Dual-Sided Cube Mosaics: Exploring the Nature of the Rubik's Cube through a New MDSI Solving Method

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Abstract

Creating mirror dual-sided inverse color (MDSI) mosaics using Rubik's cubes offers new opportunities to explore the nature of the 3x3x3 cube, providing fresh insights into its symmetry and internal structure. The MDSI method is the first comprehensive approach for solving dual-sided cube patterns and constructing artistic mosaics from cubes. The proposed algorithms have been instrumental in designing author's own artworks for several contemporary art exhibitions. The MDSI approach could be valuable for cube mosaic creators, Rubik's cube artists, and enthusiasts interested in delving deeper into the mathematical nature of the cube.

Introduction

Over the course of its 50-year history, the Rubik's cube has become a research object for mathematicians and programmers: the development of various basic, speed-solving and blindfolded solving methods, the study of the group theory, the search for God's number and optimal solutions for the cube in fewest moves, training of the supercomputer and AI, robot-solvers programming, converting images into pixel patterns in cube colors for cube mosaics… even cryptography methods. These numerous problems reveal the amazing nature of the classical Rubik's cube 3x3x3, its harmony and symmetry.

I was immersed in exploring the Rubik's cube principles when I started to create dual-sided Rubik's cube mosaics. I started to notice order and restrictions inside the cube structure. Through experimentation, creating various dual-sided mosaics, I came up with mirror dual-sided inverse color mosaics (MDSI cube mosaic) that demonstrate both the symmetrical nature of the colored elements of the cube and the limitations inherent in the cube.

As a Rubik's cube artist and educator, I create MDSI mosaics in various sizes, ranging from just four cubes for the Dual-Sided Rubik's Cube Mosaic Tutorial on Rubik's YouTube channel to my largest mosaic, consisting of 4,800 cubes. This record-breaking mosaic was created at the Ivanovo Art-Station Residence and was recognized as a Guinness World Record in 2021 [10].

In this paper I give a brief lead-in to the Rubik's cube mosaics and dual-sided mosaics in particular. A key part of the paper is a presentation of my layer-by-layer method of creating mirror dual-sided inverse color mosaics with the list of algorithms in the appendix [4]. And in the end, I share my observations over two years of working on my system. The proposed method allows Rubik's cube amateurs to create MDSI cube mosaics and better understand the nature of the cube. I assume that a theme of dual-side mosaics will give impetus to discussion among mathematicians and allow the formulation of new math problems for group theory and other research areas.

Rubik's Cube Mosaics

The Rubik's cube has six sides of different colors, but it also has several philosophical sides that Erno Rubik, inventor of the cube, discussed in his recent book "Cubed" [8], one of which is art. Six traditional colors of the cube and nine pixels on one side can form various pixel patterns. Several cubes together can form bigger pixel patterns and images. The more cubes, the higher quality pixel pictures one can create. Thus, in the end of the 20th century Rubik's cube art appeared as a part of pointillist art style in the digital pixel art branch. Franck Slama (aka Space Invader), a well-known French artist and a pioneer of a fine art school characterized by the use of the Rubik's cubes as a medium [9] has named this artform Rubikcubism.

Patterns of pixel mosaics are limited by the six colors of the cube, which makes images out of Rubik's cubes so special and recognizable. The process of creating Rubik's cube mosaics has two key stages. The first – is to develop a pixel pattern divided by cubes. Amateurs use various online services or applications to convert images into pixel patterns. It takes just a few minutes or even seconds. But the artistic quality of that kind of pattern will also be amateur. Serious artists have their own approaches to creating patterns; it is always an artistic process that can take hours or days. The second stage is physically assembling a mosaic out of Rubik's cubes. The process does not need special skills, you don't even need to know how to solve a cube, you just need to solve a color pattern on one side and it can be easily done intuitively. Created one-sided mosaics out of hundreds of Rubik's cubes look monumental, extraordinary, and attractive for spectators. Making mosaics out of several cubes (starting from four cubes to several hundreds) could be considered as a good educational method for kids in the field of math and art.

Dual-Sided Rubik's Cube Mosaics

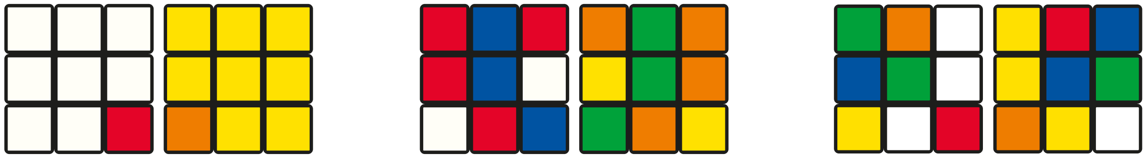
Another technique reveals the deeper potential of the cube as a twisty mechanical puzzle and art medium – dual-sided mosaics – more complex, but also more limited by the Rubik's cube nature. For instance, the central white element will always be opposite the central yellow, the red center opposite the orange, and the blue opposite the green. Accordingly, in the solved cube, the faces of paired colors will always lie opposite each other, and in the cube, there will be no elements (corners or edges) with inverse colors from opposite sides. For example, there is no green-blue edge, or white-yellow-red corner. In addition, there are only nine pixels of the same color on one cube, so in two-sided mosaics there cannot be more than nine pixels of any color on both sides of the cube. And these are just some of the basic design features of the Rubik's cube that impose significant limitations in creating dual-sided mosaics.

The number and nature of limitations for dual-sided mosaics with different images on opposite sides are so huge that it doesn't seem possible to develop a comprehensive hand-solving method, and such mosaics can only be created with a constant adjustment of patterns and finding custom solutions for each specific cube in a mosaic. At the same time, our experience in creating Rubik's cube mosaics, including dual-sided ones, allows us to discuss the possibility of creating such a system for mosaics with a mirror pattern on the opposite side in inverse colors of the Rubik's cube. A dual-sided mosaic can be compared to a woven fabric, when the wrong (back) side is a mirror image of the front side, and the colors are replaced with inverse: respectively, white – to yellow, red – to orange, blue – to green.

Following the "fabric" metaphor, I sometimes create double-sided mosaics using threads. I secure the cubes by weaving them into a grid of stretched threads, resulting in a fabric or a carpet made of cubes [10].

Color Patterns of Dual-Sided Mosaics

Pixel cube patterns may vary in the number of colors. On the side of the cube, there can be all nine elements (pieces, pixels) of the same color (full solved side), pixels of two, three, four, five, or all six colors of the Rubik's cube. When creating a mirror dual-sided inverse color (MDSI) mosaic, if there are pixels of two colors on one side of the cube (for example, white and red) then the opposite side of the cube will have yellow and orange pixels, since yellow is the inverse color for white, and orange for red (Fig. 1a). Please note that the red element on the front side, which is at the bottom right, on the opposite side will be displayed (as in an inverted mirror) in orange at the bottom left. The same principle works for dual-sided patterns for three or more colors (Figs. 1b and 1c). Examples of MDSI mosaics created by the author are presented in Figures 2 and 3.



(a) (b) (c)

Figure 1: *Examples of MDSI patterns (front and back sides):*
(a) two-color pattern, (b) tricolor pattern, (c) six-color pattern.

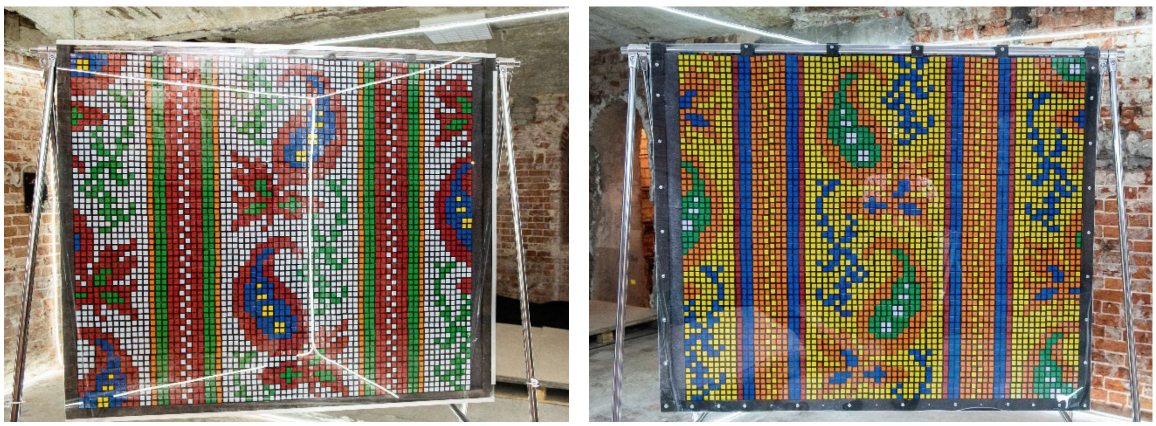


Figure 2: *Cubed Textile. Andrey Maslov. "CUBED" Exhibition. NIMLOFT Art-Centre. Ivanovo. 2021.*
Dual-sided mosaic out of 480 hand-twisted Rubik's cubes. 24x20 cubes in transparent frame.
Reference: Historical textile pattern "Chicago" produced by the Gandurin's factory
in Ivanovo-Voznesensk in 1893. [2]

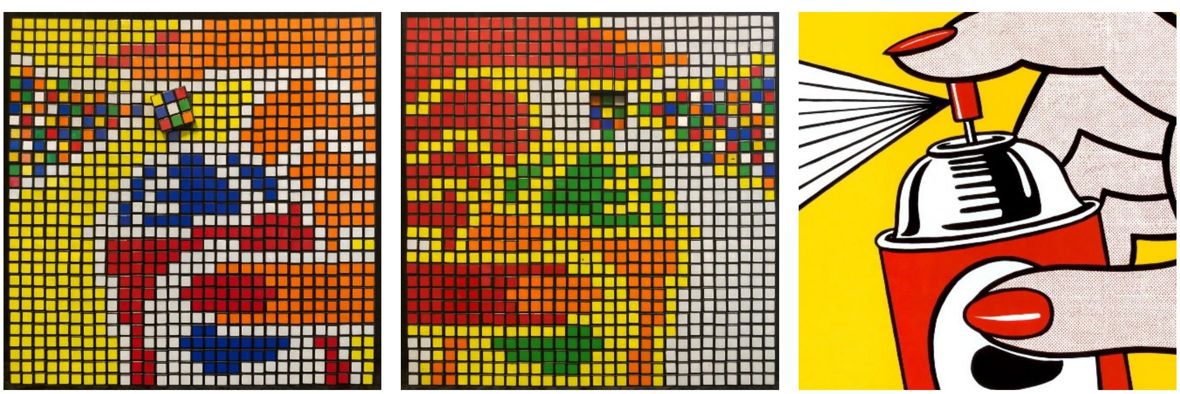


Figure 3: *Rubi-Spray. Andrey Maslov. "RUBICON" Exhibition. Levitan State Museum. Ples. 2023–2024.*
Dual-sided mosaic out of 100 hand-twisted Rubik's cubes. 10x10 cubes linked with a thread.
Reference: Roy Lichtenstein. Spray (1962). [5]

Method of Creating MDSI Pattern on the Rubik's Cube

The proposed method of creating MDSI mosaics assumes solving a pattern layer by layer (Fig. 4). It consists of four stages, each with several specially developed algorithms. The algorithms in this paper use traditional Rubik's Cube Move Notation [6, 7]. A list of developed algorithms is presented in [4].

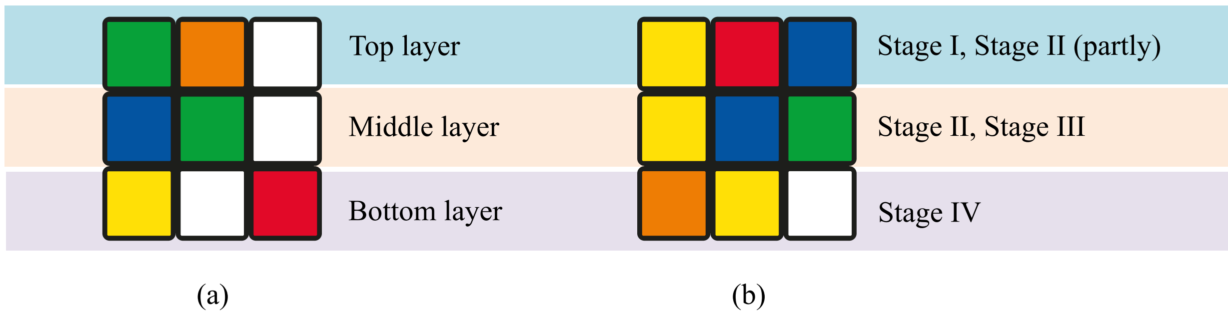


Figure 4: *Example of MDSI pattern to be solved by layers: a) front side, (b) back side.*

Stage I. Top layer

Before creating a pattern, the cube should be completely solved and have a classic color scheme: white opposite yellow, red opposite orange, blue opposite green. The solved cube should be placed so that the green side faces you and the white side faces up (classical position for scrambling in speedcubing). This is a correct position for solving any pattern. At this stage, we solve the top layer of the pattern – we set the necessary colors (of six available) on the three top slots (pixels) on the cube's front side. There are 216 possible patterns (6^3) on the top layer. So, there are 216 algorithms. However, we have reduced the number of algorithms for solving a pattern by half – to 108 ([4] List of algorithms. Stage I. Top layer). Each pattern is double-sided, and in the process of applying the algorithm, the pattern appears on both the front and back sides of the cube. Thus, one half of the 216 patterns in the top layer will be a mirror image of the other half.

The list of algorithms contains two columns of top layer patterns: 1) with a green, orange, or yellow element in the middle, and 2) with a blue, red or white element in the middle. The second column contains mirror patterns with inverse colors for linked patterns from the first column. For example, a pattern with orange-green-white combination in the first column will correspond to its mirror pattern – "yellow-blue-red" in the second column. Both these patterns are solved via the same algorithm with one exception. Correct solving a mirror pattern from the second column needs to apply a *U2* move. This move is not performed immediately after the completion of the algorithm, but *only after completion of Stage II*. So, the sequence of actions is as follows: take the cube with the green side in front of you and the white side up. Determine which algorithm should be used for the pattern and use it. If this is a mirror pattern with inverse color (from the second column), then proceed to the second stage, complete it, and make the *U2* move. It is important to note that if your pattern at the top layer has all green or all blue pixels, do not skip the algorithm corresponding to these situations. Otherwise, the further solving will be incorrect.

Several patterns at the first stage with combinations of corner colors *green and yellow or blue and white* need to perform a supplementary algorithm to create the pattern correctly. The list has 24 patterns and 12 algorithms marked with * and by color. These are cases that need to apply the supplementary algorithm *L D2 L2 D2 L2 D2 L*. Note that this supplementary algorithm is applied after completing Stage II. You need to proceed with the next algorithm at Stage II and, only after that, return to the supplementary algorithm. *U2* move (for cases where *U2* applicable) applies right after performing a supplementary algorithm.

Stage II. Middle layer. Edges

The second stage is solving the middle layer of the pattern, except the center. That means solving two edges on the middle layer using one of 36 algorithms from the second part of the list. Unlike the first and fourth stages, the second contains algorithms for all possible situations, so you don't have to perform any additional actions. Find the necessary algorithm on the list and complete it.

If your algorithm at Stage I was under *, use the supplementary algorithm *L D2 L2 D2 L2 D2 L*.

If you need to make the U2 move, do it now.

Stage III. Middle layer. Center

By default, the central piece of the pattern is colored green, since initially we took the cube with the green side facing us. If the center in the pattern is green, then the step is skipped, since the pattern is completely solved. If the center of the pattern is a different color, then use one of five algorithms from the third part of the list to set the desired color and complete the whole pattern.

Stage IV. Bottom layer

Select the required algorithm from the fourth part of the list and complete it. If the middle element of the bottom layer is red, white or blue, then, similar to the first stage, you need to make an additional move. This time use *D2*. However, it can be done right away without waiting for other stages to complete. Now, dual-sided pattern solving is complete. Check the front and back sides of the cube to make sure your solution is correct.

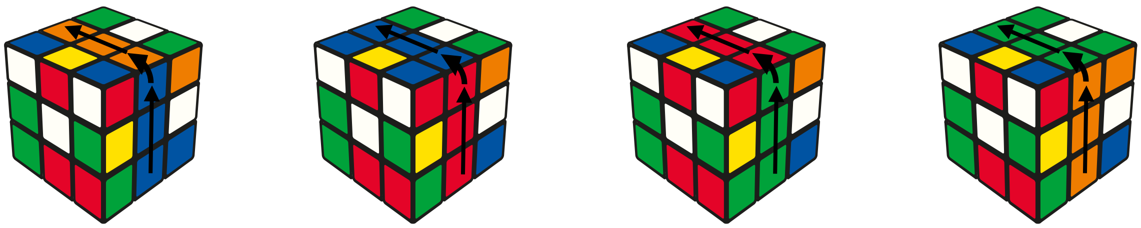
Exploring the Nature of the Rubik's Cube through Searching for MDSI Solving Methods

I started to develop a system of creating dual-sided mosaics with a method for creating simple patterns with pixels of two colors. There are 512 combinations of setting 2 colors into 9 boxes. I reduced this number to 69 because some patterns differ only by orientation of the cube. I also defined two groups of patterns: one is for neighboring colors (like green and white, 8 pairs in total) and one is for opposite colors (like yellow and white, 3 pairs in total). Thus, I developed two groups of 69 algorithms each. The guide was released by Rubik's company in 202. In 2023, this guideline was updated. Formulas were optimized by Tomas Rokicki via special software and Google supercomputer capacity [1]. That made the creation process of mosaics much faster.

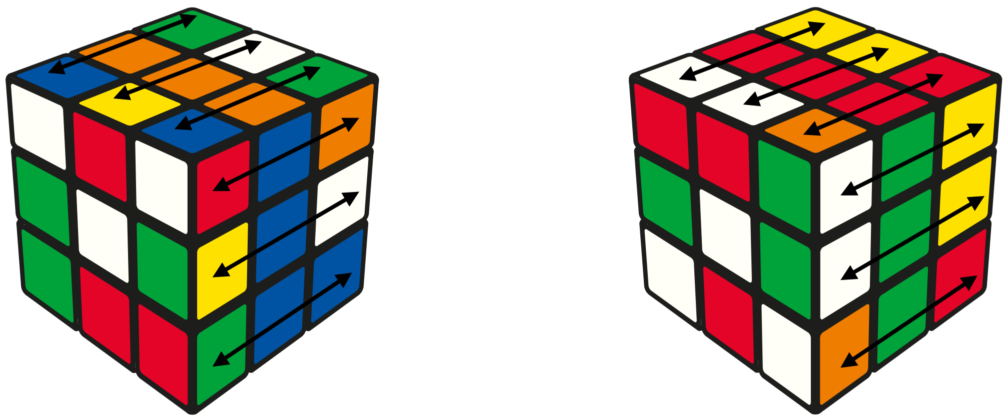
Working on two-color patterns, I understood that all combinations of two colors on the front side of the cube can be mirrored in two inverse colors on the back side of the cube and all existing MDSI patterns for two colors can be solved because the cube's nature allows it; we always have enough elements for both sides of the MDSI pattern to be placed correctly on opposite sides of the cube. The next step was to develop a method for building tricolor patterns. The same approach, as for two colors, did not suit because the number of combinations for 3 colors in 9 boxes is 19683. Even after combining similar patterns, the number of formulas will be too large, and further steps in this direction would be inappropriate. Then I developed a new approach, which was divided into three stages of solution: centers, corners and edges. I also identified two types of patterns: shared and non-shared corners.

My method for tricolor MDSI mosaics contains four groups of algorithms (231 in total) for all possible cases of tricolor patterns [3].

This work revealed the symmetry of the cube. All tricolor patterns in my system have strong symmetry of rotating elements: corners and edges. For example, white-blue-red corner will always be the opposite yellow-green-orange corner, or the yellow-red edge will always be the opposite white-orange edge. I considered the middle layer between opposite patterns as a "mirror" which reflects patterns (Fig. 5).



(a)



(b)

Figure 5: *Middle layer – "mirror": (a) Rotation of the "mirror", (b) Inverse color pairs of pixel elements: the cube in its original position (left) and the same cube flipped 180° (right).*

First, I tried to extend my method to 4-color patterns, and then to 5- and 6-color patterns. But further investigation showed that it would not make practical sense, since the number of possible cases would be too large. At the beginning I was not sure that any full-color pattern could be converted into MDSI. But exploring the cube deeper and deeper I realized that the cube nature allows it. Each element has its opposite pair in inverse colors. The only question was whether the cube mechanics could be enough to place all elements correctly. And if so, is it possible to do it not only intuitively but systematically and make the solving process understandable for MDSI mosaic creators? In the end I came out to the general system for any pattern that allows any one-sided mosaic to be turned into a dual-sided one.

I understood that my middle "mirror" layer worked only for patterns with no more than three colors. More colorful patterns sometimes need middle-layer edges to be involved in the solving process (for tricolor patterns, 4 pairs of opposite edges were enough). Thus, I needed to invent a different system that allows me to solve full MDSI patterns.

I made the assumption that the pattern can be solved layer-by-layer. I focused on edges, because corners can be replaced without destroying a pattern. I decided to mark edges according to which pieces would be permuted into the three layers of the pixel patterns. The top layer has one edge. Thus, the three edges that are reserved for the top layer are white-green, yellow-red, and orange-blue. These elements are marked with the letter "T" (Fig. 6). The middle layer has two edges and needs 6 edge elements to provide all possible variants of colors: white-blue, white-red, red-blue, yellow-green, yellow-orange, and orange-green. These elements are marked with the letter "M". The bottom layer has one edge, which means we need three edge elements for solving: white-orange, yellow-blue, and yellow-green. Possible bottom edges are marked with the letter "B". The Cube color scheme with marked edges is presented in Figure 6.

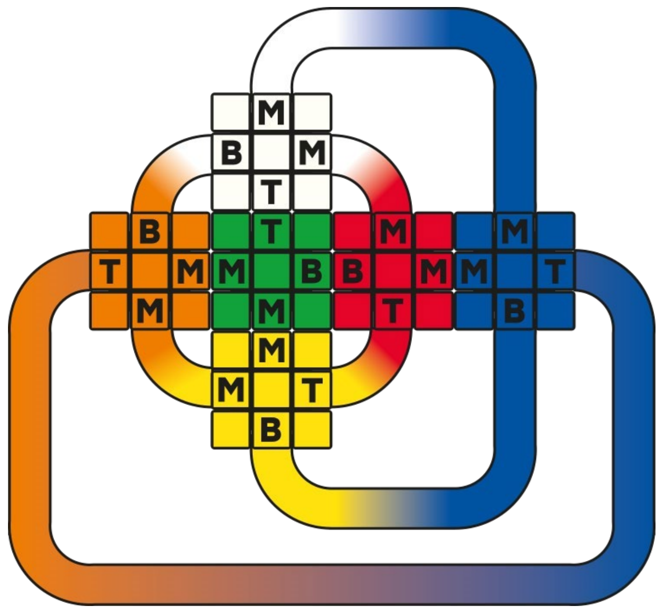
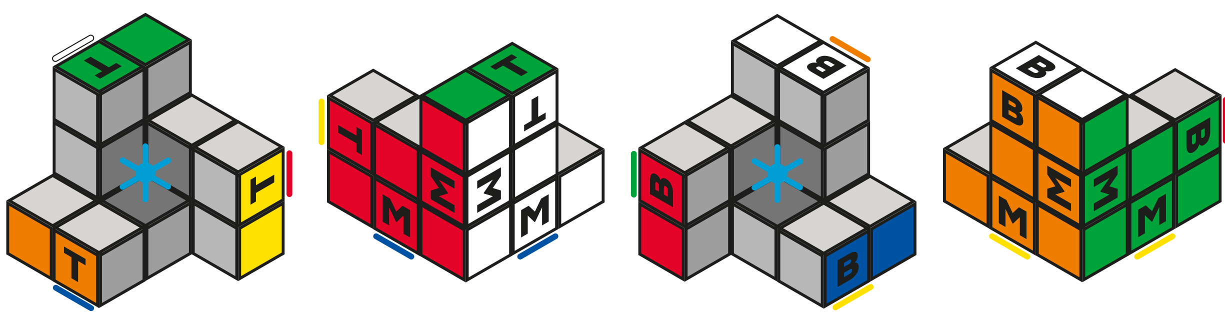


Figure 6: *Cube color scheme with letter-marked edges.*

It is difficult to discern the symmetry in this scheme as represented in Figure 6. But there is a symmetry to the group of edge pieces reserved for each layer. We can virtually divide the cube into two symmetrical parts (Fig. 7). Part 1 contains all 3 T-edges (Fig. 7a) and 3 of 6 M-edges (Fig. 7b). Part 2 – all 3 B-edges (Fig. 7c) and left 3 of 6 M-edges (Fig. 7d).



a) Front side   b) Back side   c) Front side   d) Back side

Cube Part 1. Views from opposite sides   Cube Part 2. Views from opposite sides

Figure 7: *Symmetry of the T-, M-, and B-edges of the cube.*

It should be noted that while working on algorithms, I used a specially scrambled cube as starting position (Fig. 8) – Algorithm I-1. All three T-edges are in the top level, all three B-edges – in the bottom level. Four M-edges are in the middle level, other two M-edges – one in the top, another in the bottom level. This scramble helped me to find solutions for every possible case.

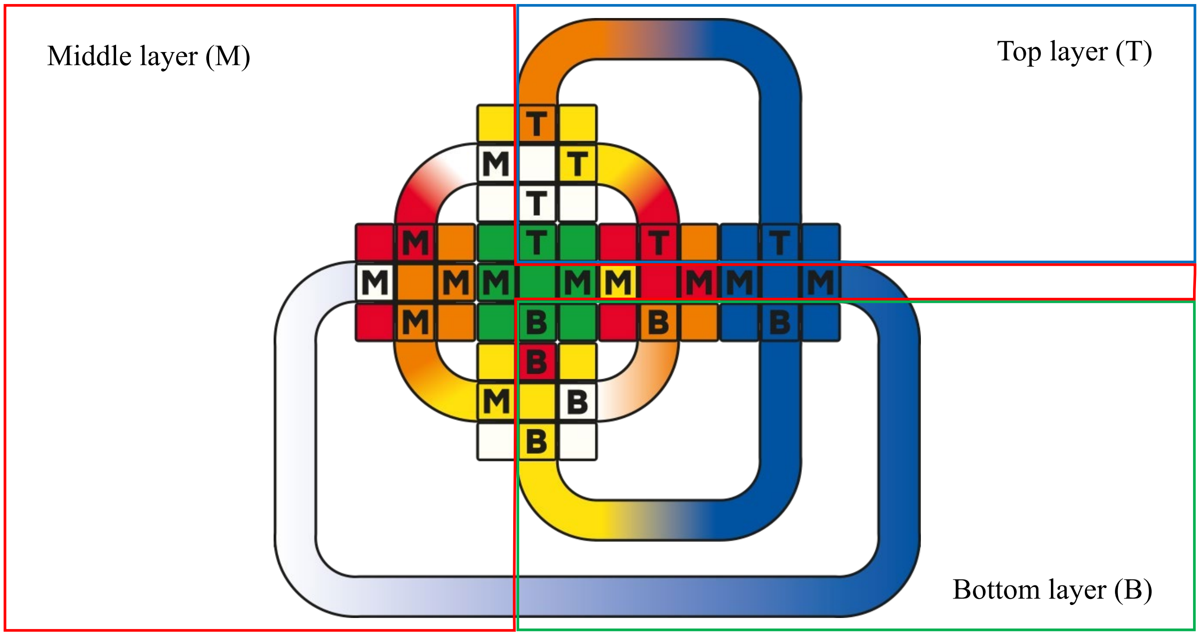


Figure 8: *Scramble of starting position of the cube.*

Conclusion

The proposed method of creating MDSI pixel images will push artistic boundaries for cube mosaic creators. It also gives new opportunities to explore the nature of the Rubik's cube. MDSI cube patterns are an example of a symmetry inside a 3x3x3 cube. Dual sided patterns group cube elements by inverse color and by its opposite places on a cube's sides. Further research of the MDSI principles may give rise to new developments in group theory and setting other math problems.

References

[1] A. Maslov and T. Rokicki. *You can create dual-sided Rubik's cube mosaics*, Ver. 2. [Optimised Formulas]. Rubik's Ltd., 2023 https://academicmaslow.com/wp-content/uploads/2023/11/rubiks_dual-sided-mosaic_guide_version-2_20112023.pdf

[2] A. Maslov. *CUBED Art Exhibition Catalog*, NIMLOFT. Ivanovo. 2021. https://academicmaslow.com/wp-content/uploads/2024/02/cubed-exhibition-catalogue-andrey-maslov-2021.pdf

[3] A. Maslov. *Dual-Sided Inverse Rubik's Cube Mosaics (Guidelines for tricolor pixel patterns)*. 2023.

[4] A. Maslov. *List of algorithms for creating MDSI mosaics*. 2024.https://bit.ly/MDSIMosaicAlgorithms

[5] A. Maslov. *RUBICON Art Exhibition Catalog*, Levitan State Museum. Plyos. 2024. https://academicmaslow.com/wp-content/uploads/2024/03/rubicon-catalogue-2024_web.pdf

[6] D. Singmaster *Notes on Rubik's Magic Cube*, London: Enslow Publ., 1981.

[7] D. Wang. J Perm. *Rubik's Cube Move Notation*. 2025. https://jperm.net/3x3/moves

[8] E. Rubik. *Cubed – The Puzzle of Us All*, Flatiron Books. 2020 https://academicmaslow.com/wp-content/uploads/2024/01/andrey-maslov_dual-sided-mosaic_tricolor-patterns-eng.pdf

[9] Invader. *Rubikcubism*. 2025. https://space-invaders.com/post/rubikcubism

[10] Sheddok, A. Maslov. *Largest dual sided rotating puzzle cube mosaic (pattern)*. Guinness World Records. 2021. https://www.guinnessworldrecords.com/world-records/643635-largest-dual-sided-rotating-puzzle-cube-mosaic-pattern