The Sound of Teaching: Soundtracks for Unit Plans in Mathematics

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Abstract

The study describes a course in which mathematics education students engaged in live composition exercises, linking musical elements with mathematical concepts. Through collaborative activities and the creation of soundtracks for teaching units, students explored algebraic and geometric relationships in an auditory medium. The findings suggest that musical engagement fosters creative thinking and offers alternative perspectives on mathematical structures, supporting a multidisciplinary approach to mathematics instruction.

Introduction

Is creativity transferable across different domains? Baer [4] argues that creativity is largely domain-specific, as demonstrated by Silvia et al. [13] and Kapoor et al. [10], who found distinct patterns of creative achievement across disciplines. However, Baer also acknowledges that some general cognitive skills, such as divergent thinking and problem-solving, may apply across domains [5]. The Honing Theory of Creativity [8] suggests that creative thinking involves restructuring one's worldview through cross-domain influences, a concept supported by Scotney et al. [12], who found that inspiration from multiple fields is more common than within-domain influences. Similarly, Harrison [9] examined creativity transfer between jazz improvisation and field hockey, concluding that while specific skills may not transfer, broader cognitive abilities like pattern recognition and adaptability can. Root-Bernstein and Root-Bernstein [11] reinforce this perspective, showing that many highly creative scientists and artists engage in cross-disciplinary thinking.

An interesting domain to explore is how creativity transfers from music to mathematics, as the connection between the subjects has long been recognized, dating back to ancient Greece [2]. Research suggests that engaging in musical activities fosters cognitive skills that are highly relevant to mathematical thinking, such as recognizing patterns, manipulating structures, and understanding proportions [2][7]. Integrating math and music may enhance students' creative thinking by improving fluency, flexibility, and originality in problem-solving [1]. However, other researchers argue that music interventions in school mathematics have only a small to moderate effect on achievement [3].

This study takes a first step in exploring the transfer of creativity from music to mathematics among education students. We examine how future mathematics teachers develop creative thinking through musical engagement and how mathematical concepts and ideas are expressed in musical life composition exercises.

Applying Live Composition to Mathematics Unit Plans

We developed a course titled *"Music and Mathematics*," which centered on live composition to cultivate creativity. The course was attended by 32 students studying mathematics education at a teacher's college, most of whom had no substantial musical background. The primary instructor was a lecturer from the music department – a pianist and composer specializing in fostering creativity through music - supported by a lecturer from the mathematics department.

The guiding principle of the course was that creativity emerges from action rather than from knowledge alone. Through live composition exercises, we established a connection between the intrinsic, universal source of creativity and mathematics. Initially, students engaged in activities emphasizing the "here and now" by moving around the room and using rhythmic tapping to express themselves. Then they were instructed to listen to their peers and translate their auditory experiences into a musical representation of the moment, transforming the group's ambient noise into structured music.

In subsequent exercises, students worked in pairs, engaging in percussive dialogues that symbolized the teaching process. Using accessible tools that required no prior musical expertise, they created musical expressions representing the dynamics. This phase of creative exploration was ultimately linked to mathematical concepts in the course's culminating assignment.

For their final project, students were required to compose and record five musical pieces to serve as a *soundtrack* for a mathematical teaching unit. The musical pieces should be designed to fulfill at least one of the following pedagogical functions: illustrating a mathematical concept or idea from the teaching unit (e.g., depicting the intersection points of functions), facilitating pedagogical explanations (e.g., addressing common student misconceptions), or supporting classroom management (e.g., signaling transitions in group activities). Each composition was accompanied by an explanation detailing its educational significance, its connection to the mathematical unit, and the collaborative creative process behind its development.

We now present a description and analysis of four examples from the students' work, focusing on musical pieces that represent mathematical concepts and ideas.

Examples from Algebra

Equality Between the Sides of an Equation

Students explored the concept of equality in equations by performing the same notes on two different instruments—first on the piano and then on string instruments. "*We experimented with various approaches to identify the most suitable sounds to represent the concept of equations, which is typically explained in class using the metaphor of balanced scales. The process involved discussions, recordings, and iterative refinements until we achieved a final composition that integrated musical harmony with clear pedagogical intent.*"

From the students' perspective, each instrumental group represents one side of the equation, with the piano on one side and string instruments on the other. The primary unifying element between these sides is melody, while the differences in algebraic expressions correspond to the sound. This approach contrasts with conventional musical perception, where the hierarchical order of musical elements establishes distinctions between sounds—one initiates, while the other responds. For musicians, expressing the concept of equality may seem more intuitive when all instruments play simultaneously.

By applying their mathematical perspective to music, the students offer a non-traditional yet valid musical interpretation. Yet an interesting contrast between the two approaches emerges when considering errors in the process of solving an equation, where the two sides are not equal. In the case of simultaneous playing, a mistake is immediately perceived through disharmony. Conversely, when applying a sequential approach, the recognition of errors relies on memory and the ability to compare previously heard sounds.

Shifting a Parabola

In a unit that teaches parabola transformations, GeoGebra is integrated to link algebraic and graphical representations while promoting inquiry-based learning and mathematical discussions. The students created a GeoGebra simulation of parabolic shifts, starting with vertical movement ($y = x^2 + c$), followed by horizontal shifts ($y = (x - p)^2$) ,and concluding with a combined transformation ($y = (x - p)^2 + c$). They presented the simulation to the performing composer, who responded in real time to the visual stimulus.

From a musical perspective, the composition represented a free interpretation rather than a structured, one-to-one translation of the mathematical transformations. Musically, the composition consists of two sections rather than three, as the composer perceives vertical and horizontal shifts as manifestations of the same phenomenon. Once movement occurs—whether vertically or horizontally—his response remains the same, possibly due to the piano's single-axis structure. The movement along both directions was expressed through ascending and descending gestures on the keyboard. Musicians may notice that the pitch rises when the function shifts downward, and vice versa. Similarly, in horizontal shifts, the pitch lowers as the function moves to the right and increases when it moves to the left. Notably, there is no explicit distinction between the x-axis and y-axis in the musical representation.

When transitioning to the combined, or floating transformation, the composer shifts to an Impressionist musical style. Instead of direct pitch ascents and descents, the tones become more fluid. The right hand remains fixed, symbolizing the coordinate axes, while the left hand moves freely across the upper and lower registers, as if painting the movement of the parabola as it floats between the axes.

Examples from Geometry

Congruence of Triangles

Identical chords are used to illustrate the concept of triangle congruence, drawing a parallel between musical structures and geometric principles. For example, each note represents an element in the Angle-Side-Angle (ASA) congruence theorem. The same chord is played in different octaves to demonstrate the existence of congruent triangles, where an octave shift signifies that the same chord is played at a higher pitch. The objective is to convey the principle of congruence by deconstructing and reconstructing chords, allowing students to develop an intuitive understanding of the concept.

The melody was played exclusively using a C major chord in its root position (C–E–G) and in its harmonic fundamental form (with all three keys pressed simultaneously). Later, triadic inversions were introduced, where the notes appeared in a different order, which may conceptually parallel different orientations of a triangle. Each chord was played twice - once in a higher register and once in a lower register (an octave apart) - to represent the different triangles.

Rectangular Prism - Surface Area and Volume

Students recorded live music to represent the distinction between the surface area and volume of a rectangular prism. "At first, we considered how we teach these concepts in our classrooms and began with the abstract concepts and definitions, focusing on the distinction between surface area as the outer layer or envelope of a rectangular prism and volume as the inner space the object occupies. When creating the musical piece, we wanted to illustrate these concepts in a tangible way, effectively becoming a human prism. We started with surface area, using distant sounds to represent the outer shell of the box, and then, as we moved inward, the sounds grew stronger and deeper, symbolizing the volume of the prism with rich and resonant tones. This piece was particularly meaningful to us because we witnessed how effectively it enhanced the teaching of this topic. Students were able to grasp the fundamental difference between surface area and volume through this multisensory experience."

What makes this piece unique is that the shaping of sound is achieved through spatial movement using percussion instruments. The students stand at the four corners and gradually move toward what is perceived as the centre of the body, which is reflected in the intensity of the sounds. The instruments enter gradually, musically representing the outline of the surface area, while the students' movement towards each other creates a sense of volume. This serves as a literal translation of the concept of sonification—the representation of a physical phenomenon through music [6].

Summary and Conclusion

This study explores the integration of music into mathematics education through live composition exercises, aiming to foster creativity among future mathematics teachers The analysis of student compositions reveals innovative representations of algebraic and geometric ideas, highlighting the potential of music-based learning to enhance conceptual understanding.

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