# Carving Complexity: Expressing Interwoven Geometric Structures in Wood

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## Abstract

Geometric sculptures carved from wood are presented to demonstrate the application of a creative discovery process utilising rhombic polyhedra. A surface tiling is applied to the faces of the rhombic dodecahedron and rhombic triacontahedron to generate three interwoven cage structures. The tiling is composed of tiles that produce a triaxial weaving pattern. These structures have been carved from a single piece of beech timber to form three completely separated but interwoven cages. These works were produced primarily with hand tools, and represent two members of an extended family of potential structures. Further family members could employ zonohedra, but would push the woodcarver to the practical limits in the mechanical properties of their medium, a topic explored within the context of the two geometric sculptures presented.

## Introduction

Wood is an excellent medium for exploring three dimensional space. A few coffee stirrers, toothpicks or lollipop sticks can be combined into the skeletons of familiar regular polygons and then into polyhedra to begin this exploration. An alternative (and rather more daunting) approach is to begin with a single block of wood and take a subtractive approach. The result is an object that was clearly once a living entity, but has been shaped by the work of the artist or craftsman. This aesthetic choice is often made for practical implements and art objects, and can also express mathematical concepts as demonstrated here.

A mathematical framework can be used to turn abstract topological or geometric ideas into a woodworking pattern - a template that can be traced or adhered onto the surface of a piece of wood to guide cuts. Bjarne Jespersen [2] utilises the rhombic solids to demonstrate how a mathematician can systematically use rhombic tiling across the surfaces of polyhedra to generate surface projections for woodcarvings. A demonstration of this method using a triaxial weaving pattern generates two novel structures (Figure 1) each consisting of three interwoven (but separated) cages. When picked up and handled, the three cages are completely separated in both sculptures and move relative to one another. The interlocking geometry prevents them from falling apart and movement produces a pleasant rattling sound.

**Figure 1:** *Triaxialla 12 (10 cm diameter, left) and Triaxiallia 30 (10 cm diameter, right)*

A planar tiling, with a rhombic unit (or tile) outlined, is shown in Figure 2, alongside the net of the 12-sided rhombic dodecahedron (RD) with this tiling applied. In the 2D planar tiling this tile takes three rotational orientations and always meets in groups of six at the acute vertices and groups of three at the obtuse vertices. Each corner defines the intersection of three units. In the 3D case, the unit cell can have a varying number of nearest neighbours, and consequently one corner of the tile may be part of a vertex that defines three, four or five faces. Tiled onto the surface of a RD, a three-fold vertex creates a distorted version of the same pattern as in the planar tiling. The RD also has three-fold vertices, and a simpler crossing (red over green) is formed. The alternating over/under pattern is preserved and the colour coding of the tiles components creates a matching rule for arranging the tiles into extended patterns.

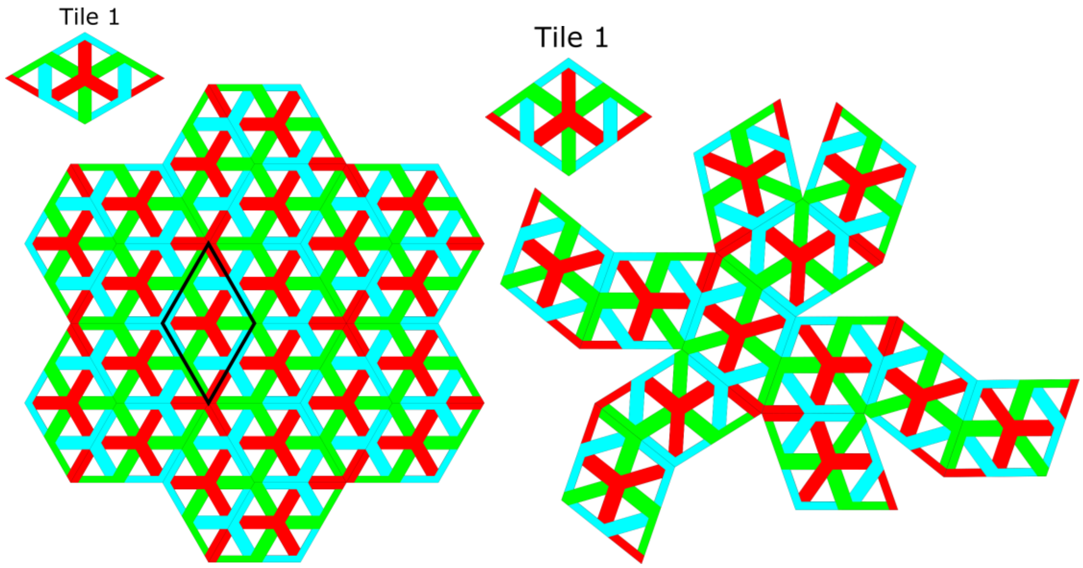


**Figure 2:** *2-dimensional tiling (left) and net for Triaxiallia 12 (right)*

To tile the plane, Tile 1 is formed of equilateral triangles and each rhombus therefore has internal angles of 60° and 120°. To form the tiles for the RD net these are distorted slightly to 70.53° and 109.47° respectively and yield a rhombus whose ratio of long axis to short axis is √2. Tile 1 is otherwise identical in both arrangements, and is the only tile necessary to cover the plane or net.

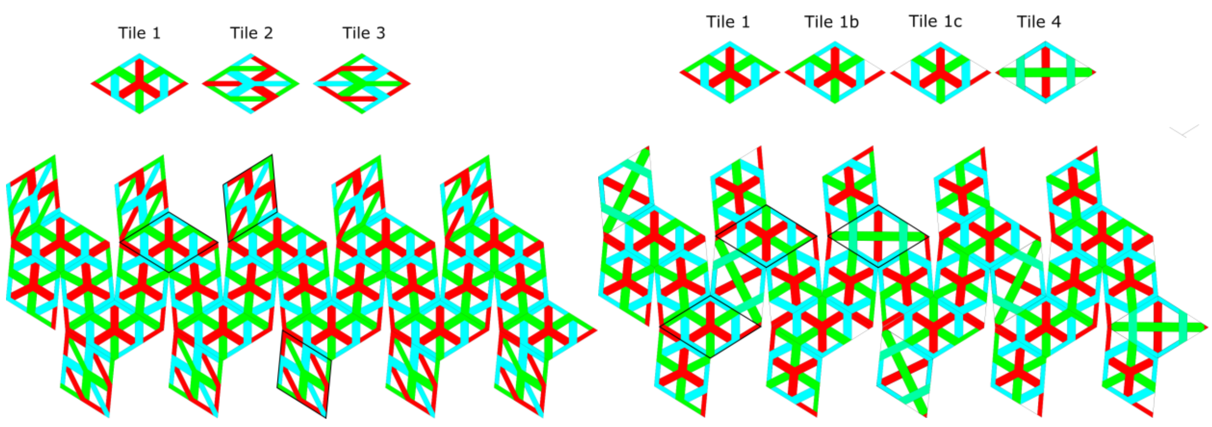


**Figure 3:** *Nets of two variants of Triaxiallia 30 with distorted tiles (left) and an alternative tile set (right)*

Figure 3 demonstrates the compromises required to generate the RT net. All tiles are ‘golden rhombi’ ie. their ratio of long axis to short axis is φ, the golden ratio. The leftmost solution uses an extra pair of tiles (Tiles 2 and 3) with weaving patterns rotated 90° around the axis of the face-diagonal length and recoloured to enforce the matching rules. These ‘distorted tiles’ give a type of crossing at the five-fold vertices that would be incredibly fragile to carve from wood but may form the basis of a future sculpture. The rightmost solution substitutes six of the 30 tiles of the RT net with Tile 4. The remaining 24 tiles also use Tiles 1b and 1c, additional, slightly altered, base tiles to simplify the five-fold vertices and provide access to tools through the larger holes in each of the six sides with Tile 4.

**Figure 4:** *Carving patterns applied to the surfaces of the RD (left) and the RT (right) respectively.*

## The Practicalities of Carving

Two practical obstacles stand in the way of the woodcarver when making these sculptures from a single piece of wood: the strength of the medium itself and access to the portions that need to be cut.

Triaxiallia 12 is physically large enough to provide sufficient entry holes in the design produced from the net in Figure 2. However, my first net for Triaxiallia 30 (using Tiles 1, 2 and 3) presented much smaller entry holes. The holes must be big enough to allow for curved chisels, gouges and other cutting tools to get behind the sections that cannot be directly accessed from the outside of the sculpture. Many axes of rotational symmetry can still be found in the resulting sculpture, but the icosahedral symmetry of the RT is lost. This is a compromise of the design that would otherwise be incredibly fragile if it could even be carved in the first place. It could possibly be overcome with a larger starting block of wood or a type of wood that is very resistant to snapping. The structure itself may be of interest to artists using other media.

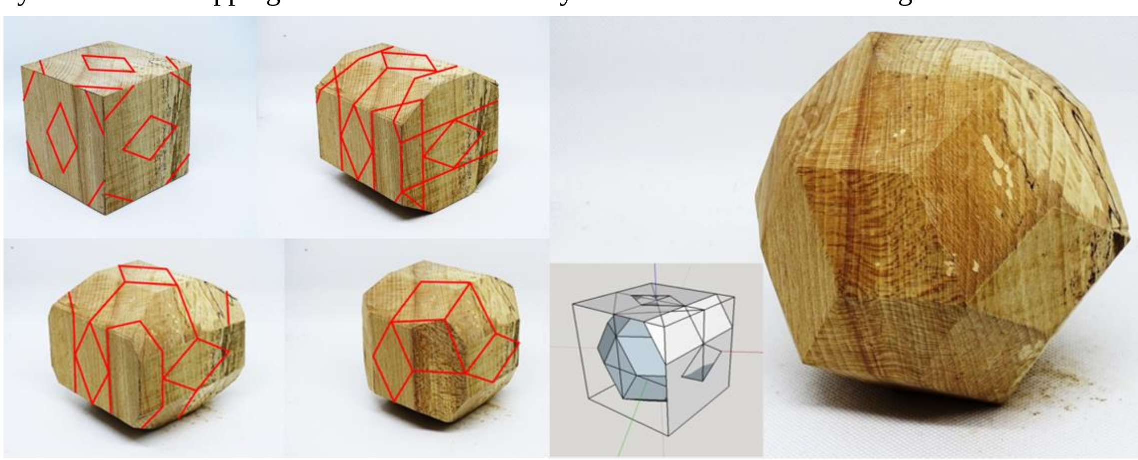

**Figure 5:** *Cutting the Rhombic Triacontahedron from a cube of spalted Beech wood*

As described in by K. J. M. MacLean [3] the rhombic triacontahedron has the unique property of containing all of the platonic solids simply by connecting the appropriate subset of vertices. Start from a cube (or cuboid) consisting entirely of right angled edges and parallel planes as most woodworking tools are optimised for these angles. Removal of wood can then begin with high precision, by reference to lines projected on the cube faces. Jespersen describes the process of cutting a RD from a cube or board, and the process can be extended to the 30-sided RT.

Starting from a cube, a RT can be inscribed by first identifying the six rhombic faces that sit in the planes of the parent cube. Further exploration of this zonohedron was presented at this conference in 2016 by Hart and Heathfield, and may help the geometric woodworker to arrive at an even more efficient method of cutting it from a single block [1]. Removing the sections of wood in the sequence shown in Figure 5 yields the starting point for applying the carving pattern net in Figure 3, demonstrated in Figure 4. This allows pilot holes to be drilled into the centre of the block.

Systematically establishing the crossings of the cages that form the final sculpture, guided by the printed pattern, allows removal of wood to continue evenly across the piece and it is advisable at this point to create shallow cuts. Try not to rush ahead on any particular section to reduce the risk of cutting too deep in one area. It can be hard to determine an appropriate depth until the whole sculpture is iterated with gradually deeper cuts. The geometry of your tools will have a role in defining this depth. Curved tools are required to reach inside the hollow form and behind the sections that need to be preserved. From starting with a single block of wood to applying the final finish to the sculpture is a project that can take several hundred hours of work, so early in the project it is worth investing a few of these hours in creating a precise polyhedron. This will help you precisely mark the surface pattern, remove material from the surface and the interior, and work gradually around the whole form to maintain its symmetry throughout.

## Summary and Conclusions

Exploring the 3D world by starting from a 2D reference provides an intuitive route to generating sculptural forms, and using shapes that can either tile the plane or form the faces of polyhedra opens up this world. Yamasaki and Sato [4] describe methods of cutting polyhedra from wood to get you started - it is one of many media that can be used to realise these forms, and presents some unique but rewarding challenges. If you are keen to experiment in this area, starting with simple forms in whatever medium you are comfortable with. This can yield amazing results, and as you progress, might spark ideas of your own!

## Acknowledgements

I would like to acknowledge the support of my parents in encouraging my early woodcarving pursuits and supporting my creative outlets ever since. To Stuart Cockram for putting me onto the works of Bjarne Jespersen by buying me his book "Woodcarving Magic" which gave me the framework to create these artworks, as well as hundreds of hours of pleasure replicating some of his own designs. Thank you Bjarne for not only creating these amazing works, but taking the time to put it into a book that makes them so accessible!

## References

[1] G. W. Hart and E Heathfield. "Rhombic Triacontahedron Puzzle." *Bridges Conference Proceedings.* Jyväskylä. Finland 2016. pp. 609-614.

[2] B. Jespersen. *Woodcarving Magic: How to Transform A Single Block of Wood Into Impossible Shapes.* 2012. Fox Chapel Publishing.

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[4] N. Yamasaki, I. Sato. *Wooden Polyhedra.* 2023. Independently Published.