A Versatile Octagonal Beam and V-Piece to Create Sculptures

Friedhelm Kürpig[1] and Tom Verhoeff[2]

[1]Professor of Descriptive Geometry, University of Fine Arts Hamburg (retired),
Aachen, Germany; kürpig.de, kuerpig@t-online.de
[2]Dept. Math. and CS, Eindhoven University of Technology, Netherlands; T.Verhoeff@tue.nl

Abstract

We describe how the accidental discovery of an interesting V-piece for octagonal beams led to various artworks.

History, Discovery, and Results

The sculpture shown in Figure 1 (left) is composed of 48 cuts of an irregular eight-sided prism. It has eight-fold symmetry and represents the current milestone of an exciting development since it began over ten years ago. This development will be presented in more detail here, from the perspective of the first author.

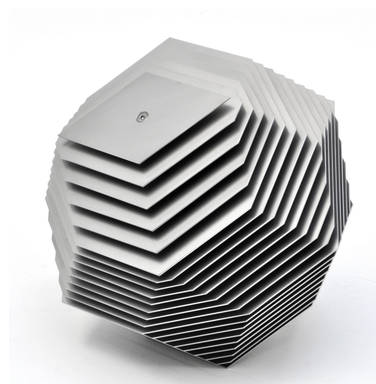


Figure 1: Rotunda*, from 48 octagonal pieces (left; maple wood, 37.5 cm diameter).*
Rhombic triacontahedron (right; stainless steel, 20 cm diameter).

My artistic activity over more than 60 years is very closely linked to different stages of my life. So I dealt with furniture construction before and after my joinery apprenticeship. During and some time after my studies in architecture, I focused on the presentation of architectural models. Parallel to my work at the University of Fine Arts in Hamburg, I developed didactic models for teaching geometry to promote the spatial imagination. In the early years of this century, the objects were conceived by support of CAD, and were realized by means of laser technology. Parallel to this development, not only the material but also the possibilities of progressing changed. In the course of time, a small joinery shop became a precision mechanical workshop in which, in addition to wood and plastic, metals and stainless steel could also be processed. Due to the different materiality, the aesthetic of my objects also changed, which has become increasingly important for me.

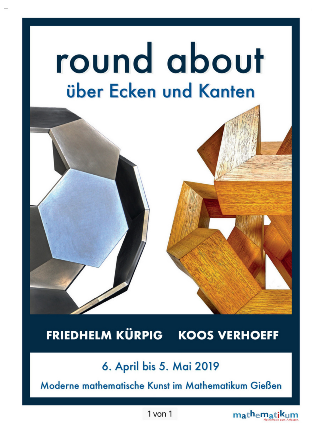
In 2014, it was time for a change of subject in my artistic work. For four years, I had dealt with polyhedra that were sliced at right angles to a symmetry axis and recomposed with gaps. As any regular or semi-regular polyhedron has normally three different symmetry axes, it can be presented in three different ways depending upon the radial symmetry of the axis concerned (Figure 1, right). These polyhedral objects were part of a group exhibition at the Mathematikum in Giessen (D), in 2014, and had thus found their end (also see [1]).

In search of another way of representing polyhedra, I came across the Hamilton cycles. These are cycles that run through all polyhedral vertices once and then return to the starting point. A friend and colleague from the Netherlands gave me the advice to contact Koos Verhoeff, who had worked extensively in this field. After exchanging several emails, I visited Koos Verhoeff in his studio in January 2015.

The reception was warm-hearted and the impression of standing in front of Koos' life's work was simply overwhelming and inspiring at the same time. Koos and I agreed on a return visit which, however, could only take place in 2017 due to adverse circumstances. In the meantime, I dealt with cycles on polyhedra and found various ways to present them. As Koos worked almost exclusively with wooden beams, I chose pipes and sheets made of aluminium to realize my ideas.

In November 2017, I was finally able to welcome Koos and his son Tom into my studio. There was an intense exchange of ideas which was the reason to make the proposal to Professor Beutelspacher of a double exhibition in the Mathematikum. The topic of this exhibition should be the cycles on polyhedra because the comparison of different constructions and materials would result in a wide-ranging whole. Also the name for the exhibition was quickly found because the term "round about" was the characteristics of all exhibited sculptures (Figure 2, left).

Tom made the additional proposal for this exhibition to present a joint work that on the one hand should express the different materials and techniques and on the other hand form a unity in the sense of geometry. The proposal was approved by everyone and the search for a suitable object could begin. After a further exchange of ideas by emails, the joint work gradually began to take shape. It should have two self-contained cycles that would be interconnected, as a Whitehead link (topologically inseparable but with linking number zero), into a single unified sculpture, whereby each cycle should differ in its materials and construction methods. It therefore seemed appropriate that Koos would use wood and I stainless steel. The result was a decagonal cycle of stainless steel, which was penetrated by an octagonal cycle cut of solid wenge-wood and glued, while the decagonal cycle in skeleton construction consisted of ten single prisms held together by permanent magnets. In this way, the two cycles could be assembled into a unified sculpture.



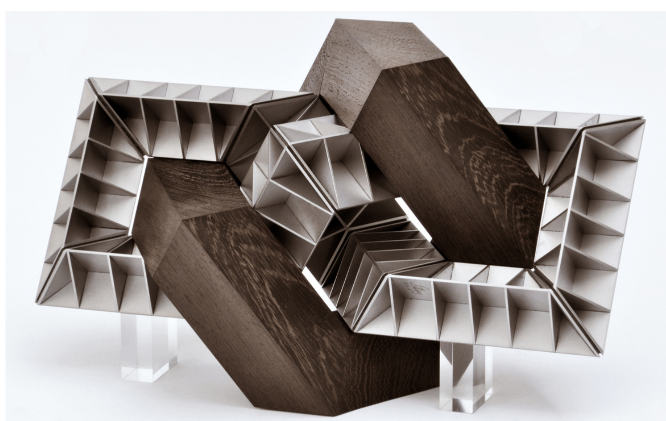


Figure 2: *Cover of catalog for* Round About *exhibition at the Mathematikum in Giessen, Germany (left). Joint work at this exhibition (right; stainless steel and wenge wood, $h \times w = 35 \times 21$ cm).*

The *Round About* exhibition in 2019 was a great success, but just a few months later a time began that was characterized by bad events and disasters and that left little room for creative work. First came the pandemic and then in July 2021 a flood that put both my house and my studio under water. The work to eliminate the flood damage and the recommissioning of the studio lasted almost until the end of 2023. Only then could the work on new geometric objects be resumed and it started again where it was interrupted three years ago.

The work on the "joint work" was already the starting point of considerations that not only wooden beams with rectangular or rhombic cross-sections are suitable for the constructions of cycles. The sides for the rectangular ones or the diagonals for the rhombic ones must be in the ratio of $1 : \sqrt{2}$. This is the only way to ensure that cutting the beam at an angle of 45° to its axis results in square cut faces [2]. These squares are the contact surfaces of the neighboring parts of the cycles. As I have a close relationship with Descriptive Geometry, I realized that there are an infinite number of rectangular prisms that have a square as sectional figure cut at a defined angle. In comparison to the rectangular respectively rhombic prisms, however, only the prisms can be considered whose edges also have an angle of 45° to the horizontal contact surface.

I first decided to investigate the possibilities of eight-sided prisms. In order to have equal proportions as with the rectangular and rhombic prisms, I chose a regular octagon as the horizontal contact surface that was inscribed in the square of the rectangular or rhombic prisms. Then the prism edges were determined at an angle of 45° to the contact surface. The intersection at right angles to the edges then results in the cross-section of the eight-sided prism (Figure 3, left). According to this cross-section, the wooden profile must be produced exactly, which is, even for an experienced joiner, a great challenge. That is why I constructed a device, which put the wood profile in the right position when planing with the machine.

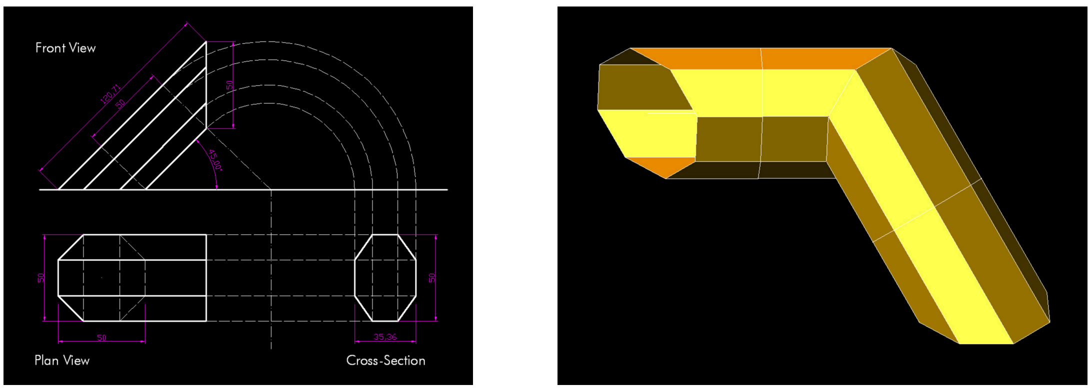


Figure 3: *Irregular octagonal cross section with regular octagonal contact surface (left). Two octagonal beams connected at regular octagonal contact surface (right).*

The eight-sided beams (profiles) could now be produced in sufficient numbers and only had to be brought to the correct length by miter cuts. The cut faces were regular octagons, which served as contact surfaces of the individual prism parts with each other. These could each be twisted by 45° against each other and there was always an exact transition from one octagon to the other, as it was already the case with the squares (Figure 3, right).

AutoCad supported me in the search for new cyclical structures. The "align" command makes it possible to assemble 3D objects in space. To do this, three start and three destination (target) points must be determined one after the other in order to connect the objects. During such an operation, an indefinable something suddenly arose on the screen, which suggested an incorrect operation (Figure 4, left).

A screenshot of this creation allowed a later investigation, which then brought to light amazing properties. It had three regular octagons, with two octagons having a common edge through which a mirror plane ran. This mirror plane halved the opposite octagon and was at the same time the intersection plane of the two eight-sided prism parts. The whole thing resembled a Siamese twin, and had the function of a three-way connector. This V-shaped piece could be obtained from two simple sections of the eight-sided prisms by milling them to the mirror plane. It was only necessary to determine two angles by means of Descriptive Geometry, first the angle of inclination of the prism axis to the mirror plane, then the torsion angle of the mirror plane around the prism axis. However, these two angles had to be adjusted at the same time. This could only be done in a three-axis vise, so that the intersection plane of the two prism parts could be milled

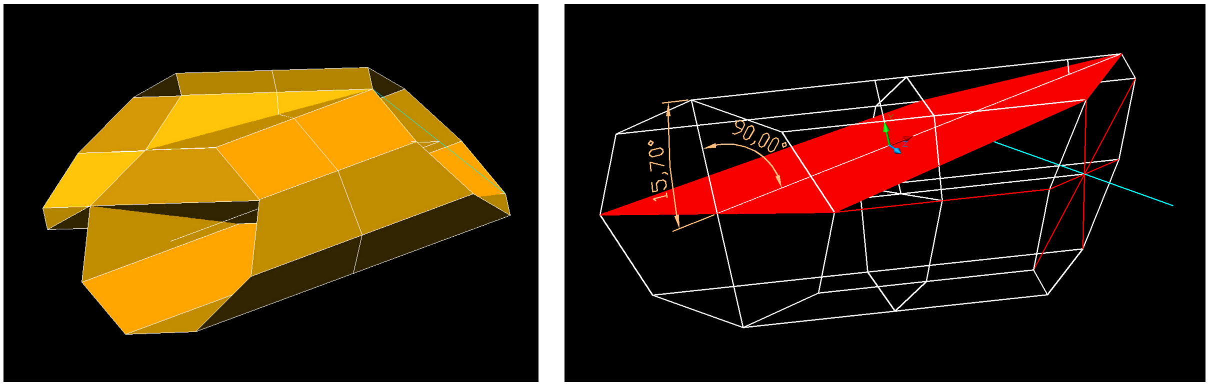


Figure 4: *Unintended alignment of two octagonal beams led to the V-piece (left). Half a V-piece is obtained by milling off the top of an octagonal beam (right).*

in a single process. After that, the two parts only had to be glued together, which was done in a mould made for this purpose to prevent the parts from slipping.

Now a fixed but detachable connection of the individual components had to be found, both with each other and with the V-pieces. Such a connection could only be established by permanent magnets together with small steel cylinders. Therefore, all octagons were provided with concentric holes, in which magnets or steel cylinders were inserted. This resulted in a modular system of eight-sided prisms cut in trapezoid or parallelogram form, combined with V-pieces and short angular (degenerate trapezoidal) pieces.

These components were and are always an invitation to creative play and a challenge to the spatial imagination. In recent months, many interesting but also bizarre sculptures have been created, of which a small selection is shown in Figure 5. See the supplementary material for more pictures.

Figure 5: *Various artworks from octagonal beams and V-pieces: oak wood, $h \times w = 20 \times 32$ cm (left),* Lantern*, maple wood,* 22×17.5 *cm (middle), maple wood,* 17.5×17.5 *cm (right).*

References

[1] R. Fathauer and N. Selikoff (Eds.). *Bridges Enschede 2013 Art Exhibition Catalog*. Tessellations Publishing. https://gallery.bridgesmathart.org/exhibitions/2013-bridges-conference/friedhelm-kurpig

[2] T. Verhoeff and K. Verhoeff. "The Mathematics of Mitering and its Artful Application." *Bridges Conference Proceedings*, Leeuwarden, Netherlands, July 27–31, 2008, pp. 225–234.