Fabric Origami Based on an 8-Fold Rotational Symmetric Tessellation with Irregular Polygons and Multi-Valent Vertices

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Abstract

This article discusses the process of creating of a piece of fabric origami art that is based a design that exhibits 8-fold rotational symmetry. It focuses on how to create a sewing diagram for fabric origami. The tessellation demonstrated here consists of irregular polygons and vertices that have valencies of more than three.

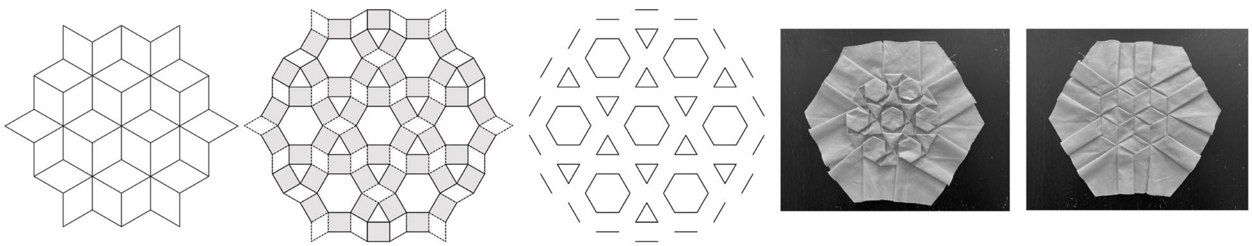
Introduction

Fabric origami is the art of using the fabric manipulation and origami techniques to alter the texture of the original fabric to create a new fabric that is endowed with novel visual and tactile characteristics. The art is pioneered by Palmer and Rutzky [1]. Previously, I explored a tessellation grafting technique to generate sewing patterns consisting of polygons for fabric origami [2], to generate crease patterns from sewing patterns [3], as well as showcasing a variety of examples of fabric origami art in various scales [4]. To make fabric origami, the corners of the polygons are sewn together, and then the gathered fabric is pleated and flattened. In many of the examples demonstrated previously, either the corner of irregular triangles or other polygons with rotational symmetry were sewn together. In this article I will show a new example of fabric origami tessellation in which corners of irregular polygons can be sewn together. The overall design exhibits 8-fold rotational symmetry.

Background Information on the Fabric Origami Tessellation Grafting Method

The key to an interesting fabric origami design is to create sewing patterns. Previously [2,3,4] I discussed a technique I devised to generate patterns for fabric origami tessellations. Here, a tessellation refers to a finite edge-to-edge tiling by either regular or irregular polygons. Edge-to-edge means that each edge of a polygon tile meets exactly one edge of another so that each vertex of the tiling must also be a vertex of each of the polygon tiles that meet there. To graft such a tessellation, one starts by figuratively cutting apart adjacent polygons that share an edge and expanding the space between them to allow either rectangles or parallelograms to join the previously adjacent polygons. In addition, new polygons are inserted around the vertices of the original tessellation. If a vertex in the original tessellation has valence of three, the polygon connecting the new separated vertices must be a triangle. In general, for vertices that are n-valent in the original tessellation, the inserted polygons much be n-gons in the new grafted tessellations [2].

The new grafted tessellations can be then used to produce a sewing diagram for fabric origami. Figure 1(a) shows the original rhombille tessellation. Figure 1(b) shows its grafted tessellation, the polygons of the original tessellation here are drawn in dashed lines, and solid lines are new lines drawn in the grafted tessellation (this applies to all figures in this article). The polygons shaded gray are the rectangles (or parallelograms) inserted during the tessellation grafting process. The sewing diagram in Figure 1(c) is the result of eliminating the original rhombi and the inserted rectangles. To make the fabric origami from the sewing pattern, the corners of each of the triangles, and the corner of each hexagon, are sewn together respectively on the back side of the fabric. On the front side, the fabric is carefully pleated and flattened to create a new tessellation (Figure 1(d)) and on the back side, the sewn fabric's seams are now reflecting the original rhombille tilling (Figure 1(e)).



(a) (b) (c) (d) (e)

Figure 1: *A simple fabric origami example.*

Previously, I have concluded that for grafted tessellations of this kind to generate sewing patterns that can be sewn and pleated flat, the inserted polygons between the edges of original adjacent polygons need to be either rectangles or parallelograms that are made of two congruent isosceles triangles [4]. To create the grafted tessellation as in Figure 1(b), the rhombi in the original tessellation are scaled at the geometric center by an arbitrary scale factor. The edges between the rhombi form rectangles that are shaded in gray. While this method works for tessellations that are made of polygons with rotational symmetry, such as the rhombi in Figure 1, and the Voronoi tessellations in which each vertex has valency of three [2], it doesn't work for other tessellations in which there are vertices of valency greater than three and the polygons in the tessellation are irregular. In the following, I will show such a fabric origami example that has 8-fold rotational symmetry.

A Fabric Origami Example Based on 8-Fold Rotational Symmetry

Figure 2(a) shows an example of tessellation with 8-fold rotational symmetry. It is worth noting that in this tessellation, there are both regular and irregular polygons and the vertices have valency three, four, five, six and eight. To graft this tessellation, one can attempt to scale the polygons in the tessellation from their respective geometric centers to create spaces between the polygons as shown in Figure 2(b). However, this method doesn't result in a sewing pattern that can be used for fabric origami in this example. Notice that the gray quads that are drawn between the polygons in Figure 2(c) are not necessarily rectangles or parallelograms that are made of two isosceles triangles, a required condition for fabric origami to be pleated flat.

Previously, I also discussed the primary and dual relationship in tessellation grafting [2]. A given tessellation that is called *primary* has vertices, edges, and tiles, and these can be understood as vertices, edges, and faces in a particular planar embedding of a graph. We denote the number of the vertices by V, its number of edges by E, and its number of faces by F. The unbounded external region is not considered to be a face in this particular planar embedding. An edge that belongs to only one face is a border edge, and we denote the number of border edges as E_b. The *dual* tessellation can be understood as an interior dual geometric configuration that consists of V' vertices, E' edges, and F' faces and has a vertex for every face of the primary tessellation, and edge for every non-border edge of the primary tessellation, and a face for every interior vertex of the primary tessellation. In addition, the edges of the dual are perpendicular or parallel to the corresponding edges of the primary tessellation. To graft a tessellation, the vertices of polygons in the primary tessellation can be spaced apart by inserting the corresponding polygons in the dual tessellation. Whether or not a primary tessellation has a dual tessellation depends on this difference: $2F-(E-E_b+3)$. If the difference is 0, there is exactly one solution for the dual graph, which means that one can find exactly one dual tessellation of the primary tessellation. If it is greater than zero, there is more than one solution, and if less than zero, no solution is possible unless for very special condition.

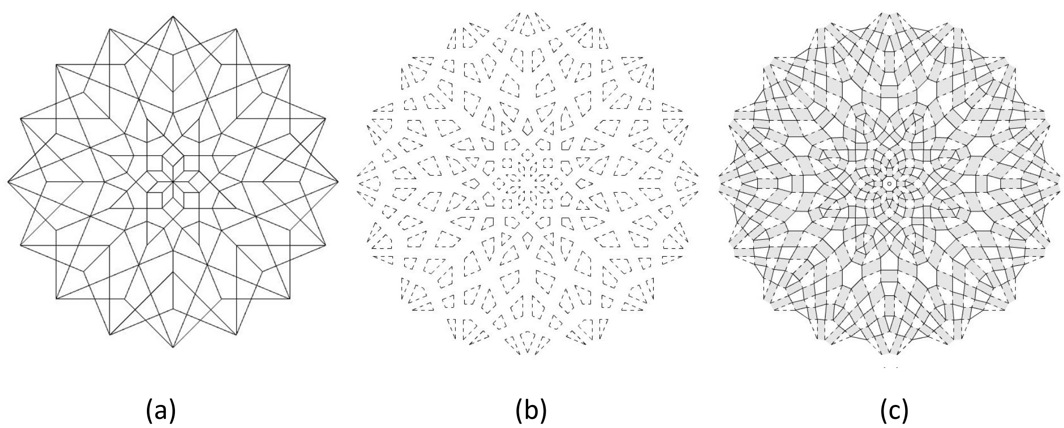


Figure 2: *A failed attempt at grafting the 8-fold rotational symmetric tessellation.*

The 8-fold rotational symmetric tessellation in Figure 2 has $F = 168$, $E = 336$, $E_b = 32$, and therefore, $2F-(E-E_b+3) = 29$, resulting in more than one solution for the dual tessellation. To produce one of the dual tessellations of the 8-fold rotational symmetric primary tessellation, I started out by drawing edges that are perpendicular to the eight rhombi at the center of the tessellation and then continued outwards. The positions and the lengths of the edges were determined by the 8-fold rotational symmetry. Figure 3(a) shows part of the dual tessellation in red that is drawn in relation to the primary tessellation. Figure 3(b) shows the completed dual tessellation (in solid lines).

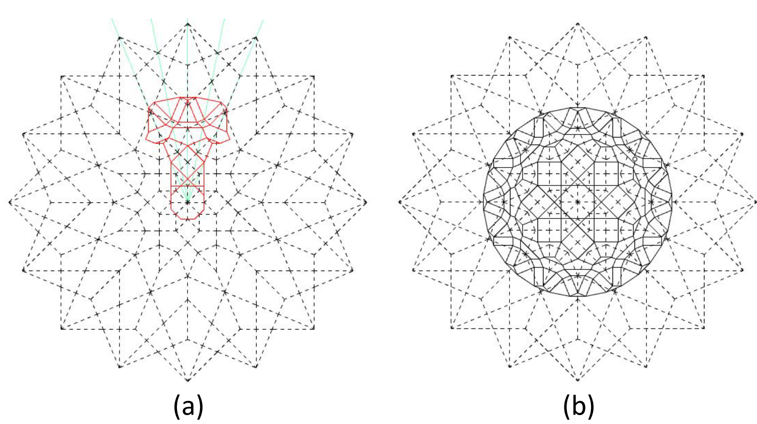
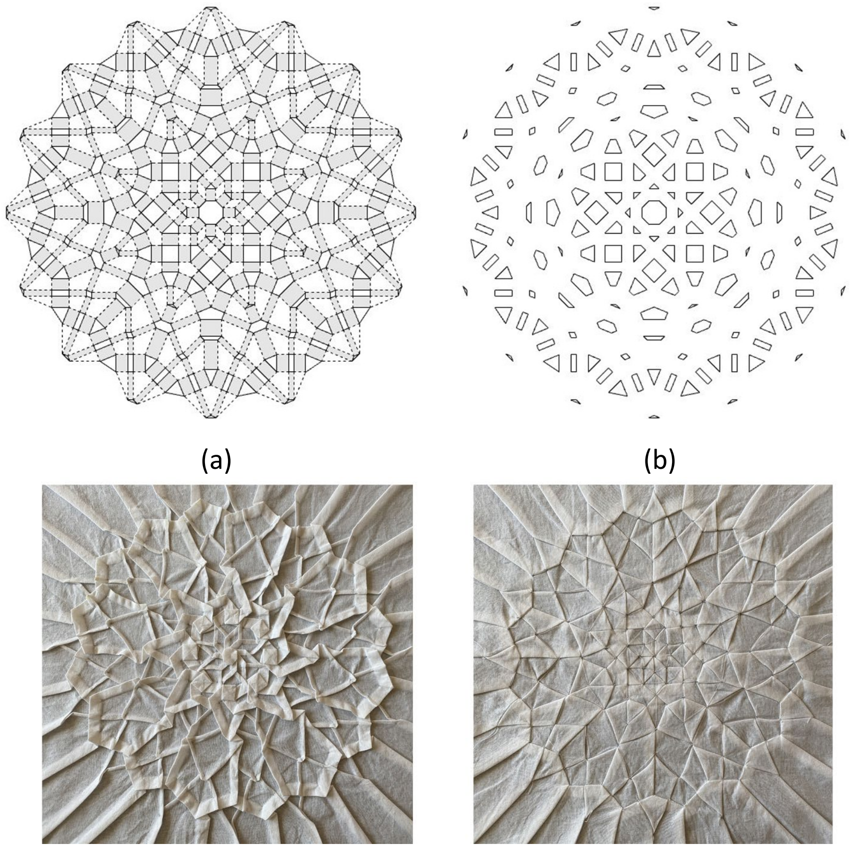


Figure 3: *A primary and dual relationship of the 8-fold rotational symmetric tessellation.*

Figure 4(a) shows an example of a successfully grafted tessellation using the dual tessellation from Figure 3(a), and Figure 4(b) shows the sewing pattern by eliminating the dashed lines and the shaded rectangle. And finally, Figures 4(c) and 4(d) show the front and back of the finished fabric origami.



(c) (d)

Figure 4: *An example of fabric origami with 8-fold symmetry.*

Conclusions

This article discussed an example of fabric origami tessellation with irregular polygons and vertices that have valencies of more than three. There are many more tessellations that can be explored for fabric origami. Furthermore, the crease patterns that can be generated from the fabric origami could also be studied.

References

[1] J. Rutzky & C. K. Palmer (2011). *Shadowfolds: surprisingly easy-to-make geometric designs in fabric*. Kodansha International, 2011.

[2] J. Wu. “Grafting Tessellations for Fabric Origami.” *Proceedings of Bridges Aalto 2022: Mathematics, Art, Music, Architecture, and Culture,* Aalto University, Espoo, Finland, 2022, pp. 375-378.

[3] J. Wu. “Grafting Tessellations for Fabric Origami.” *Proceedings of Bridges Richmond 2024: Mathematics, Art, Music, Architecture, and Culture,* Virginia Commonwealth University, Richmond, Virginia, 2024, pp. 423-426.

[4] J. Wu. “The artistry and algorithms in fabric origami tessellations.” *Journal of Mathematics and the Arts,* 18:3-4, 2024, pp. 201-221.