Transformable Surface Mechanism with Single Scissors Units

Seri Nishimoto[1], Maya Kraft[2] and Tomohiro Tachi[3]

[1] The University of Tokyo, Japan; s-nishimoto@g.ecc.u-tokyo.ac.jp
[2] Universität Innsbruck, Austria; maya@kraft.work
[3] The University of Tokyo, Japan; tachi@idea.c.u-tokyo.ac.jp

Abstract

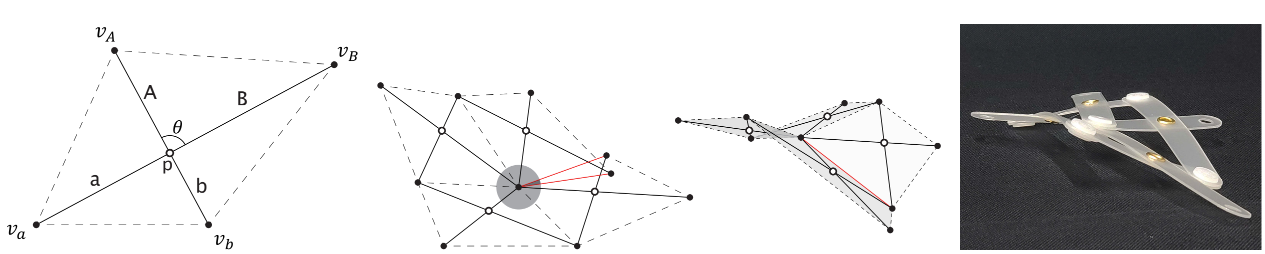
Connecting copies of a single scissors unit into a grid forms a surface which deploys nonuniformly where the units will be open at various angles. When these units are laid out in a two-dimensional pattern, the entire mechanism transforms into a three-dimensional surface. This paper presents the conditions for arranging a single type of scissors unit in a two-dimensional pattern and shows some design examples.

Introduction

Assembling scissors units in a sequence forms a one-DOF mechanism [4], but tessellating them in a plane, i.e. a scissors grid, forms an overconstrained system, so it does not generically transform in the plane except for a special case, e.g. scissors grids based on self-similar tiling of parallelograms or cyclic quadrilaterals [3]. However, when we allow the mechanism to transform out of plane using thin elastic slats, we obtain more varieties of mechanisms [2] that deform out-of-plane through in-plane metric changes. Such shape-shifting surfaces are a great source of inspiration for kinetic sculpture and transformable architecture. Several studies solve inverse problems that aim to deploy to target surfaces by assembling units with bespoke lengths [1, 5]. This paper presents a novel finding that even repeating the same scissors unit can deform out of plane and deploy into a nontrivial surface. We describe the conditions under which the mechanism can be deployed and introduce examples made using elastic sheet materials.

Geometry and Compatibility

The scissors unit consists of two thin slats connected by a pivot. Each slat is modeled as a length-preserving straight line as it is stiff in the in-plane bending or compression. The design parameters of this unit are the lengths of the four arms that extend from the pivot, denoted as a, A, b, B. The opening angle θ is a parameter that determines the deployment state of the unit (Figure 1a). Next, the scissors grid is constructed by tiling the scissors units. The pivot point is denoted as p, and the endpoints are v_a, v_A, v_b, v_B. In the vertical direction, v_a and v_A, v_b and v_B are connected. In the horizontal direction, v_a and v_b, v_A and v_B are connected. The flexibility in out-of-plane bending is modeled by the rotation of units around the endpoints.



(a) (b) (c)

Figure 1: *(a) One scissors unit, (b) Scissors connection and 3D shape, (c) Scissors with elastic slats.*

The variation in the angle θ of a scissors unit propagates through adjacent units both vertically and horizontally. However, the system becomes overconstrained when the units are assembled into a closed loop. That is, generically, the side lengths (dashed lines in Fig 1a) of adjacent scissors units fail to align as they transform, causing the mechanism to lock up. However, if the scissors surrounding the shared endpoints adhere to the compatibility described in the next paragraph, they transform synchronously. As θ varies, the sum of the sector angles around the vertices deviates from 360 degrees, causing the mechanism to shift out of plane (Figure 1b and 1c).

The compatibility of the scissors units around a closed loop is as follows. When connecting units in one direction, the opening angles of each unit propagate as two units share the same distances between the end points of the arms. This relation can be expressed by a first-order function if the cosine of the angle is used as the variable.

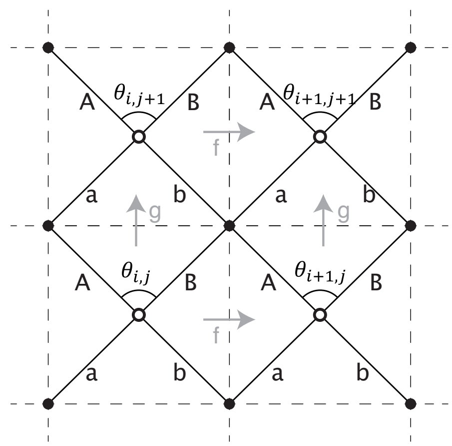


Figure 2: *Propagation of opening angle.*

First, consider the propagation function $f : \cos\theta_{i,j} \mapsto \cos\theta_{i+1,j}$ between horizontal units. This can be obtained using the cosine law on the shared distance as

$$b^2 + B^2 + 2bB\cos(\theta_{i,j}) = a^2 + A^2 + 2aA\cos(\theta_{i+1,j}).$$

Then

$$f(\cos\theta_{i,j}) = x\cos\theta_{i,j} + y \tag{1}$$

holds, where

$$x = \frac{bB}{aA}, \quad y = \frac{b^2 + B^2 - a^2 - A^2}{2aA}.$$

Similarly, the propagation function $g : \cos\theta_{i,j} \mapsto \cos\theta_{i,j+1}$ can be expressed as

$$g(\cos\theta_{i,j}) = z\cos\theta_{i,j} + w \tag{2}$$

where,

$$z = \frac{AB}{ab}, \quad w = \frac{a^2 + b^2 - A^2 - B^2}{2ab}.$$

For the loop to close, it is necessary that the following holds

$$\cos\theta_{i+1,j+1} = f(g(\cos\theta_{i,j})) = g(f(\cos\theta_{i,j})). \tag{3}$$

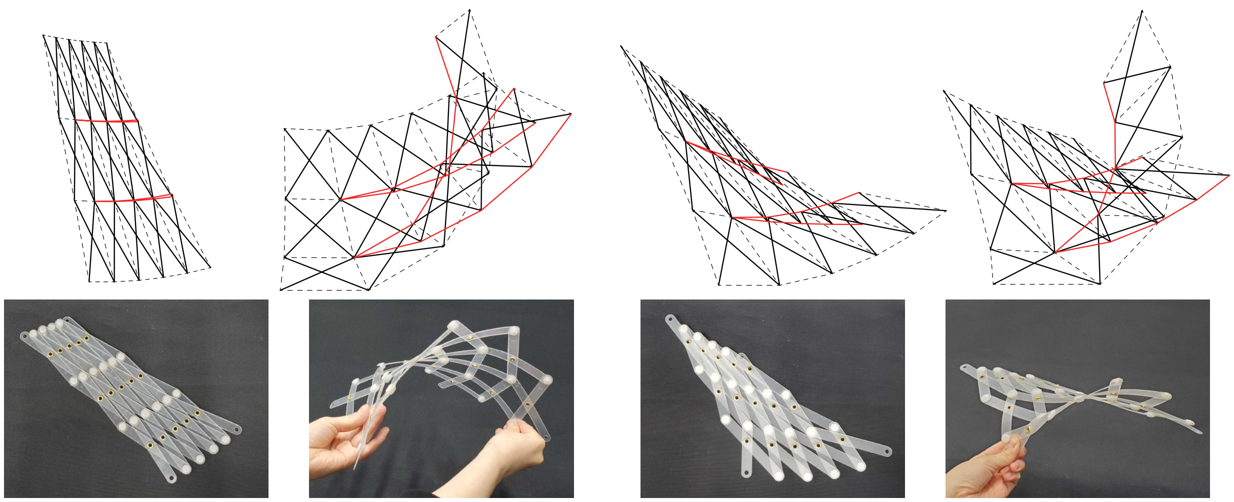
Solving this gives

$$a + A = b + B \quad \text{or} \quad a + b = A + B. \tag{4}$$

These equations are equivalent to the condition of linear collapsing in the vertical or horizontal direction, respectively.

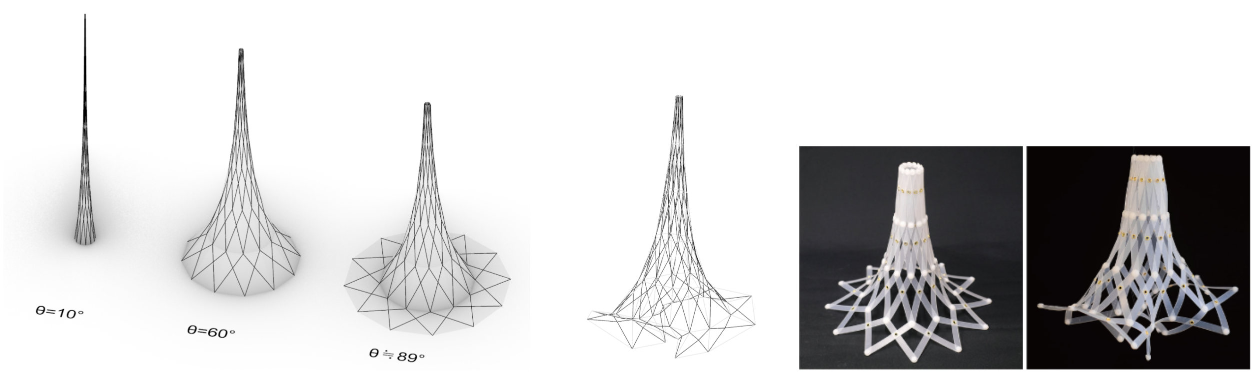
Examples

We made some models by laser-cutting polypropylene sheets and connecting them with eyelets and plastic screws. The development and three-dimensional shape are shown in Figure 3. The model cannot be collapsed into a straight line due to interference of the parts.



(a) **(b)**

Figure 3: *Development and physical models. (a) Arm length ratios: a=9, A=8, b=10, B=7. (b) Arm length ratios: a=2, A=2, b=3, B=1.*



(a) **(b)** **(c)**

Figure 4: *A Mechanism with isoceles trapezoid units ($a = b = 7$, $A = B = 4$). (a) Geometrically obtained rotationally symmetric shape where θ is the opening angle of the bottom row. (b) Asymmetric shape obtained by dynamic relaxation simulation (angle of the bottom row $\theta \approx 122°$). (c) Physical model.*

When the scissors have $a = b$ and $A = B$, the unit shape becomes an isosceles trapezoid, and the opening angle remains constant in the horizontal connection. Such scissors can be connected rotationally symmetrically, resulting in a negatively curved surface like a horn. The limit of the rotationally symmetric arrangement is when the sum of the angles between the slant sides of the trapezoids at the ends exceeds 360 degrees. Bar-based elastic simulation of the deployment shows that it is possible to expand beyond this symmetry with frills similarly to [2] (Figure 4b). The physical model is shown in Figure 4c.

Future work

As we increase the number of units, the entire structure approaches a smooth surface (Figure 5). We conjecture that there is a subdivision scheme in which the surface approaches a pseudosphere (constant Gaussian curvature), but it is unknown for a general case.

From a design perspective, this mechanism has the advantage of being able to construct curved surfaces with only a few types of parts, making it effective as a modular system for deployable surface structures.

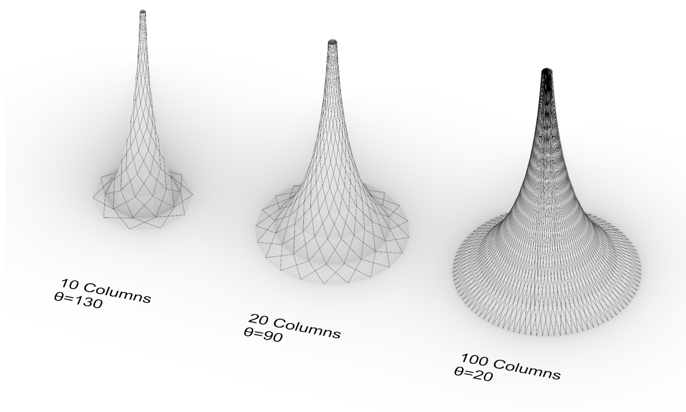


Figure 5: *Mechanisms with different number of columns by the same scissors unit (a=b=4, A=B=3).*

Acknowledgements

We thank Rupert Maleczek, Klara Mundilova, Fuki Ono, Kiumars Sharifmoghaddam, and other SOG 2025 participants for helpful comments and inspiration. This work is supported by JST FOREST JPMJFR232T, JST ASPIRE JPMJAP2401 and JSPS KAKENHI 24KJ0637.

References

[1] S. Nishimoto and T. Tachi. "Transformable Surface Mechanisms Based on Bending-active Scissors Structures." *Journal of the International Association for Shell and Spatial Structures*, vol. 65, no. 4, 2024, pp. 268–276.

[2] F. Ono, H. Kamijo, M. Kase, S. Nishimoto, K. Sempuku, M. Shigematsu, and T. Tachi. "Growth-induced transformable surfaces realized by bending-active scissors grid." *Architectural Intelligence*, vol. 3, no. 1, 2024, p. 21.

[3] H. Segerman and K. VanDeventer. "Self-Similar Quadrilateral Tilings and Deployable Scissor Grids." *Proceedings of Bridges 2022: Mathematics, Art, Music, Architecture, Culture*, 2022.

[4] N. Temmerman and L. Alegria Mira. "Development of a sustainable construction system for temporary structures." vol. 150. 07 2011. pp. 285–296.

[5] N. Toyonaga and L. Mahadevan. "Additive design of 2-dimensional scissor lattices." *arXiv preprint arXiv:2410.14124*, 2024.