Genuine Pretending: A Philosophy for Mathematics and Art

Edmund Harriss

Department of Mathematical Sciences and School of Art,
University of Arkansas, USA; eharriss@uark.edu

Abstract

One of the challenges of doing mathematical art is that it seems to fit into a societal blind spot, the popular perception of both subjects appearing to have little intersection. Bridges participants know this is not at all correct, but how did it come about? More importantly how can understanding that help us all to make better mathematical art? In this talk I will discuss joint work with the late Roger Antonssen, applying the notion of Genuine Pretending (based on the work of Hans Georg Moeller) to mathematical art. I will present several examples of my own work to bring these theoretical considerations into practice, in particular how ideas from differential geometry can be both used to control digital machines to make art, and be explored themselves as the content of artwork.

Introduction

Interdisciplinary work can often seem to occur in the space between disciplines rather than in their intersection. That is especially true of mathematical art, which to many seems to sit not just between distinct disciplines, but quite distant ones. Relating the work to either math or art can often feel like a challenge, as discussed by the Illustrating Mathematics Community [1] and by George Hart [7] in recent papers in Notices of the American Mathematical Society.

In this paper I hope to delve into aspects of philosophy and culture to consider a possible approach to this challenge. The underlying motivational question is how I try to structure my thinking to make better mathematical art, in the hope that this will be of use to others.

The term "Better" here is used to take full advantage of its rather vague definition. It is very hard to come up with definitions of "good" art that takes in all the possibilities, but within a piece or specific collection of ideas it is often not hard for the artist themselves (or a critic) see how it can be improved. Yet the consideration of "better" lies at the heart of this paper. How do we reconcile the multiple approaches and even contradictory ideas that people have about what can make better art, mathematics, or mathematical art?

One note on language here, I use both the terms "mathematical art" and "mathart". When I use "mathematical art" I am refering to the broad space of possible work that brings ideas from mathematics into an artistic setting or vice versa. I use the term "mathart" to refer specifically to the work done at Bridges and related spaces, such as the art shows at the Joint Mathematics Meetings. "Mathart" is a particular (though broad) culture of "mathematical art".

Genuine Pretending

Many solutions proposed to the tension felt between math and art, especially by more mathematically inclined scholars, have tried to resolve things by building a unifying framework that encompasses both the approaches of mathematics and art. A good example of this is [2] which grounds both in the shared sense sense of aesthetics. Though these often provide fascinating frameworks to work in, they have never proved completly satisfying to me. The tension itself, the resolution of the tension, and the mathematical and artistic consideration on their own, all seemed to be separate worlds that revealed more by their communication than as part of a unified whole. A different approach is to instead build a new discipline of mathematical art, perhaps built around a community like Bridges. Yet creating new disciplines just spreads the problem out, with new interdisciplinary options lying between the new discipline and art on one side and mathematics on the other.

The Genuine Pretending approach leaves the disciplines as they are and instead considers the individual worlds they represent. The notion of Pretending says that either framework can be taken on, and should be considered as if it is the only one. The competing notion of Genuine asks for awareness of the different worlds and aims to find the consistency between interpretations of the distinct disciplines, while also aware (and informed) by the differences.

This approach resolves a different issue, that even within disciplines there is not necessarily consistency. The question of the difficulty of defining art is almost a cliché. In mathematics there is a desire to find a unified approach. Yet, as an example, the way mathematicians consider mathematics in terms of proof and theorems, is quite different from school mathematics, often more centered on applying algorithms.

Even in research mathematics there are differences of approach between topologists, algebraists and analysts, although the strange way in which results in one field can sometimes prove results in another pushes back on these divisions. The goal of the field of category theory has much in common with Genuine Pretending as it aimed to make that process clearer, to build bridges between different ways of approaching mathematical problems, rather than to unify mathematics into a single approach. This is in explicit contrast to the previous goal of set theory that had tried to unify mathematics into one form.

We learn to accept the inconsistencies and multiplicities of these disciplines and in many cases manage to productively see and use them as coherent, distinct systems. Genuine Pretending allows our approach to work at multiple levels, rather than as a pure hierarchy. This becomes increasingly valuable when discussing interdisciplinary work, as thanks to the slightly hidden multiplicities within a discipline, there are many ways that two can be combined, depending on what is taken from each discipline involved.

Genuine Pretending acknowledges these potential differences, in this case that two people (I will call them "you and I") might have very different ideas on what constitutes mathematical art. Our ideas might even be contradictory to each other. Yet we do not stop there with a sort of "anything goes" freedom of opinion. Both your and my theories or approaches must be taken on seriously and robustly as if they are the only one. By taking your approach seriously in this way I might learn something directly, but also find things that are relevant to my system or approach.

This has the implication that we do not have to discard, and should even embrace the rich systems of understanding created by disciplines. Their cultural survival in many ways shows the power of their approaches. Yet the fact that no discipline became the unique approach in academia reveals our need for multiple approaches.

Similarly attempts to create universal systems give other ways to build connections between the ideas we employ. In part the work presented here came from reading [2] and trying to get clarity for myself on how, though it resonated well with much of my own thinking, there were specific points that left me uncomfortable. Thus the disagreement helped me to develop my own thoughts. Similarly we do not need to reject but can embrace and take the role and standards of specific communities, Bridges being a great example, alongside the associated notion of mathart.

To add a definition to this, as that is something appreciated in the sorts of mathematical worlds that appear at Bridges. The fundamental object here might be considered a *world*, or world of thought. This is a somewhat coherent system of thinking about a topic. Worlds share some of the properties of sets including intersection (creating a world in its own right) and being subsets (where one world exists within another). Union however is trickier as simply combining two worlds might violate the "somewhat coherent" property. Examples of worlds include academic disciplines and academia itself, the culture of individual countries, and often of regions within those countries, also corporate culture and the cultures of certain industries. At a smaller scale the thoughts and ideas of an individual (or at least their coherent sub-parts!). Worlds can also be created by a community coming together around a common goal and can be institutional or transitory. Much like the definition of "set", this definition should be taken more as an attempt to give a word to an idea than something completely clean.

Genuine Pretending comes from the Genuine and full adoption of the rules and norms of different worlds. Yet it keeps an awareness that other worlds exist and perhaps greater insight will come from seeing how they agree and disagree. It thus provides a space to think about the multiple worlds involved in something without violating the integrity of those worlds as individual spaces.

Genuine Pretending in Mathematical Art

A work of art can be seen as creating its own world. That world might sit comfortably or uncomfortably in various other worlds, and in this case the two significant ones are the worlds of mathematics and of art. A work of mathematical art should embrace how it might be considered in the world of mathematics and in the world of art, in the community of mathart and in other spaces.

A playful counterpoint to this serious approach is the understanding that these worlds, the individual pieces, the broader work of the artist, the disciplinary and cultural interpretations are all at some level Pretense. They should be taken on seriously but can also be put aside. An artwork might be considered stronger or weaker in distinct worlds, but those responses reveal only the content that can be shifted into those worlds from the equally valid world of the piece itself. A specific notion might only be partially captured, or a concrete idea interpreted with whimsy and beauty rather than precision. There should be a delight in how a work both succeeds and fails as it is observed and critiqued through different approaches.

In fact for a community like Bridges this grounding can be particularly valuable. The mix of research mathematicians, working artists, teachers, makers and many others that make up Bridges, make defining a distinct audience very hard. A framework that brings the strengths of all these approaches, and works through recognition and dialogue would help the community to stay vibrant. It is worth noting that this is not an attempt to redefine or even change Bridges, but to bring out some of the aspects of how as an organisation and community it is already successful.

Some Cultural and Mathematical History

These ideas do not come from nowhere, but from their own worlds. I am currently writing a longer version of these ideas which will go into more detail on the scholarly rather than the practical side. That work was joint with the much missed Roger Antonssen.

In what follows I will attempt to give some summary of the cultural trends and ideas that come into this approach, and also imply a need for it. Genuine Pretending comes from Hans-Georg Moeller and Paul J. D'Ambrosio's reading of the Zhuangzi (庄子) [10], one of the central books to Taoism. Their work also draws on a cultural history of the twentieth century. That proposes a shift away from a model of Sincerity, where the goal for a true life is to match one's inner life to the roles and descriptions one has in society. The western challenge to this, called Authenticity, states instead that a true life comes from the reverse, finding the inner self and bringing that into the world in roles and descriptions, a process of "finding ones true self". Sincerity can be identified with Confucianism in Chinese culture, so Taoism, as a reaction to that, has often been identified with Authenticity. Moeller and D'Ambrosio instead discuss how the reaction of Taoism is quite distinct, instead of reversing the relationship between roles and self it rejects the relationship, and the idea of inner self. They instead propose Genuine Pretending where everything is considered a role with some level of Pretense, and yet each role is taken on with deliberate purpose and a goal to ensure reasonable consistency between roles.

This is a cartoon of some complex ideas. I encourage reading Moeller and D'Ambrosio work. There is probably as much learning in how my cartoon is wrong as in how it resonates with the book. I do think though, that this story provides a rich space to consider mathematics and art, both broadly in the twentieth century and in their relationship in mathart, mathematical art, and artistic mathematics.

In many ways mathematics can be seen as the clearest example of the approach of Sincerity, and in contrast art can be seen as the embodiment of Authenticity. The cultural moment when these are seen as opposing forces thus pulls mathematics and art apart. This resonates through notions of objectivity vs subjectivity and the many battles around postmodernism. This framing leaves little room for an intersection, yet the Bridges community and others prove that there is rich space between the two.

In many ways this mirrors developments in the history of modern mathematics. The nineteenth century revealed the deep unity of the subject with notions of geometry, algebra and analysis/infinity all being brought to bear on the same problems [4]. The quest to put this all on firm foundations is well documented, and was faced with the challenge that a single contradiction would cause a foundational system to collapse. Gödel's theorems revealed that there would always be something a little arbitrary to the axioms of mathematics [5], yet there was significant agreement on set theory with Zermelo Frankel axioms. Within this framework, mathematical branches bloomed with many different concepts and approaches. Yet these approaches had the habit of connecting in deep ways, so a big problem in one area might be resolved by techniques that seemed to be built for a different purpose. Category theory (see for example [9] or [3]) was developed not to reunify the distinct fields but to give robust communication between them.

The Genuine Pretending approach to mathematics, art, and mathematical art mirrors this approach from category theory, looking as much at the relationships between ideas as the ideas themselves and how they are revealed in different disciplines, and the further relationships between those disciplines.

Examples/Exercises

The multiple levels of Genuine Pretending can be applied even at the level of a paper like this. The previous section for example was part of the story of the whole paper, but was also playing the role of showing that the ideas discussed here are not simply the thoughts of an individual, but are part of a broader collection of ideas related to different traditions, that are deep worlds in their own right.

The next section of this paper, also takes on several roles. One aspect is for me to show off my work. Related to that, is to try to demonstration my own artistic practice and thus justify my rather declarative pronouncements above. It is up to the individual reader to decide whether the quality of the work justifies considering my more philosophical ideas, or alternatively means they can easily be ignored.

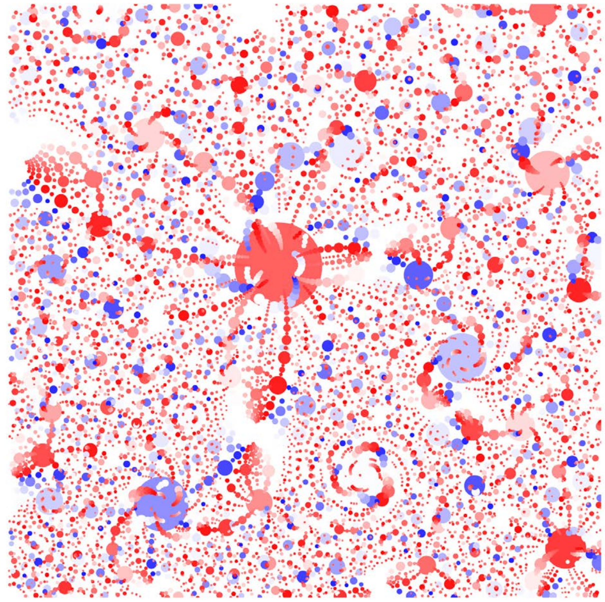
In each example I will also try to give some detail on the worlds and relationships I was thinking about in creating the work, but if the Genuine Pretending approach is compelling, they hopefully also provide exercises to see how aspects of each work resonates with your own conceptions and worlds around mathematical art. In particular I have been very deliberate in my use of the word "world" below to denote the meaning defined above. It should be noted that much of the world of my work is somewhat abstract, creating meta worlds that themselves work as bridges between worlds.

Example: Illustrating Mathematics

Illustrating Mathematics is a community that I am active in, that grew out of community building (the intersection with Bridges is significant) and a semester program at ICERM (a mathematics research institute, and part of Brown University) in 2019. The term was chosen explicitly to go beyond the notion of mathematical visualistaion and imagery to take in the broadest notion of the term "illustration".

The goal of Illustrating Mathematics is to make images, objects and experiences that accurately reflect

(a)



(b)

Figure 1: *Examples of mathematical illustration. (a) A wood and brass model of the Barth Sextic, joint work with Silviana Amethyst. Made using 5-axis CNC (8x8x8 inches). (b) A patch of an algebraic starscape, the complex algebraic numbers close to a complex root of* $x^3 - x - 1 = 0$.

mathematical ideas. It can be thought of as a dual to mathematical modeling. Both create a world of their own from the relationship between the physical world and the abstract world. The distinction comes in which is considered primary. In modeling, if there is disagreement it is the ideas in the abstract world that must be considered and improved. In contrast, in Illustrating Mathematics, disagreement must be remedied or explained by considering the object or experience in the physical world.

For both there is a clear importance to the clarity and reliability of the map between the physical and the abstract. A couple of specific examples can hopefully show what this means. Figure 1(a) shows a model of the Barth Sextic (joint work with Silviana Amethyst). This model is a 3D form whose surface consists of all solutions to a polynomial. It can be 3D-printed, but this model is machined in wood, with the toolpaths for the machine computed directly from the algebra defining the form. That abstract world of algebraic surfaces is thus linked as directly as possible to the precise and limited world of a five axis CNC machine.

Building clear and readable relationships between a mathematical world and illustrations means that they can be used, not just to communicate mathematics that is already understood, but even to reveal structure that is not known to mathematicians. In other words they can be a powerful tool in mathematics research. An example of this are algebraic starscapes (with Steve Trettel and Kate Stange) [6], such as Figure 1(b) which shows complex cubic algebraic numbers. Some structure is revealed that no one can yet describe precisely.

Example: Curvahedra and Zipform

Having close relationships between abstract worlds and the physical world can have other benefits, even the creation of new manufacturing techniques. The Zipform technique uses ideas from differential geometry including the framing of curves (where a coordinate frame is placed at every point along a curve, so that one of its coordinate directions is the curve tangent) and developable surfaces to create T-beams along any three dimensional curve. This was developed to create the Curvahedra sculpture in Gearhart courtyard at the University of Arkansas (Figure 2), a joint work with Emily Baker.

Figure 2: *The David and Jane B. Gearhart Curvahedra sculpture in the courtyard of Gearhart Hall, outside the Honors college at the university of Arkansas. Joint work with Emily Baker. (8' sphere)*

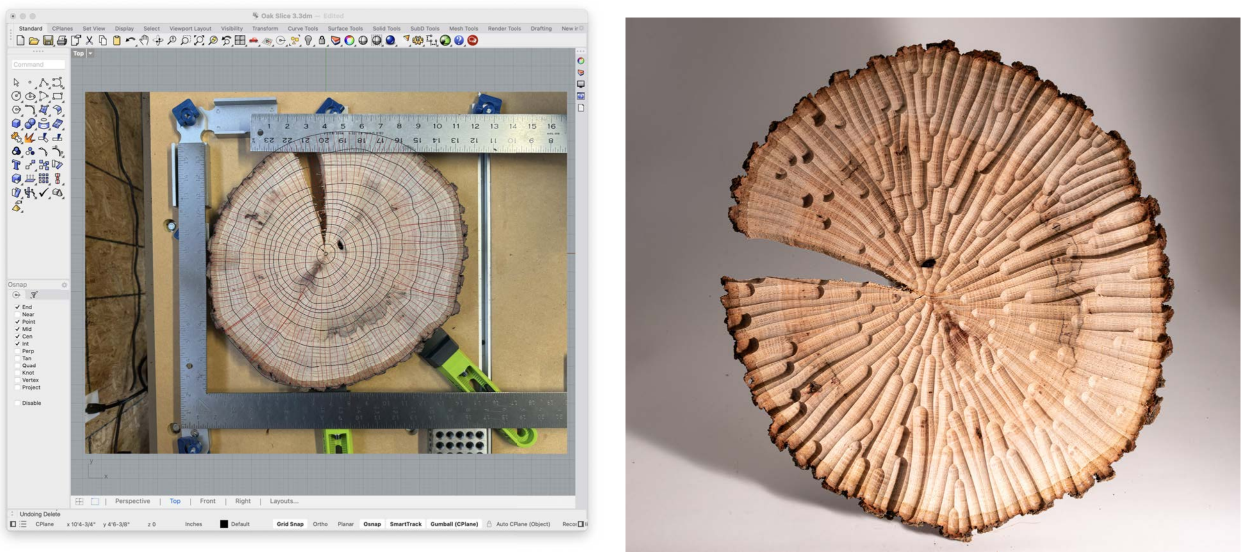
Figure 3: *The lighthouse for the Mathemalchemy exhibit. Joint work with Emily Baker and Sabetta Matsumoto. (28x28x84 inches)*

These techniques required close consideration of the world of differential geometry and also the world of steel construction, that my colleague Emily was able to bring. The effectiveness of the technique comes from discovering a clear point, where the two worlds grew close. We could map curvature in the mathematical domain both to curves cut in flat steel and the way a steel sheet can be bent. The assumption that a flat sheet of steel can effectively be modeled (and provides an illustration of) developable surfaces was key to this approach.

By working in the abstract mathematical world we could then ask what else might be possible with this technique and the robust bridge allowed attempts to actually construct the results. Beyond the initial work (and joined by Sabetta Matsumoto) we were able to create a very differently shaped surface (Figure 3), the lighthouse for the Mathemalchemy project [11].

The same techniques were then used to create formwork for optimised concrete beams, going from creating mathematical art to an engineering application [8].

Example: Gradient of Grain



(a) *Toolpaths generated and registered onto CNC.* **(b)** *Final Piece*

Figure 4: *Gradient of Grain piece 1 of 17. CNC carved slice of oak with every path staying at right angles to the grain lines. (15x15x2 inches)*

In much of my work, wood provides a great medium to create geometry, the natural pattern of the grain adds just enough to geometric forms so they do not feel cold or purely formal. I wanted to take this further, to work with the world of the wood itself. The natural grain pattern provided an excellent source of geometry to use, bringing the wood into the abstract world.

This idea was realised in the series Gradient of Grain (Figure 4). To make this, a slice of oak was placed into a CNC and photographed. The CNC or Computer numerical control machine is a router attached to three linear axes, capable of moving it to any point in a region. It thus provides a way to move the router precisely along any path you can draw. The image was brought into software, registered to scale, and the grain lines drawn on top. Then an algorithm created paths expanding out from the center staying at right angles to the grain. The photograph and resulting paths are shown in Figure 4(a).

In this case we bring in the world of the tree, as recorded in the grain lines. The process itself is part of the work, as the grain lines are converted into paths. The manufacturing method is also considered as wood cuts cleanest at right angles to the grain. This allows the piece to have dramatic vertical changes between the individual cut paths without splintering the wood.

Conclusion

Mathematical art should draw deeply on both of its disciplines, but can also live comfortably in a spaces between them, in worlds that contain them both and in the relationships between those worlds and ideas. Genuine Pretending gives a way to embrace the potential of these various approaches and ideas, without sacrificing the powerful thinking that occurs within more established worlds of thought. At its best it can use the methods of art to reveal more about and better communicate mathematical ideas, while also bringing in deep mathematical thought as a tool to inspire artists.

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