $\phi$ .

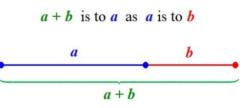


Figure 1. Definition of the golden ratio.

Manipulating the equality of ratios yields the equation:  $\phi^2 = \phi + 1$ . Solving for  $\phi$ , we obtain the value,  $\phi = \frac{1+\sqrt{5}}{2} = 1.6180339...$  In addition, multiplying this equation on either side by  $\phi^n$  we obtain that for all integer values of n,  $\phi^{n+2} = \phi^{n+1} + \phi^n$ . The golden ratio first appears in a proof in the *Elements* as a ratio of the sides of a rectangle. Such a rectangle, called a golden rectangle, is a geometric figure that will feature in this workshop.

The golden ratio was first connected to artistic expression by Leonardo da Vinci's illustrations for Luca Pacioli's book, *Divina Proportione* (1509) [16]. But it was not until the 20<sup>th</sup> century that other artists began to use patterns involving the golden ratio in their work [12]. Poets, attracted at first to the mystique surrounding the golden ratio, have also started to incorporate its mathematical properties into their craft [6].

The three golden ratio poetry writing techniques explored in this workshop were introduced in the papers: "Experimenting with the Golden Ratio in Poetry" [6] by Sarah Glaz, and "Modulor Poetry" [9] by Lisa Lajeunesse. The workshop assumes no prior knowledge of either the mathematics or the prosody involved. However, participants interested in more information on the golden ratio, more sample poems, as well as other ideas for writing golden ratio poems, are encouraged to read these papers.

Please bring writing materials, and, optionally, a device for accessing an online syllable-counter.

### Poems Whose Syllable Count Follows the Decimal Expansion of $\phi$

The first activity of this workshop involves writing poems whose syllable count per line follows the decimal expansion of the golden ratio. The decimal expansions of other irrational numbers, such as  $\pi$ , e, and  $\sqrt{2}$ , have been used to construct poems that follow the sequence as syllable count per line, or other counting patterns associated with the poem [6, 8]. The first poet to consider the decimal expansion of  $\phi$  was Radoslav Rochallyi [17]. Rochallyi used a modified decimal expansion of  $\phi$  as word or syllable count per line,

stopping at the 6th decimal place and rounding the last number from 3 to 4. In her paper [6], Sarah Glaz used the decimal expansion of  $\phi$  to 24 decimal places (without rounding) as syllable count per line for her poem "1.618033988749894842204586 Waterlilies." Since the decimal expansion of any irrational number is infinite, the poem's syllable counting pattern uses only an approximation of the number itself, but it may, of course, go to any desired length. For your information and use, below is the decimal expansion of  $\phi$  to 130 decimal places:

 $\phi = 1.6180339887$ 4989484820 4586834365 6381177203 0917980576 2862135448 6227052604 6281890244 9707207204 1893911374 8475408807 5386891752 1266338622...

Given the time constrains of the workshop, we will write poems whose syllable count follows the decimal expansion of  $\phi$  to only 7 decimal places (without rounding). This is the golden ratio equivalent of a *fib* which follows the Fibonacci sequence to 6 places [8]. We will call such a poem a *goldie*. Like the fib, part of the goldie's charm is brevity. Below are examples of goldies written by the authors of this paper.

The Golden Book	First Crocus
by Sarah Glaz	by Lisa Lajeunesse
The	first
book of questions was sent	crocus stirs a longing
to	for
me at birth – I opened it and	soft breezes and birds' wistful calls
found delights	grasses bow
and sorrows	scented green
inscribed inside in equal measure.	memories of snow cradle her bloom

Workshop activity: Participants will write goldies. After completion, participants will be invited to share their draft poems and receive feedback from workshop participants.

If time permits, we will consider longer poems with golden ratio syllable counts per line — either following the decimal expansion of  $\phi$  to more decimal places, or linking several goldies.

# Poems Structured by the Fibonacci Sequence and $\phi$

The Fibonacci sequence is the sequence made up of the following numbers, called Fibonacci numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

Starting with the number 2 in the third position, each number in this sequence is constructed by adding the previous two Fibonacci numbers. The connection between the Fibonacci sequence and the golden ratio relevant to this workshop is the following: Denote the  $n^{\text{th}}$  Fibonacci number by  $F_n$ ; then the ratios  $\frac{F_{n+1}}{F_n}$  converge to  $\phi$ , as n goes to  $\infty$ . In other words, as n increases, the ratios of consecutive Fibonacci numbers become a better and better approximation of  $\phi$ . We can use this connection to construct poems in which approximations of the golden ratio appear in more than one way. These are two-stanzas poems in which the number of lines, as well as the syllable counts, of the two stanzas are consecutive Fibonacci numbers. For such a poem, both the ratio of lines and the ratio of syllables of the two stanzas are approximations of  $\phi$ . In addition, the uniformity of line spacing, and the double line-space between the stanzas, ensure that the point that cuts the vertical line segment (which starts at the top of the poem's text and ends at its bottom) in the golden ratio, falls in the stanza break (with the longer segment in the upper part). Thus, the poem's presentation on the page visually represents a ratio that approximates  $\phi$ , where the top stanza is the duvision-line symbol).

This poetic form was introduced by Sarah Glaz in [6], which also includes her sample poem "Golden Days." "Golden Days" has two stanzas with 13 and 8 lines, and 89 and 55 syllables, respectively.

We call a poem with such a structure a *golden fibble* (rhyming with *nibble*, for the nibble from the Fibonacci sequence it uses in its structure). Given the time constraints of the workshop, we will write golden fibbles with small Fibonacci numbers for both line and syllable counts. Below are examples of golden fibbles written by the authors of this paper.

Sleight of Hands	Golden
by Sarah Glaz	by Lisa Lajeunesse
Light and shadows	Sunflowers, turmeric
complement each other on the wall,	brush strokes and butterflies
and the wall breathes joy:	autumn leaves, warblers, embers that glow
birds in flight, long-eared bunnies,	Sun's rays at golden hour
perky roosters on their morning crow.	sparkling on glistening sand
A child's world:	tall honeyed grasses bent to wind's will
make believe and magic,	my mother's summer dress
before the shadows multiply out of control.	face pressed against her skirt

Both golden fibbles have the following counts: The first stanza has 5 lines and 34 syllables, the second stanza has 3 lines and 21 syllables. The ratio of lines is 5/3 = 1.666 and the syllable ratio is 34/21 = 1.619.

Workshop activity: Participants will write golden fibbles. After completion, participants will be invited to share their draft poems and receive feedback from workshop participants.

## Poems Constructed on the Modulor Tiles Covering a Rectangle

The Swiss born French citizen Le Corbusier (1887-1965), was one of the most influential architects of the  $20^{th}$  century. In the mid 1940s, he formalized a series of measurements based on the golden ratio which he named *Le Modulor*, and introduced in detail in his book by the same title [10]. Le Corbusier's *Le Modulor* consists of two geometric sequences that extend infinitely in both directions. These sequences are called the *blue* and *red* series and use  $\phi$  as common ratio.

The *blue* series' terms are given by  $b_n = 2d\phi^n$  and the *red* series terms by  $r_n = d\phi^n$  where *n* is any integer. When placed in increasing order we have alternating blue and red terms.

 $\dots, b_{-4}, r_{-2}, b_{-3}, r_{-1}, b_{-2}, r_0, b_{-1}, r_1, b_0, r_2, b_1, r_3, b_2, r_4, b_3, r_5, b_4, r_6, \dots$ 

In architecture dimensions are influenced by human size, so Le Corbusier chose d = 1.130 metres, being the height of the navel of a 6-foot man, assuming the navel cuts the man's height in the golden ratio. In the context of poetry, fixing a value for the factor d is unnecessary and distracts from the main feature of the series, which is the recurring ratios between various terms and their additive properties. For the purpose of our discussion, we assume that d = 1, but when used to construct poems, the poet can choose any value of d that facilitates typesetting so long as the proportions are preserved.

A *modulor rectangle* is any rectangle with length and width measurements chosen from terms of the blue and/or red series. Several modulor rectangles are shown in Figure 2 with length and width dimensions labeled on the axes.

Because the golden ratio satisfies the additive property,  $\phi^{n+2} = \phi^{n+1} + \phi^n$ , each of the red and blue series also satisfies this additive property:  $b_{n+2} = b_n + b_{n+1}$  and  $r_{n+2} = r_n + r_{n+1}$ . This is illustrated visually for the red series in Figure 3.

Due to this property, it is possible to find countless tilings of a modulor rectangle using smaller modulor rectangles. The tiled rectangle may be a golden rectangle or another modulor rectangle in any one of numerous proportions.

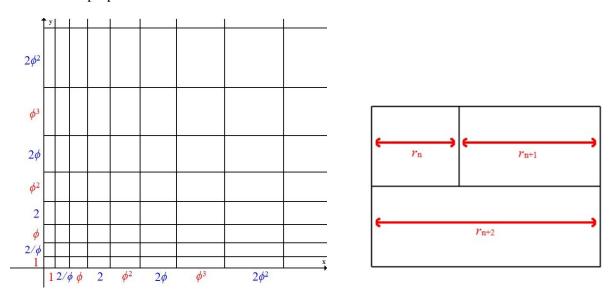


Figure 2. Modulor rectangles.

Figure 3. Additive property of the red series.

To obtain a *modulor poem* we begin with a modulor tiling of a rectangle. A separate poem is placed inside each tile. The poems are written and ordered in such a way that new poems are created when the words in more than one tile are combined to make a larger poem. To combine poems, read either from left to right across sequences of two or more adjacent rectangles, or from top to bottom across boundary lines of tiles, or as a combination of horizontal and vertical boundary line crossings, provided the boundary lines' alignments allow reading across.

This poetic form was introduced by Lisa Lajeunesse in [9], which also includes her sample poem "Come to Dust." "Come to Dust" is written on a  $2\phi^3$  by  $2\phi^2$  golden rectangle tiled with 8 modulor rectangles, and may be deconstructed into more than 22 sub-poems.

Given the time constrains of the workshop, we will write modulor poems on tilings that use fewer modulor tiles. Figures 4 and 5 on the next page are examples of modulor poems written by the authors of this paper on rectangles with 3-tile coverings.

Sarah Glaz's "Monarch Butterfly" (Figure 4) is composed on a golden rectangle with width (at top)  $2\phi^2$  and length  $2\phi^3 = 2\phi^2 + 2\phi$  (along the side). It is tiled with 3 modulor rectangles: at the bottom is a golden rectangle of size  $2\phi^2$  by  $2\phi$ ; and at the top, the  $2\phi^2$  by  $2\phi^2$  square is split into two equal size rectangles of size  $\phi^2$  by  $2\phi^2$  each.

Lisa Lajeunesse's "Parallel Universe" (Figure 5) is composed on a modulor rectangle with width (at top)  $2\phi^2$  and length  $\phi^3 = \phi^2 + \phi$  (along the side). It is tiled with 3 modulor rectangles: at the bottom is a rectangle of size  $2\phi^2$  by  $\phi$ ; and at the top, the  $\phi^2$  by  $2\phi^2$  rectangle is split into two squares of size  $\phi^2$  by  $\phi^2$  each.

Each of the two poems, "Monarch Butterfly" and "Parallel Universe," can be deconstructed into 7 subpoems: In addition to the sub-poems written on each individual tile and the entire poem read across all 3 tiles, one can read a sub-poem horizontally across the top two tiles and also two additional sub-poems read vertically from each top tile to the bottom tile.

when ready to emerge defenseless he sensed her eyes like anxious ghosts	shed the hard chrysalis in full morning sunlight hovering close a sheathing glance
blurring brightness	may dazzles
Metamorphosis!	gale winds blow strong
in all directions	turbulent clouds roll
uncontrolled	in space
a dew drop	
on newly stretched-out wings	
can affect precarious balance	
nevertheless –	
he will do it alone	

Figure 4. "Monarch Butterfly" by Sarah Glaz.

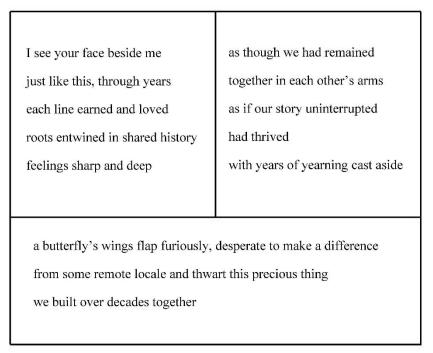


Figure 5. "Parallel Universe" by Lisa Lajeunesse.

Workshop activity: Workshop leaders will provide participants with a few sample modulor tilings to choose from for the composition of their modulor poems. After writing, participants will be invited to share their draft modulor poems and receive feedback from workshop participants.

If time permits, we will discuss tilings with a larger number of modulor rectangles and the construction of poems across complex modulor tile covers that include combinations of vertical and horizontal readings of sub-poems.

## **Concluding Remarks**

Traditionally, the main role the golden ratio played in poetry has been as a metaphor for great beauty, or for the underlying principle of aesthetic perfection. See refences [1, 2, 7, 13] for examples of this use of  $\phi$  in poetry. Using the numerical and geometric properties of the golden ratio to structure poems is a relatively new enterprise. In addition to the already mentioned papers, interested participants can find other ideas for  $\phi$  driven structure of poems in [3, 5, 11, 14, 15]. A beautifully written book about the mathematical and artistic history of the golden ratio is [12].

We hope that this workshop will inspire participants to explore the many potential uses of the golden ratio in their own poems.

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