Exploring Variable-Based Generation of Girih Patterns: A Comparison of Radial and Polygonal Methods

Delara Razzaghmanesh[1] and Ethem Gürer[2]

[1]Department of Informatics, Faculty of Architecture, Istanbul Technical University, Istanbul, Turkey; razzaghmanesh@itu.edu.tr
[2]Department of Interior Architecture, Faculty of Architecture, Istanbul Technical University, Istanbul, Turkey; egurer@itu.edu.tr

Abstract

Radial and polygonal methods are fundamental techniques for generating Islamic geometric patterns, and they are widely used by scholars to create and analyze a variety of patterns. Despite their widespread application, the underlying principles and key variables of these methods are distinct. This paper compares the two methods by exploring their respective variables in the generation of two prominent Girih patterns in Islamic architecture. By manipulating these variables within each method, diverse patterns are produced, demonstrating the flexibility and variation achievable through algorithmic design. The study utilizes Grasshopper programming to implement the methods, illustrating how adjusting variables in both approaches can lead to entirely different pattern outcomes.

Introduction

Islamic geometric patterns have long fascinated scholars due to their intricate designs and mathematical sophistication. While historical scrolls and treatises contain drawings of these remarkable patterns, they often lack detailed explanations of the methods used to generate them. “The aesthetic character of a given geometric design is greatly determined by the method used in its creation” [2]. Therefore, a generative methodology serves as a key criterion for gaining a deeper understanding of Islamic geometric patterns.

The complexity of the patterns suggests an advanced understanding of mathematics and geometry. Among these patterns, *Girih* patterns hold a special place, particularly in Iranian architecture [10]. Traditional methods for designing Girih patterns typically involve polygonal, radial, or grid-based approaches, each relying on distinct underlying variables.

This study employs algorithmic design, utilizing the computational power of Grasshopper to explore the variables behind polygonal and radial methods systematically. Grasshopper is a visual programming language and environment that runs within the Rhinoceros 3D modeling software developed by Robert McNeel & Associates [9], and it is used as the visual programming environment to implement the pattern generation algorithms. Grasshopper was selected for this study due to its robust parametric modeling capabilities and seamless integration with Rhino, allowing both geometric control and algorithmic flexibility. Two prominent Girih patterns in Islamic architecture, including *Hasht-Chahar-Lengeh*, and *Kond-e-Do-Panj,* are generated by both polygonal and radial methods. By altering the variables within each method, it is possible to generate entirely new patterns, offering insights into the creative potential of computational tools in reinterpreting and extending traditional Girih designs. This approach highlights the interplay between mathematics and aesthetics and provides a framework for innovation in the study and generation of Islamic geometric patterns.

Related Works

Girih is a Persian word, which means “knot”. In Iran, “*gereh-sāzī*” is the term for “making knots”. Altering the angles in the Girih pattern gives rise to the distinctions between *Tond* (acute), *Kond* (obtuse), and *Shol* (loose) Girih [11]. This categorization is somewhat similar to Bonner's classification of patterns into acute, obtuse, and median families [2]. The repeat unit in Girih patterns plays a crucial role in their overall structure and composition. To construct the main unit of the pattern, symmetrical operations such as reflection, rotation, and translation are applied to the repeat unit. These transformations ensure that the pattern maintains its continuity and coherence while preserving its underlying mathematical principles.

In the modern era, many scholars employ various methods to construct Girih patterns. The Polygons in Contact (PIC) method, or Polygonal Technique, was first described by Hankin [6], and it is the most widely used technique for the derivation of patterns. The distinctive feature of this methodology is the employment of a polygonal tessellation that acts as a substructure from which the geometric pattern is derived. In this technique, strategic points of a polygonal tessellation, such as the midpoints of each polygonal edge, are used to locate main pattern lines, and then the tessellation is discarded. From a single underlying tessellation, various designs can be generated based on the angles of the applied pattern lines. Bonner [2] used the polygonal technique for generating different geometric patterns. The underlying tessellation for this technique can consist of either regular or irregular polygons. In addition to the polygonal technique, geometric patterns can be produced from other design methodologies. Since 1970, some authors have used the radial technique as a dominant historical design methodology [5], [4], [14], [3]. In this method, patterns can be drawn with a ruler and compass, and through the intersection of the radii and arcs, the lines of the pattern are drawn. This technique is one of the most common methods among Iranian masters such as Sharabaf [13], Lorzadeh [11], and Hely [7].

In contrast to the traditional methods of drawing Girih, some scholars employed new and generative methods and reinvented the drawing instructions for Girih patterns [1]. More recently, Razzaghmanesh and Gürer [12] analyzed and generated Girih patterns based on graph-theory definitions.

This study used algorithmic design to employ polygonal and radial methods to generate two Girih patterns, comparing the underlying variables of each method and evaluating their creativity levels in generating new patterns through altered variables.

Girih Generation with the Polygonal and Radial Methods

Girih patterns can be categorized based on the number of sides of their stars to eight, ten, twelve, and fourteen-sided Girih [8]. One of the important types of ten-Girih is "Kond-e-Do-Panj" or "*Om-al-Girih*" (mother of Girih), which is a Kond Girih. This name signifies that the other ten possible Kond Girihs originate from this Girih type [11].

For generating the Kond-e-Do-Panj Girih based on the polygonal method, the polygonal tessellation in this pattern should be understood. Since the pattern features a 10-sided *Shamseh* (Islamic star), the decagon serves as the key geometric shape, around which regular pentagons and irregular hexagons are arranged (see Figure 1(a)). By using and connecting the vertices, the midpoints of the edges, and the lines connecting the centers of each polygon to its vertices, the main shapes, such as stars and interlocking motifs, are generated, as illustrated in Figure 1(b).

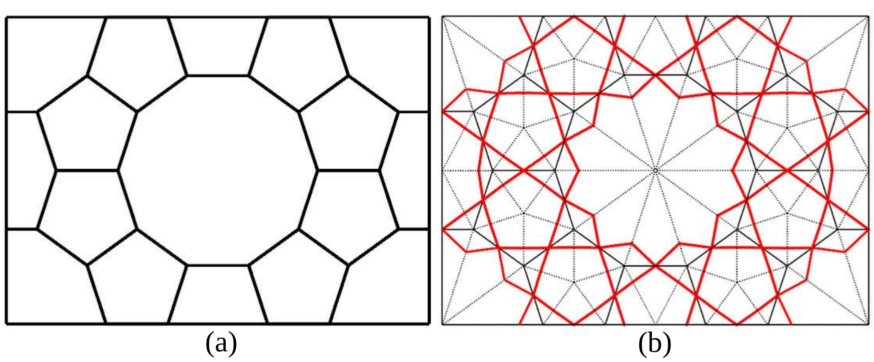
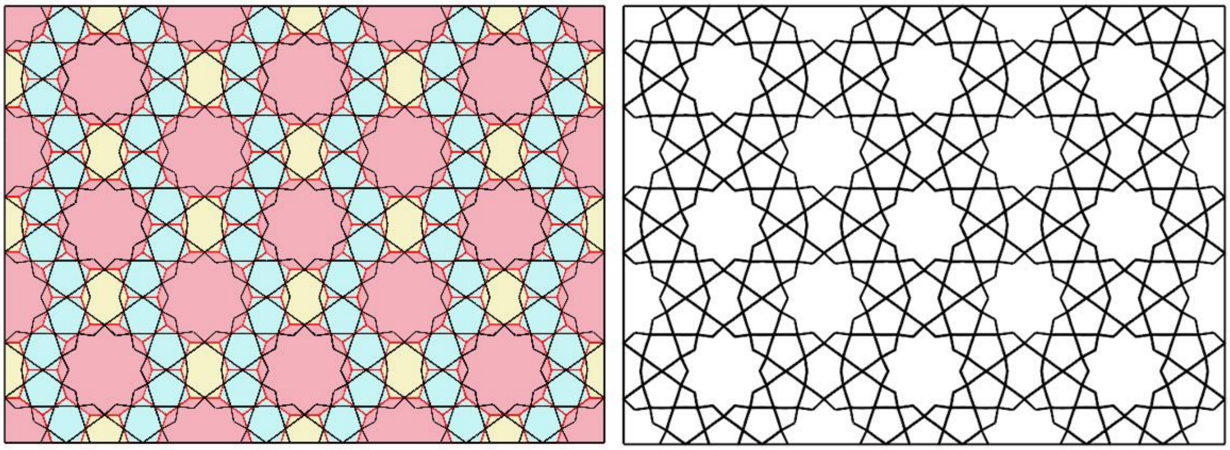


Figure 1: *The polygonal method to generate Kond-e-Do-Panj Girih: (a) the underlying polygonal tessellation, (b) the main shape of Girih drawn inside the polygonal shapes.*

The entire pattern is generated by arranging the main unit in the x and y directions, illustrated in Figure 2.

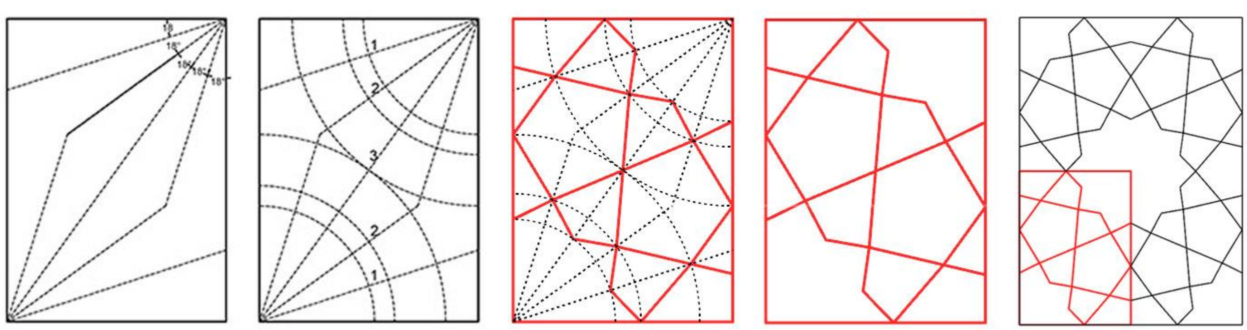


(a) (b)

Figure 2: *The entire Kond-e-Do-Panj Girih: (a) the underlying polygonal tessellation, (b) the entire Girih shape without tessellation.*

To generate the Kond-e-Do-Panj Girih using the radial method, the process begins by drawing two perpendicular lines. The right angle is divided into five equal segments, each measuring 18 degrees (90°/5 = 18°). The width of the rectangle is optional, while the length of it is determined by the third division line, which is a diagonal.

Next, circles are drawn with centers at two opposite corners of the rectangle and a radius extending to point 3 from the segment divisions. Two additional circles, with smaller radii than the first, are drawn. The radii of these circles are determined by the perpendicular bisectors of the rectangle's width, which intersect the division lines. These radii serve as adjustable parameters, allowing for the generation of diverse pattern variations (see Figure 8). The intersections of these circles and the division lines define key construction points, as illustrated in Figure 3, revealing a quarter of the Kond-e-Do-Panj (Om-al-Girih) pattern. By performing three rotations, the main unit of the Girih pattern is formed.



(a) (b) (c) (d) (e)

Figure 3: *Radial method for generating Kond-e-Do-Panj Girih: (a) dividing the space between two lines into five segments, (b) drawing three circles, (c) connecting the intersection of circles by division lines, (d) the quarter of unit is displayed, (e) rotating the quarter unit to get the main unit.*

To generate the Hasht-Chahar-Lengeh Girih using the polygonal method, the underlying tessellation must first be defined. This tessellation consists of regular octagons, squares, and isosceles triangles. The center of each octagon is connected to all its vertices. Then, two additional lines extend to every second vertex from a selected point along these connecting lines, forming the Shamseh motif within the octagons. In the squares, the center is connected to the midpoints of all four sides. From a designated point on these lines, two additional lines extend to the corresponding vertices, generating a four-pointed star within each square, as presented in Figure 4.

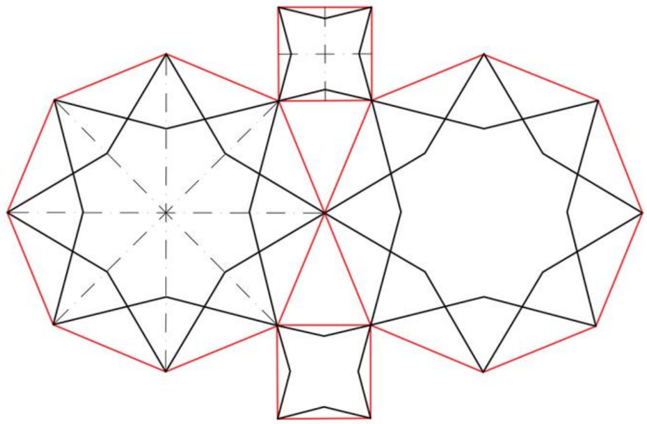
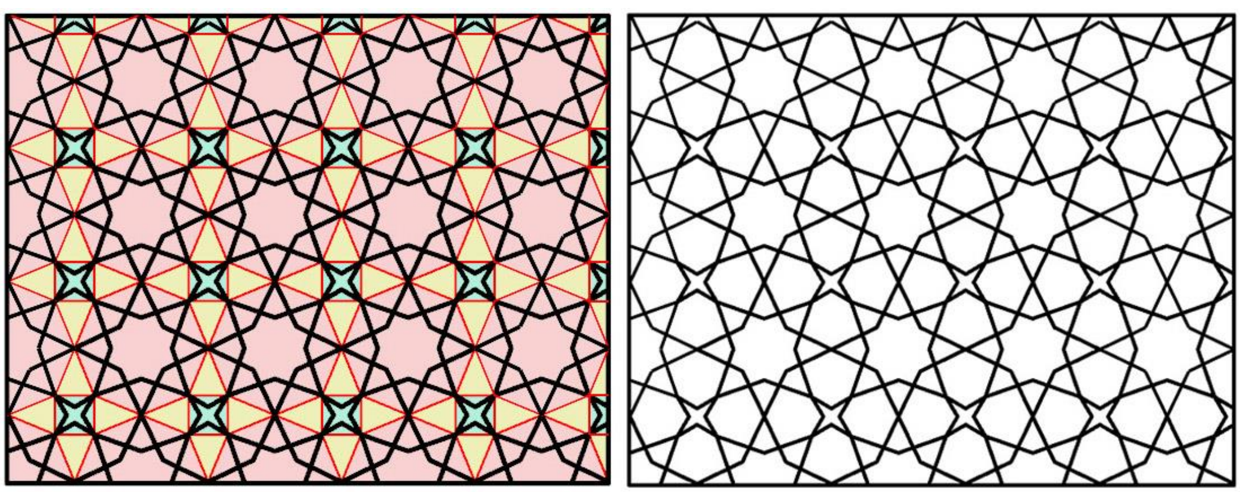


Figure 4: *The underlying polygons and the method of generating Hasht-Chahar-Lengeh Girih with the polygonal method.*

Unlike Bonner's method [2], which relies on rotated octagons and fixed 45-degree pattern lines without using subsidiary isosceles triangles, our approach introduces additional geometric elements to maintain uniformity and connectivity between tiles. This allows for a more flexible and scalable generation of the patterns. The entire pattern and its underlying polygonal tessellation are illustrated in Figure 5.



(a) (b)

Figure 5: *The entire Hasht-Chahar-Lengeh Girih: (a) the underlying polygonal tessellation, (b) the generated Girih pattern.*

Generating the Hasht-Chahar-Lengeh pattern using the radial method is illustrated in Figure 6, which begins by defining the repeat unit, which in this case is a square. One of the square's diagonals is drawn, and the two opposite sides of the square are connected. From a selected point on one side, a line is extended to a point on the diagonal, ensuring that when mirrored along the diagonal, one-quarter of the Shamseh motif is formed. By rotating this quarter motif three times, the complete Girih unit is constructed. Repeating this unit along the x and y directions generates the full Girih pattern.

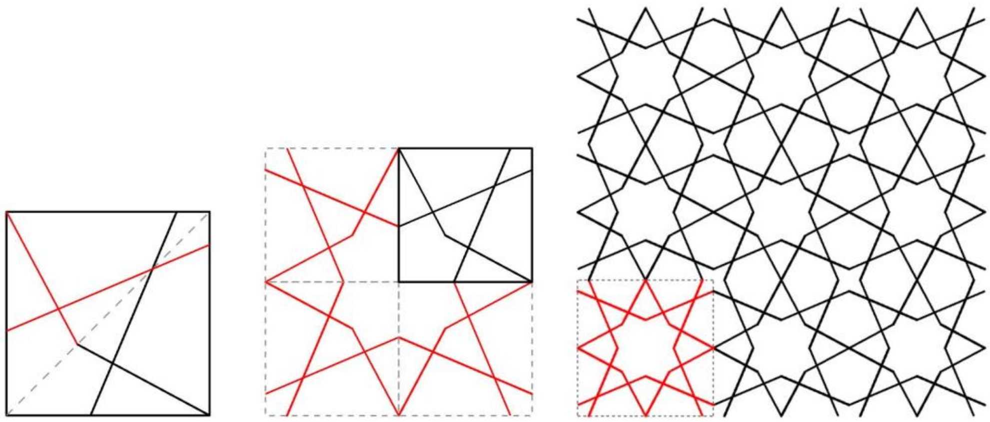


Figure 6: *Algorithmic steps for the generation of Hasht-Chahar-Lengeh Girih based on the radial method.*

New Patterns

The variables in the polygonal and radial methods are distinct. By altering the variables in each method, entirely new patterns can be generated.

Some of the patterns generated by altering the variables in the polygonal method for the Kond-e-Do-Panj Girih are illustrated in Figure 7. The variables are the position of the vertex connections.

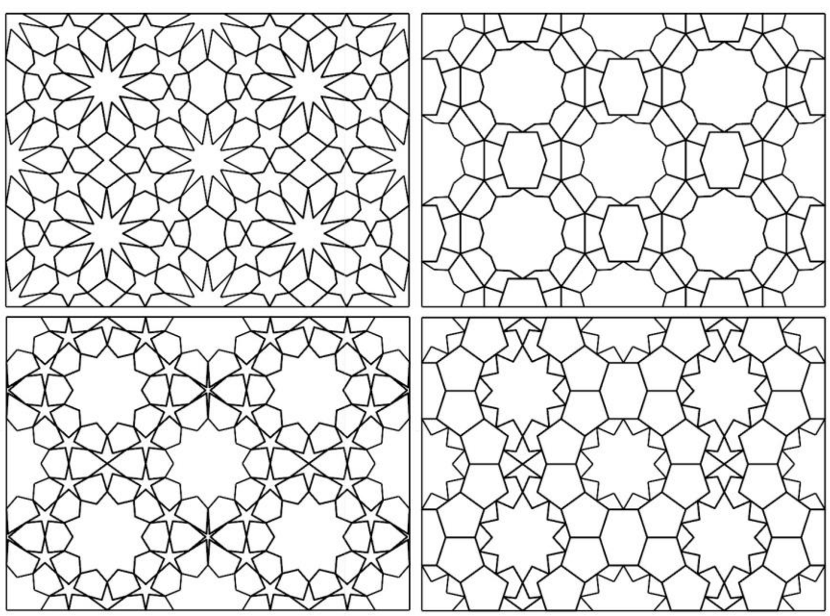


Figure 7: *Patterns generated by altering the vertex connections in Kon-e-Do-Panj Girih within the polygonal method.*

Figure 8 illustrates some patterns generated by altering the variables in the radial method for Kond-e-Do-Panj Girih, specifically the radii of the circles.

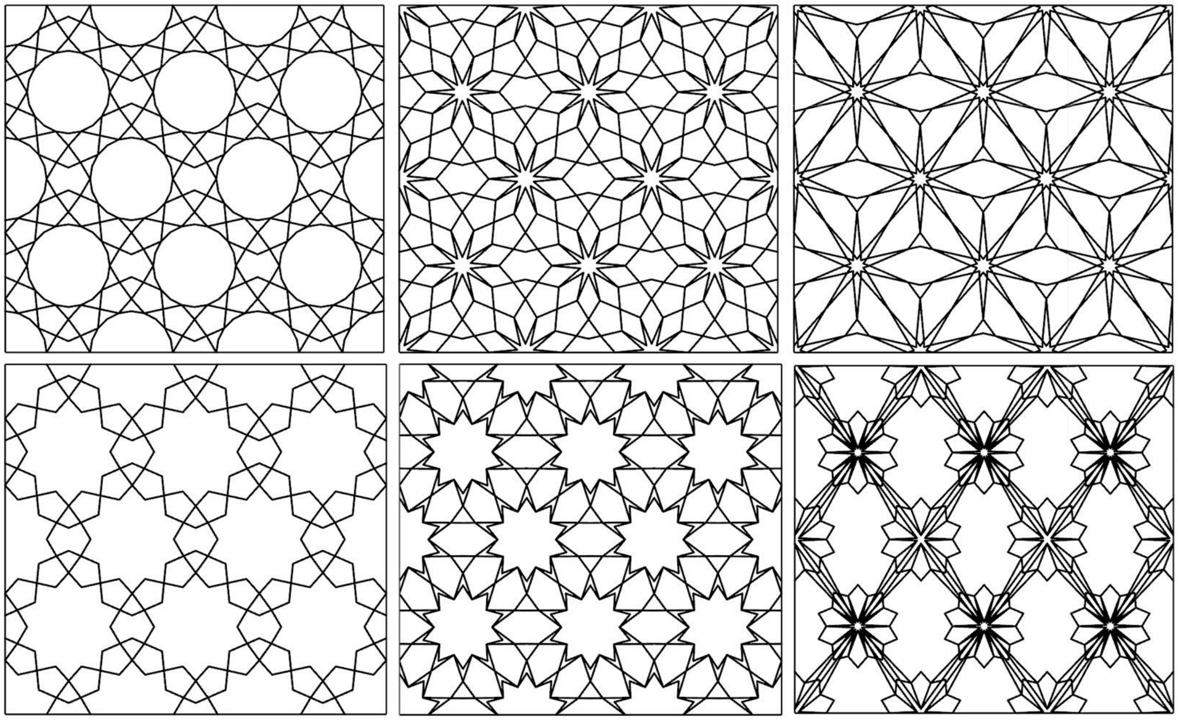


Figure 8: *Generated patterns by altering the radii of the circles in the radial method for Kond-e-Do-Panj Girih.*

Figure 9 illustrates the patterns generated in the polygonal method for the Hasht-Chahar-Lengeh Girih by modifying the contact angles of motifs within the polygonal tessellations.

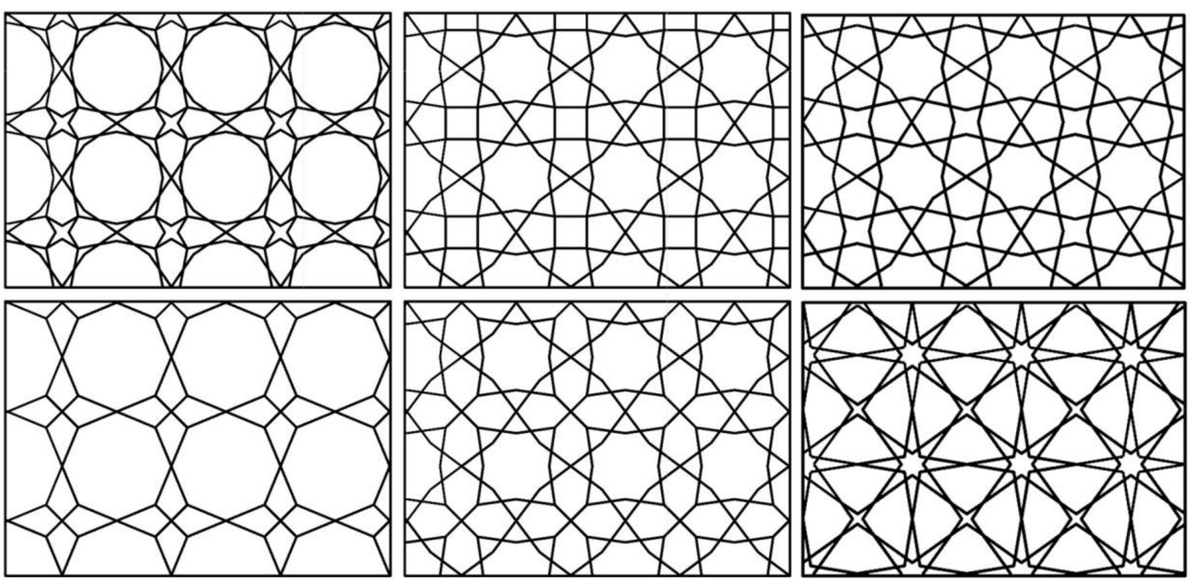


Figure 9: *Generated patterns by altering the position of vertex connections in Hasht-Chahar-lengeh Girih within the polygonal method.*

Figure 10 presents the patterns generated in the radial method for the Hasht-Chahar-Lengeh Girih by adjusting variables, specifically the positions of the lines.

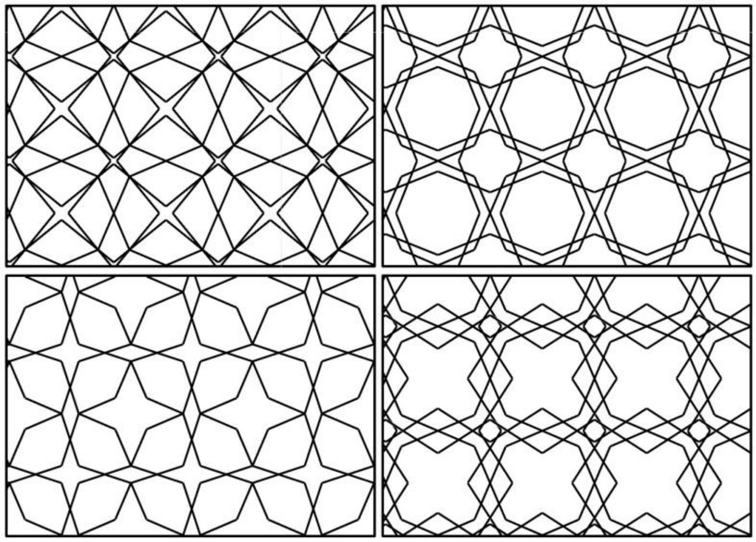


Figure 10: *Patterns generated by altering the variables in Hasht-Chahar-lengeh Girih within the radial method.*

Summary and Conclusions

Girih patterns are among Islamic architecture's most distinctive and intricate geometric designs. Due to their complexity and the mathematical principles embedded within them, generating these patterns requires a structured and transferable computational methodology. Two key approaches for deriving such geometric patterns are the polygonal method and the radial method, both of which play a significant role in the construction of Girih designs.

This study employs both methods to generate two widely used Girih patterns in Islamic architecture. The approach is generative, meaning it not only reproduces the original patterns but also allows for the creation of entirely new designs by altering the variables within each method. The research aims to compare these two methods by analyzing their underlying variables and assessing their effectiveness in generating variations of Girih patterns. The drawings were created in Grasshopper, leveraging its parametric capabilities to define and manipulate the variables systematically.

The generated patterns in both methods differ due to variations in their underlying variables and distinct construction approaches. In the polygonal method, the primary factor influencing pattern generation is the type of polygons used in the tessellation. Modifying the polygon types alters the overall structure and leads to different motif arrangements. Additionally, the contact angles, specifically, how the center points are connected to vertices or side midpoints, affect the internal motifs, allowing for the generation of different families of patterns, such as acute, median, and obtuse families.

Conversely, in the radial method, the key variables include the number of divisions and the way lines are connected. These factors directly impact the pattern's complexity, symmetry, and overall visual composition. By adjusting these elements, variations in density, intricacy, and motif appearance can be achieved.

The patterns generated using the polygonal method exhibit a higher degree of systematic organization, as the contact angles within each tessellation grid are adjusted in a controlled manner. This results in a more structured and predictable arrangement of elements. Additionally, compared to the radial method, the polygonal approach produces patterns with greater symmetry, as the consistent geometric relationships between components, such as equal angles and uniform connections, reinforce symmetrical arrangements throughout the design. It can also be stated that the radial method offers greater novelty in pattern generation, as it is based on the connection of segment points to form the motifs. By altering the positions of these points, a wider range of unique patterns can be created, some of which would be difficult or even impossible to generate using the polygonal method. This flexibility allows for more innovative and diverse design variations.

It is important to note that not all Girih patterns can be generated using both methods, as certain geometric and structural constraints may prevent their accurate construction. Additionally, altering design parameters does not always result in a valid Girih pattern, as some variations may deviate from the fundamental rules and characteristics of traditional Girih designs.

This comparative analysis highlights the strengths and flexibility of both methods in the generative design process, demonstrating how computational approaches can be used not only to reconstruct historical patterns but also to innovate new geometric compositions within the framework of Islamic art.

References

[1] A. Azizi Naserabad and A. Ghanbaran. "Presenting a Method to Generate Existing and Novel Girihs (Islamic Geometric Patterns) for All Systems and Families Based on 7-fold Patterns." *Computer Aided Design*, 2023.

[2] J. Bonner. *Islamic Geometric Patterns: Their Historical Development and Traditional Methods of Construction*. New York: Springer, 2017.

[3] E. Broug. *Islamic Geometric Patterns*. London: Thames & Hudson, 2008.

[4] K. Critchlow. *Islamic Patterns: An Analytical and Cosmological Approach*. London: Thames & Hudson, 1976.

[5] I. El-Said. *Islamic Art and Architecture: The System of Geometric Design*, ed. T. El-Bouri & K. Critchlow. Reading UK: Garnet, 1993.

[6] E. H. Hankin. *The Drawing of Geometric Patterns in Saracenic Art*. Memoirs of the Archaeological Society of India, 1925.

[7] S. A. Hely. *Gireh and Arch in Islamic Architecture*. Kashan: Seyed Akbar Hely, 1986.

[8] M. H. Kasraei, Y. Nourian and M. J. Mahdavinejad. "Girih for Domes: Analysis of Three Iranian Domes." *Nexus Network Journal: Architecture and Mathematics*, Vol. 18, no. 1, 2016, pp. 311–321.

[9] Robert McNeel & Associates. (2024). *Grasshopper – graphical algorithm editor*. Available at: https://www.grasshopper3d.com/

[10] G. Necipoğlu. *The Topkapı Scroll: Geometry and Ornament in Islamic Architecture*. Santa Monica, CA: Getty Center for the History of Art and the Humanities, 1995.

[11] M. RaeesZadeh and H. Mofid. *Ehyā-ye Honar Hā-ye Az Yād Rafteh: Mabāni-ye Me'māri-ye Sonati Dar Iran Be Revāyat-e Ostād Hossein-e Lorzādeh* [Revival of Forgotten Arts: Basics of Traditional Architecture in Iran as Narrated by Master Hossein Lorzadeh]. Tehran: Moula Publications, 2011.

[12] D. Razzaghmanesh and E. Gürer. "Computational design techniques for generating specific Girih patterns: A particular focus on algorithmic design and graph theory." *International Journal of Architectural Computing*. 2024;0(0). doi:10.1177/14780771241304882

[13] A. Sharbaf A. *Gireh and Karbandi*. Tehran: Cultural Heritage Organization, 2006.

[14] D. Wade. *Pattern in Islamic Art*. Woodstock, N.Y.: Overlook Press, 1976.