# Hyperboloid on a Circular Knitting Machine

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## Abstract

I share a method for creating a knit hyperboloid art piece using a very basic circular knitting machine. This technique is approachable and fairly inexpensive compared to other methods for creating a knit piece of mathematical art.

## Introduction

In this paper, I describe a method for using a basic hand-cranked circular knitting machine to create a knit hyperboloid art piece. The simplicity of the technique combined with the affordability of a hand-cranked knitting machine make this a very approachable piece of knit mathematical art for anyone.

## The Knitting Machine

Spool knitting frames, which consist of pegs around a circular frame, have been used mainly by children to create small knit tubes since at least 1535 [6]. The knitting machine was invented in 1589 by William Lee [1, 2]. Variations of this knitting machine were widely used, but it was not until 1816 that Marc Brunel designed the first circular knitting machine [2].

Modern, programmable knitting machines are capable of using multiple threads to create complex color changes and multitudes of stitch patterns [5]. These flatbed knitting machines have truly impressive capabilities and have been used to create a variety of interesting mathematical art pieces in the past [5, 7, 8].

In contrast, the knitting machine used for this project is very similar to the original circular knitting machine. It is hand-cranked, has a fixed number of needles, and is capable of only one stitch. Color changes can be accomplished by manually changing the yarn. Creating a mathematical art piece from this machine, or anything other than a tube of specific diameter, is an interesting challenge. The specific machine I used is a 48-needle SENTRO knitting machine. It can be purchased very inexpensively online; ours was purchased for an elementary school science fair project. Other similar machines could also be used for this project.

## The Hyperboloid of One Sheet

The hyperboloid of one sheet is a 3 dimensional surface that is widely seen in mathematical art as in Figure 1 [4, 3]. It is a quadric doubly-ruled surface with a standard form of $(x/a)^2 + (y/b)^2 - (z/c)^2 = 1$, where the parameters $a$, $b$, and $c$ determine its shape [3]. The hyperboloid is likely popular in mathematically art partially due its simplicity of construction and in part due to the unintuitiveness of being able to construct an aesthetically pleasing curved surface using only straight lines. George Hart's 2023 Bridges paper provides an excellent overview of this surface and its use in mathematical art [3].

## Knitting a Hyperboloid

This entire project was knit as a single tube with the ruled hyperboloid strands woven in by the machine simultaneously.

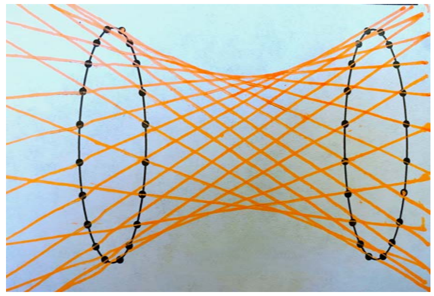


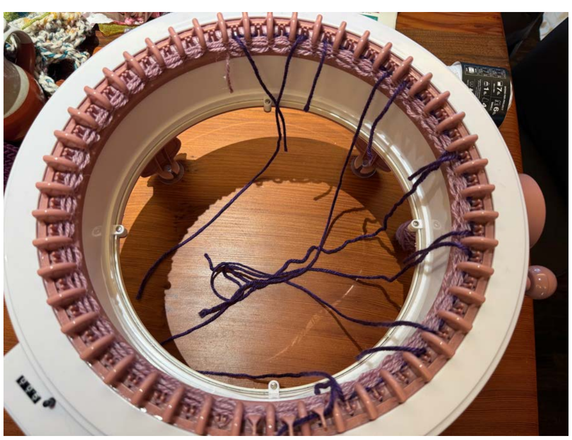
**Figure 1:** *A hand drawn hyperboloid from the author's 2023 paper [4]*

For the hyperboloid, I used two contrasting colors of medium weight yarn (worsted). The yarn needs to be reasonably stretchy. This weight of yarn will ensure optimal behavior in the knitting machine. The stretchiness is necessary for the yarn to behave well in the knitting machine and for the final project to be able to stretch over the frame and have a nice curved shape.

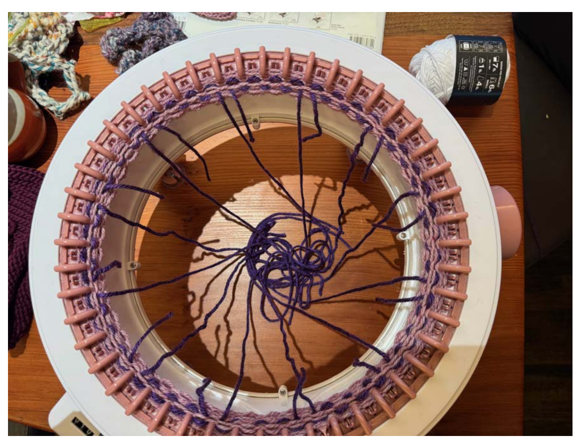
The primary yarn was used to create the exterior curved tube shape, and the contrasting color was used to create the ruled hyperboloid on the interior.

I began by cutting 12 ~60 cm lengths of the secondary yarn. Working with a 48-needle machine, this results in spacing the ruled lines 4 stitches apart. I cast the primary yarn onto the knitting machine as per the knitting machine directions. As there can be oddities at both edges of the knitting project, I opted to knit 4 rows before inserting the secondary strands.

On row 5, I inserted all of the secondary strands one at a time as in Figure 2a without ever removing the primary yarn. This means that for this row each stitch had doubled yarn. Each secondary strand was added next to the primary yarn, knit with for 4 stitches, and removed from the yarn tensioner. The loose ends of these strands were left to dangle in the middle of the knitting machine. They will be picked up one at a time later. Their insertion and removal enables us to make something that is topologically not a tube. It is important to be careful at this step to make sure that all of the strands are knit with the correct number of stitches.



**(a)** *In progress*



**(b)** *Completed*

**Figure 2:** *Row 5 of the knitting involves adding in the 12 strands that will define the hyperboloid.*

Rows 6-65 were knit normally with only the primary yarn. The precise number of rows knit can vary depending on personal aesthetic preferences. For this project, I wanted the outer tube to be wider than the inner line hyperboloid to create a contrast between the outer tube and the inner hyperboloid and make the resulting topology clearly not just a simple tube so I knit more stitches here. This also enables an interesting dynamic effect when the bottom ring of the final project is rotated. An interesting variation would be to use fewer stitches and turn the whole structure inside out so that the rulings are on the outside and the tube would hopefully take on more of the hyperboloid shape itself.

Row 66 is the most difficult row to knit. In addition to all of the difficulties encountered with originally inserting the secondary strands, it is important to be extremely careful to place all of the secondary strands so that they all go the same amount around the circle and do not tangle.

I first untangled and placed each secondary strand on top of the sewing machine on needles 4, 8, 12, etc. such that they went straight up along the knit tube as in Figure . In the knit tube, the individual stitches want to go straight up and down, so I wanted to incorporate a twist into how these strands were placed when they were knit back in. Therefore, I carefully moved each strand 20 stitches around the circle as in Figure .

I knit each strand back into the fabric for 4 stitches by re-inserting the tail into the top thread feeder and leaving enough tail for it to just touch the end of the thread tensioner. This left just a bit of tail after 4 stitches to weave in at the end of the project as in Figure .

**(a)** *Lined up* **(b)** *Shifted 20 stitches* **(c)** *Completed close up*

**Figure 3:** *Row 66 requires carefully lining up and knitting the other ends of the 12 strands into the piece.*

Rows 67-70 were knit normally with only the primary yarn. I cast off by removing the yarn from the feeder, cutting a long tail and collecting every top loop off of the machine and onto the tail.

The top and bottom of the resulting tube was stretched onto 27 cm loops of #8 copper wire. This was accomplished by feeding the wire through the loops created by the first and last rows of stitches.

At this stage, it is still possible to relatively easily adjust the lengths of the secondary color strands by pulling them lightly. Once they were adjusted as desired, I weaved the ends back in the opposite direction, this makes them much more difficult to accidentally pull out.

The finished project is displayed by hanging it from one of the wire rings. As you can see in Figure 4 the ruled hyperboloid created by the secondary color strands is distinct and separate from the outer tube forming an “eye” effect. The final art piece is interactive; twisting the bottom ring changes how the art piece looks and creates a brief oscillation effect.

## Conclusions

This project began as a personal challenge to come up with something truly interesting that could be made with a very basic knitting machine. I wanted to make something that took advantage of the machines strengths, yet also used it in an unexpected way.

This project serves as a proof of concept for a method of creating interesting patterns with a basic knitting machine by weaving in secondary colors and then skipping stitches to create thread lines within the fabric that do not necessarily follow the topology of the primary fabric sheet. Immediate extensions of this

**Figure 4:** *The completed knit hyperboloid model.*

project are to create a doubly ruled hyperboloid using the same method, and to knit fewer stitches in the middle part of the project and inverting it such that the ruled lines are on the outside and the whole art piece is a better approximation of a hyperboloid.

A similar effect could have been obtained by knitting a tube with the machine and then sewing on the secondary yarn strands by hand as a final step. I find this to be both morally and aesthetically less pleasing than having the extra strands knit in by the machine. Freed from the constraints of using a simple circular knitting machine, one could create a more perfect hyperboloid shape by hand knitting or crocheting a hyperboloid using stitch counts. The use of secondary strands as rulings could still be incorporated. As the focus of this project was trying to stretch the limits of the basic knitting machine, this was not explored thoroughly.

This project was fun and inexpensive enough that it could be done as a workshop if one was willing to purchase and transport sufficient knitting machines. While I concluded that that was impractical, I do appreciate how convenient the final knit hyperboloid is for travel. It packs up quite small and has no assembly required.

## References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