Halftoning with Layered CMYK Color Truchet Tile Mosaics

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Abstract

The use of halftone dots of cyan, magenta, yellow and black (CMYK) is ubiquitous in the printing of color images. An alternative technique with an engaging aesthetic makes use of a family of generalized Truchet tiles, created with the rule that each tile edge is divided into three segments of two alternating colors that also divide the tile face to create motifs that connect across tile boundaries. The motifs are bounded by straight lines and circular arcs and the members of the resulting tile family exhibits a range of area ratios for the two colors. A CMYK image is made with the tile set as follows: 1) an original color image is digitally processed to produce four CMYK color channel images, 2) for each color channel image, a mosaic is constructed by replacing groups of 2×2 pixels in the original image with a single tile, selected from the pool of available tiles according to both the local average and gradient of the color value, where 3) black on white tiles are used to create a mosaic for only the black (K) channel, while white on color is used for cyan (C), magenta (M) and yellow (Y) channel mosaics. As in conventional halftone color printing, the composite color mosaic image is constructed by overlaying the four individual color channel mosaics.

Introduction

Halftone processes, as introduced in the mid 1800s, employed small dots to adapt the continuous tonalities of photographs to the mechanical printing systems available at the time[8]. In duotone halftones, tonality is controlled locally through the use of a regular grid of ink dots of adjustable size, printed on a contrasting background. Print color images are created by layering offset duotone halftones created for respective color channels in cyan, magenta, yellow and black (CMYK), each derived from the original image. Bosch and others [6, 2, 1, 11, 12, 7] extended duotone art through the use of sets of two-colored Truchet tiles in which rules and optimization algorithms are applied to create figurative mosaics. By replacing the notion of a "tile" as an opaque and tangible object of finite thickness with one that is infinitely thin and semi-transparent, mosaic representation of full color images can be achieved with Truchet tiles by adapting concepts from halftone color printing.

Truchet tiles are well known for the diversity of larger scale patterns that can emerge through rotational orientation and arrangement on a grid [9]. While Sebastian Truchet's classic tile is a square divided on a diagonal into two triangles of different colors, many variants now bear his name, with the common thread of contiguous abstract geometric designs that span multiple tiles via the original triangles or more complex motifs that align at tile boundaries. Representation of color images is explored in this study through the use of a particular category of Truchet tiles, readily described using an edge classification system introduced by Virolainen [10]. Each edge of Truchet's original tile has one of two colors, A and B, so that a single tile can be represented, going around the square's perimeter, as A-B-B-A, where the dashes represent corners. The Truchet tile is generalized through subdivision of the tile edges in a systematic way, and for the two complementary sets of tiles considered here, the tile motifs divide each square tile edge into thirds, with the notation ABA-ABA-ABA-ABA and BAB-BAB-BAB-BAB, respectively. The Truchet mosaics presented in this study are constructed with a family of 20 ABA square generalized Truchet tile motifs, as well as the complementary BAB family (Figure 1). Each tile type is constructed with motifs that either connect the center third of the tile edge segments to the center third of another edge via a straight or quarter circle line, or terminate at the edge segment in a semi-circle [5, 3]. For the black color channel mosaic only, solid white tiles are allowed, in addition to the ABA set. Image construction exploits two aspects of the tile set: 1) the average color value (grayscale) varies between members of the tile family and 2) some of the included motif types exhibit asymmetries such that rotational orientation of the tile in the mosaic can be selected to approximately align the color gradient on its own surface with the local gradient in the original image.

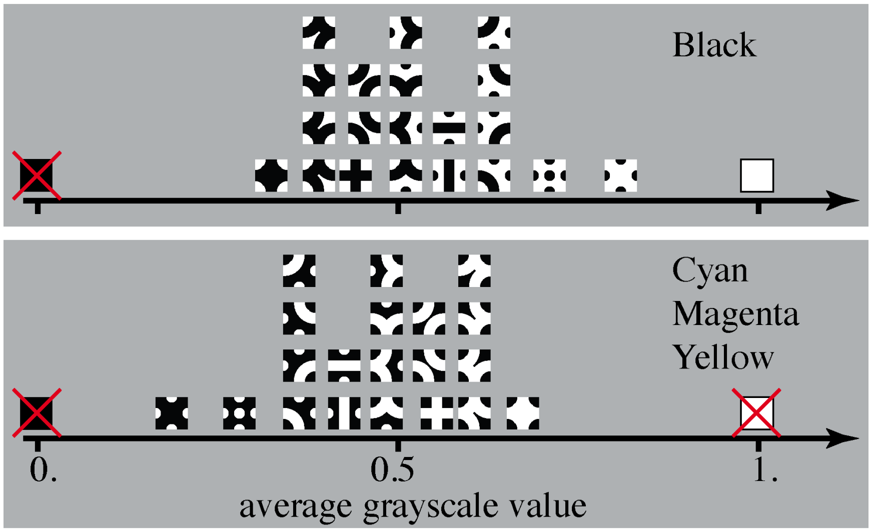


Figure 1: *Sets of black/white ABA and BAB tiles arranged by average grayscale value. The ABA set (top) is used for the black color channel only, while the BAB set (bottom) is used for the cyan, yellow and magenta channels. Tiles with a red "X" are omitted from the tile sets used in mosaic creation.*

The method of creating CMYK color images with Truchet tiles described herein is an adaptation of four-color halftone techniques commonly used in printing of photographs [8]. Original color images are decomposed into a set of four images, one for each of the four CMYK color channels, which are then converted individually to halftone. The four color-channel halftone images are then superposed to reconstruct a halftone version of the original color image. The Truchet tile version follows the same procedure, with tiles replacing dots on a grid, and with local color saturation approximated through the selection tile motifs from among those in either the ABA or BAB family according to the ratio of the tile surface areas covered by the two colors. As in the traditional halftone, the tiled images produced for each of the four color channels are layered, either digitally or with ink, to create a recognizable mosaic version of the original image. The multi-layer tile mosaics also exhibit an abstract geometric structure on the spatial scale of the tiles themselves.

The creation of color figurative mosaics with ABA and BAB Truchet tiles presented here exploits several key techniques. First, C, M, Y and K color channel images (depicted in grayscale) at a reduced resolution are obtained from a photo or other digital image using a commercial software routine (Figure 2). Second, for each color channel, a black and white (for now) mosaic is constructed by replacing groups of 2×2 pixels in the original image with a single tile, selected from the pool of available tiles according to both average grayscale value and grayscale gradient. Finally, the ABA tile set is used for the black (K) channel only while the BAB set is used for cyan (C), magenta (M) and yellow (Y) channels. As in conventional halftone color printing, the color mosaic image is constructed by overlaying the four color channel mosaics, each in its original color (C, M, Y or K). The resulting layered color mosaic is generally faithful to the original image, but with an abstract geometric fine scale structure of interconnecting features resulting from the alignment of neighboring tile motifs. In addition to 2D digital art, examples include an adaptation to mokuhanga, traditional Japanese wood block printing using transparent watercolor paints, using laser engraving to carve a set woodblocks, one each for printing the four color channel mosaics.

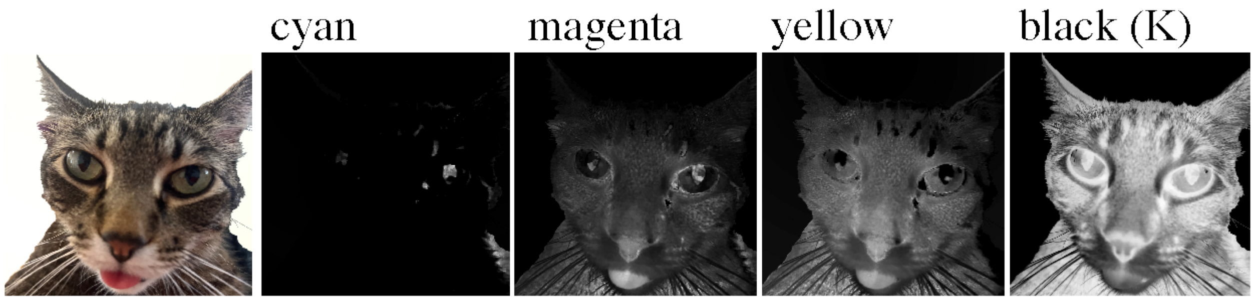


Figure 2: *Original photo and output of digital CMYK color separation function, displayed here as grayscale negatives. Mosaics are created for each of the four color channels.*

Complementary Tile Sets for Improved Space Filling

The use of complementary tile sets ABA (K) and BAB (C, M, and Y) in creating color channel mosaics provides a means for more effective "space-filling" in the resulting layered mosaic. Figure 3 illustrates layering with small mosaics in red, black and white. For reference, we assign the white areas to be the "A" color, while either black or red is used as the "B" color. Using these definitions, the top row of mosaics in Figure 3 includes red or white motifs on a white background (ABA), and on the right, white motifs on a red background (BAB). When ABA mosaics with red and black tiles are overlaid (bottom left), a large fraction of the mosaic remains white and a large proportion of the red is covered by black. As a result, although recognizable figures can be produced using layered CMYK ABA mosaics, colors are severely muted and the image has a high proportion of white. In contrast, when otherwise identical ABA and BAB mosaics are layered, the resulting composite mosaic fills the space completely with no white showing (bottom center), and when mosaics of random ABA (black) and BAB (red) tiles are overlaid (bottom right), high proportions of both red and black result.

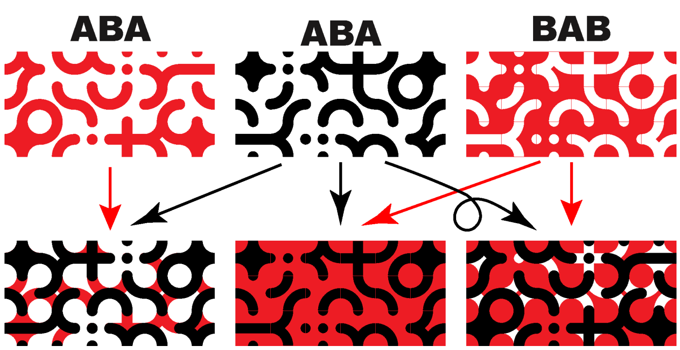


Figure 3: *Top row: example mosaics with red/white ABA tiles, black/white ABA tiles and red/white BAB tiles. Bottom row: two overlaid mosaic layers with red/white ABA or BAB tiles combined with black/white ABA tiles.*

Tile-level Gradients

Gradients in color value over a spatial scale longer than the width of a tile can be generated with a sequence of neighboring tiles with motifs chosen to match the local average grayscale value in the original image [12, 7]. A further improvement in gradient fidelity that takes advantage of gradients internal to tiles with motif asymmetries is introduced as follows. Motifs with asymmetries allow for either two or four distinct tile rotational orientations, illustrated in Figure 1. All distinct rotational orientations are included in the tile sets, such that a tile can be selected for alignment with with gradients in the region internal to the image region replaced by the tile. The original image is resampled at a resolution such that each tile represents four (2×2) pixels (Figure 4). For each tile type, average grayscale values are evaluated separately for each quadrant of the motif. Tile selection for each 2×2 set of pixels proceeds through an optimization process that involves computing the sum of the least squares differences between tile and pixel grayscale value for each the four pixels. The sum will always be greater than zero and will be exactly equal to zero in the unlikely case of a perfect grayscale match between each of the four pixels and their corresponding tile quadrants. Least squares sums for each 2×2 pixel array are computed separately for each tile candidate, and the "best fit" tile is that which produces the sum closest to zero. When multiple tile types produce the minimum (or close to the minimum), a random selection within that group is made. In this way, tile selection simultaneously optimizes both average grayscale value and local grayscale gradient.

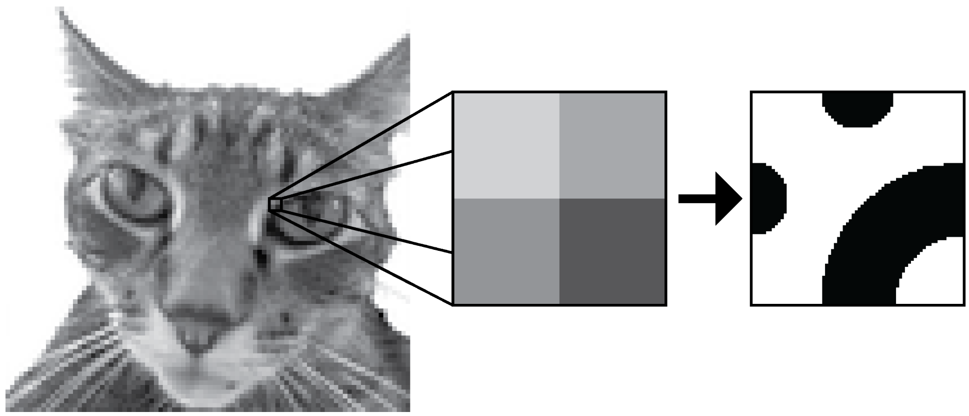


Figure 4: *Example 2×2 set of pixels from the image black color channel and replacement tile, selected from the tile pool as having optimized average grayscale value and gradient.*

Image Construction via Layering of Color Channel Mosaics

Using the tile selection method described above, four mosaics are constructed, one for each of the four CMYK color channels of an original image. Each color channel image is resampled at a pixel resolution that is twice the desired number of tiles in each dimension of the resulting mosaic, and one tile is substituted for each 2×2 block of pixels. The composite full color mosaic image is constructed by overlaying the four individual color channel mosaics. The reconstruction process is illustrated in Figure 5.

A full color mosaic presented in Figure 6 (with added background) highlights several features of the techniques introduced in this paper. First, the interplay of the four colors in the composite mosaic creates a rich color spectrum, as in conventional CMYK color halftones. In this example, tiles on a 45×45 grid represents a 90×90 pixel version of the original image. At each grid location in the composite image, four tiles, one for each color channel, are superposed, with a transparency that enables blending of colors. The optimization of tile orientation enhances the fine detail in the cat image, such as the whiskers, and the white and black rings around the eyes. And while there are no large areas that are completely white (the BAB tile set used for C, M, and Y channels omits the all-white tile), regions that span multiple tiles and are nearly solid black *can* be seen. The black and red background surrounding the cat is not generated from an image; it is a decorative filler constructed with tiles of the same size as those used in the cat image. Each background tile is a composite of a randomly-selected complementary pair of ABA (red) and BAB (black) tiles.

The homogeneity across tiles of the black regions is made possible by combining complementary tile

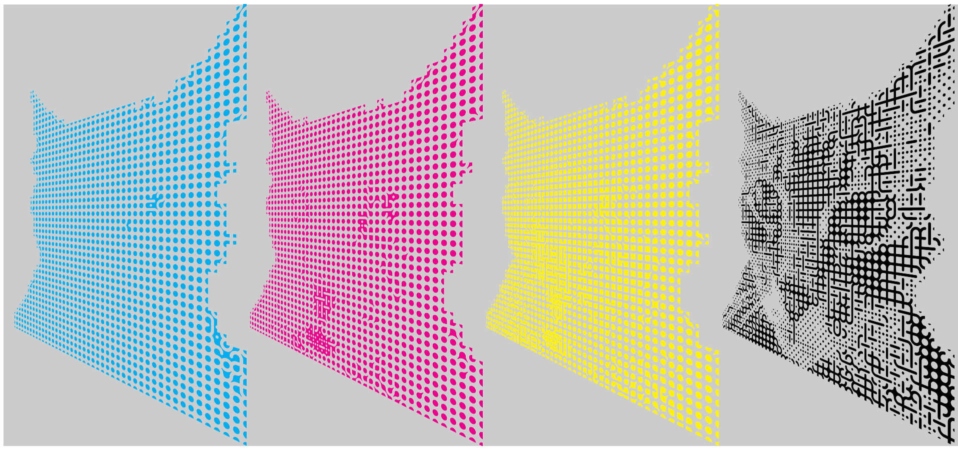


Figure 5: *Mosaic versions of the four color channels are overlaid to create the final image.*

types that spatially separate black (ABA) from the other three colors (BAB), smoothing the graininess in the mosaic image associated with the tile motifs. This is because where cyan, magenta and yellow color motifs overlap, the digitally combined result appears as a very dark gray that “fills in” the white spaces in the ABA tiles used for the K channel, such that the resulting square tile appear as solid black. To make this effect stand out, the motifs on the ABA tiles were thinned slightly to create a visible gap between them and areas where the C, M and Y motifs coincide with one another to produce dark gray. These gaps create thin white or colored circular arcs within black areas; they can be seen more easily in a magnified section on the right in Figure 6.

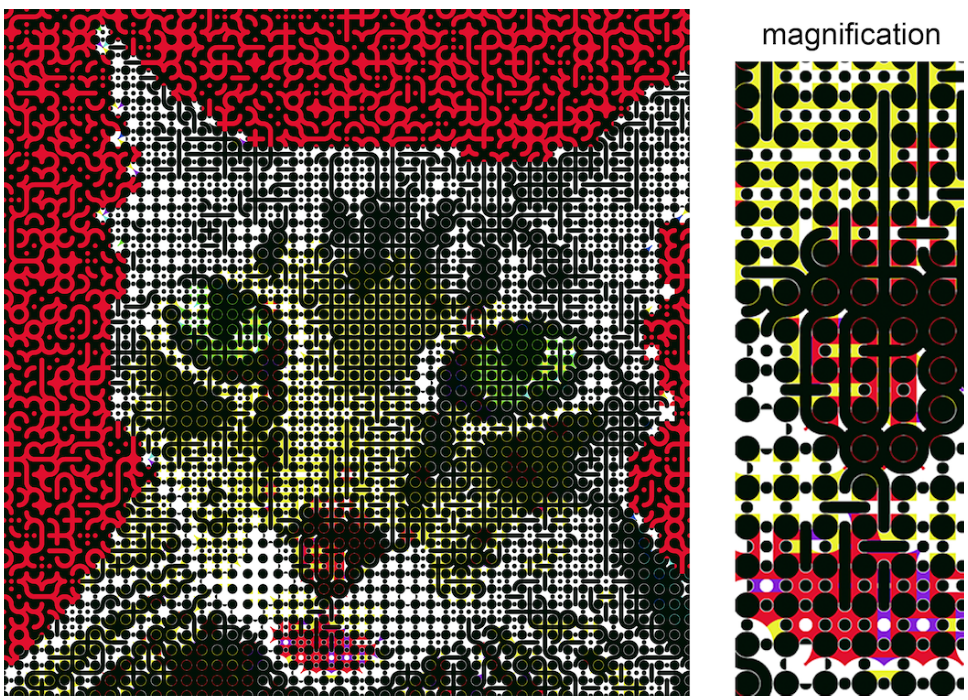


Figure 6: *Digitally reconstructed image using four layered CMYK color channel mosaics. Zoomed region highlights both the abstract design and the tile level color blocks.*

It is notable that in conventional color halftones, the black channel is also handled differently from the other three channels to enhance spatial separation of the black dots from those of the other colors. While there is overlap between halftone dots of different colors, spatial separation between colors is achieved to some extent through the choice of the relative rotational angles of the dot grids. While the angles between the C, M, and Y grids are typically 15 degrees, the black channel dot grid is separated from the others by 30 degrees, with the upshot that the overlap of the black dots with dots of other colors is relatively lower[8].

Total Tile Count: Aesthetic and Practical Considerations

"How many tiles to use?" is a perennial style-related question in the design of figurative mosaics, when faced with the tradeoff between capturing image detail and maintaining finite tile size and manageable tile number. In addition, the discrete nature of the tiles is part of the aesthetic charm of mosaics, so that ideally the viewer perceives both the large scale image and the smaller scale detail of the individual tiles. This tension, as well as the fidelity of the Truchet color halftones to the original image, is explored in Figure 7a, in a sequence of composite mosaics in which a multicolor image (top), with structure at multiple scale lengths, is depicted with different tile counts corresponding to 35, 55 and 100 tiles across the width of the image. We observe that with 100 tiles across the width (bottom pane), the smaller details in the original image are rendered well in the mosaic, and color gradient transitions are relatively smooth. Graininess not present in the original image is also evident, reflecting the shapes of the tile motifs. In particular, the background consists of black dots, with spacing equal to the width of the tiles. That "polka dot" background could be eliminated by adding a solid white (no motif) tile to the BAB tile sets used for the C, M, and Y color channels. As the tile count is reduced, so is the fidelity to the original image. At only 35 tiles across the width, the 7-letter word depicted is surprisingly readable, while the funky patterns produced by the tile motifs become more apparent.

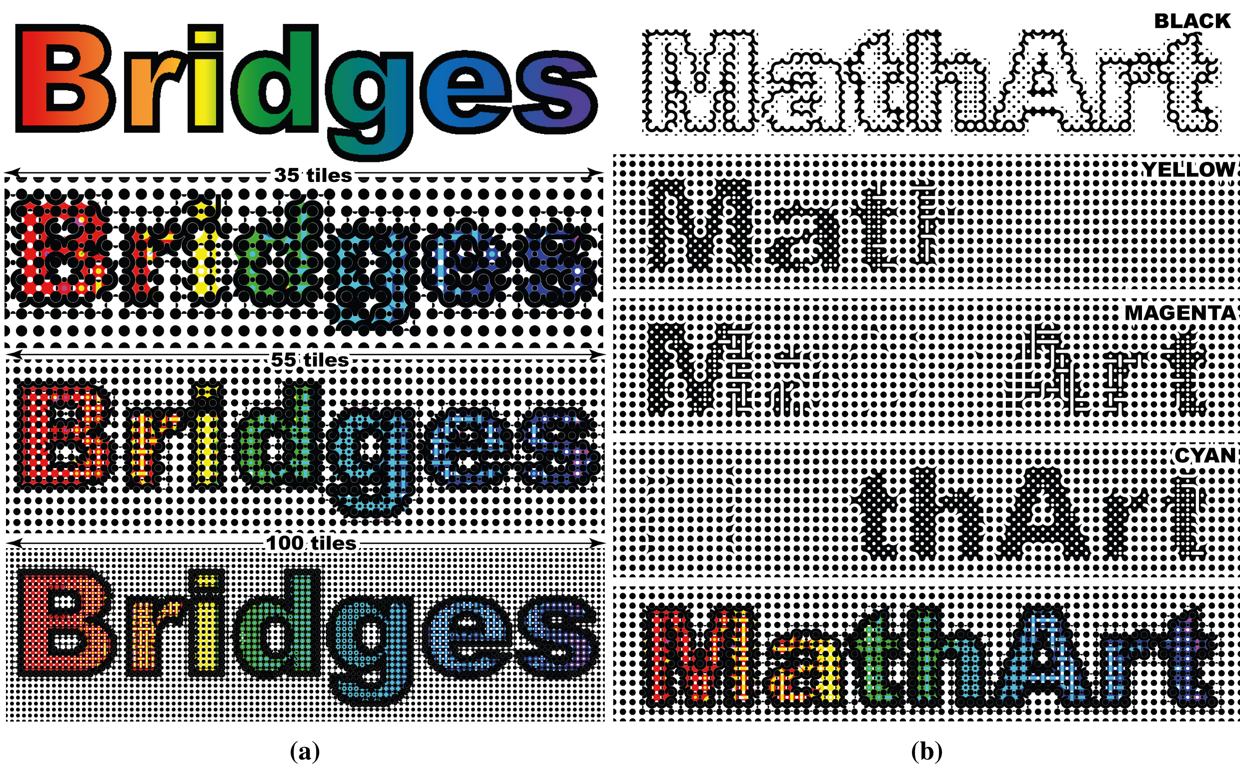


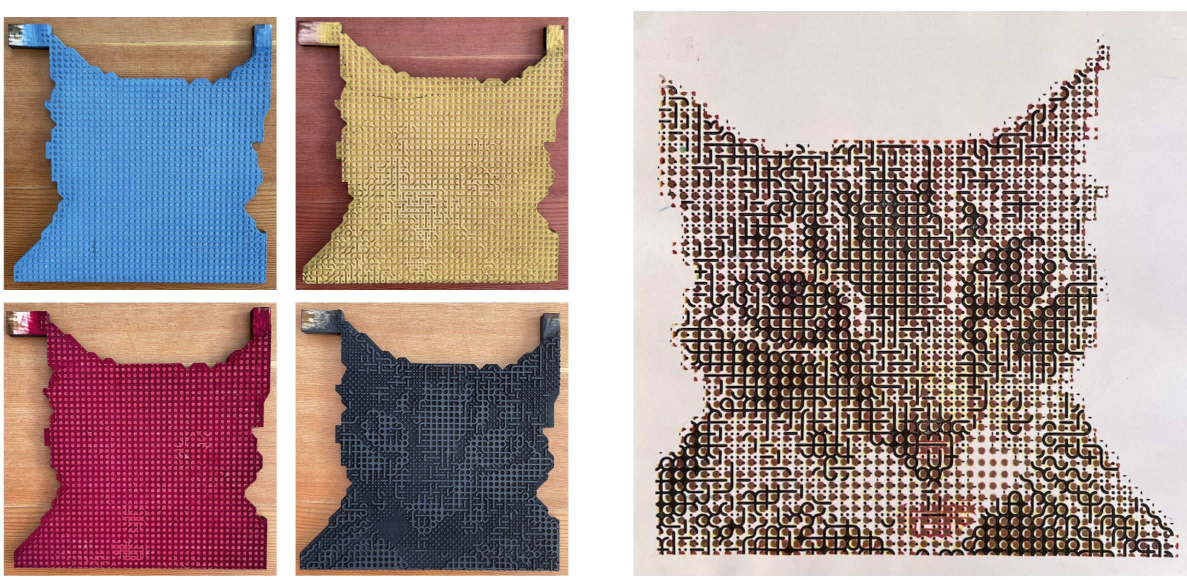
Figure 7: *(a) Image (top) depicted with resolutions of 35 tiles, 55 tiles and 100 tiles across the full width, and (b) mosaic CMYK color channels (65 tiles across) rendered individually in black and white and combined in color in the composite image.*

To illustrate how intermediate colors arise in composite mosaics, Figure 7b shows each of the four color channel mosaics (in black and white) comprising a particular composite image, depicted with a width of 65 tiles. The black areas in each of the color channel mosaics shown are converted to the channel color when the composite image is constructed. On the left side of the "M" for example, both the yellow and magenta channels use tiles motifs with a high proportion of black, so that area appears red in the composite image. Moving to the right side of the "M," the yellow channel continues to select motifs with higher average grayscale while the magenta channel transitions to motifs with lower black content, to produce a more orange color in the composite. The perception of orange by the viewer is actually produced by adjacent small areas of red (where magenta and yellow motifs overlap) and yellow, so that the impression changes depending on the chosen tile resolution.

Figure 7b also provides an illustration of how fine details, in this case the black letter outlines, are produced by the tile selection algorithm. In this example, the black outlines in the original image are approximately one tile wide. Looking at the black channel mosaic image, one can see that the tile motif and tile rotational orientation changes with location along the letter outline. In addition, one can also see that the three other color channels play a role in defining the black letter outlines. Because the C, M and Y color channels use the BAB tiles while the black channel uses ABA tiles, the C,M and Y channels "fill in" the white areas left in the outlines in the black channel mosaic to create a more distinct line in the composite mosaic.

Concluding Remarks

The mosaics presented up to this point were computer generated and as such, lend themselves to printing using conventional modern technologies, such as laser, inkjet, and dye sublimation printers. Adaptation to other art media is also possible, and the example of mokuhanga style woodblock printing [4] is highlighted in Figure 8.



(a) **(b)**

Figure 8: *(a) Photos of laser engraved 9" by 9" woodblocks, stained by inks used in the mokuhanga printing process, and (b) photo of woodblock print made with the four woodblocks.*

Mokuhanga is globally recognized through the well known work of Japanese printmaking masters, including Hokusai and Hiroshige. Mokuhanga is well-suited for Truchet color halftoning due to the techniques employed for accurate registration when printing successive blocks as well as the use of low-opacity water-based inks that enable color blending of the printed layers. While mokuhanga woodblocks are traditionally hand carved, in this study, the Truchet patterns were carved into the surface of each wood block via computer-controlled laser engraving, for both speed and accuracy. Figure 8a shows four shina plywood blocks carved with Truchet mosaics, one for each color channel, and Figure 8b is a photo of a print produced after printing with each of the four blocks.

In closing, I would like to express my deep gratitude to Mike Lyon for inspiration, community and mokuhanga instruction, and to the entire awesome team at the Anderson Ranch Art Center in Snowmass Village, Colorado.

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