A Math Puzzle Art of HANAGRAM

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Abstract

This paper presents three facets of HANAGRAM: Math Puzzle, Art Puzzle, and Puzzle Art. HANAGRAM, invented by the author in 2022, offers a unique combination of logical thinking and art thinking. Logical thinking puzzles involve Number-Fill-in puzzles, akin to the popular Sudoku. Art thinking puzzles, on the other hand, focus on image appreciation by interpreting and naming the figures created through coloring. In this paper, I will share the journey of how I created HANAGRAM and developed it from a puzzle into a form of art, from the perspective of a puzzle creator.

Introduction

Are you familiar with the Flower of Life? In Japan, it is recognized as part of sacred geometry patterns. HANAGRAM [3] is a puzzle inspired by the Flower of Life. It features a flower-shaped base of 42 equilateral triangles that are filled with the numbers from 0 to 9, following the rule of no duplications in rows. Colors are then added to create figures that inspire imaginative images. Invented by the author in 2022, HANAGRAM combines the logical thinking of number-fill-in puzzles with the creative exploration of image appreciation. Stimulate both your left and right brain with this puzzle and unleash your creativity! Figure 1 illustrates the overall process of playing HANAGRAM as both a Math Puzzle and an Art Puzzle.

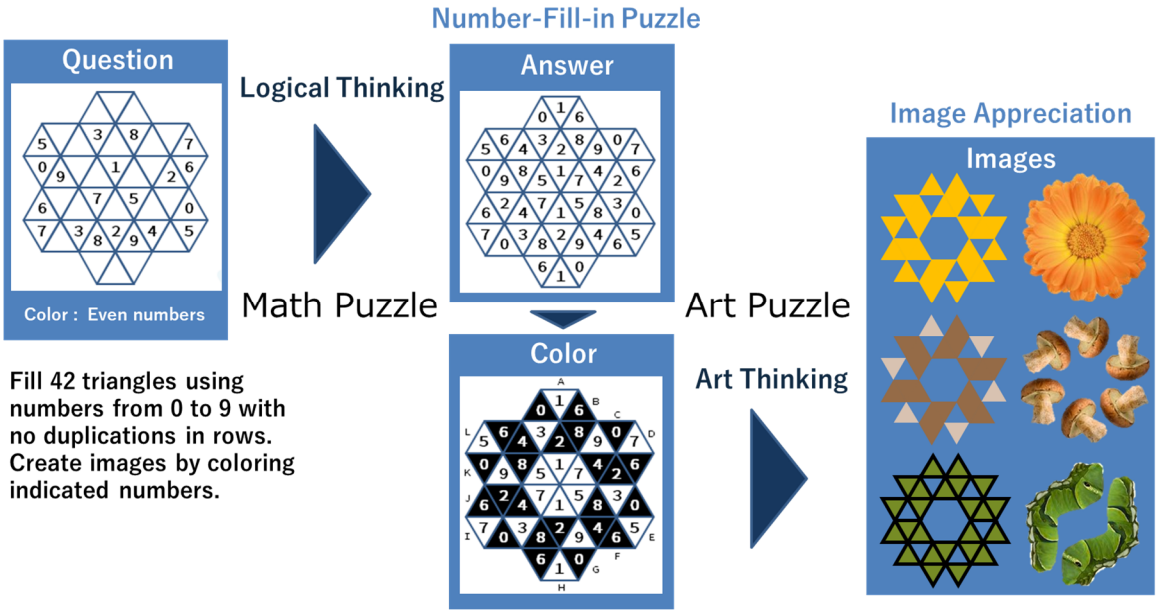


Figure 1: *HANAGRAM - Logical Thinking and Art Thinking Puzzles on Flower of Life.*

HANAGRAM is composed of just three elements: the base figure, which consists of flower-shaped 42 equilateral triangles; the numbers from 0 to 9; and the indications for coloring by numbers. Yet, the world that can be expressed through HANAGRAM is astonishingly vast. The possibilities for image appreciation are limitless, shaped by the player's experience and imagination. It is truly fascinating to see various creatures emerge from the original Flower of Life figure. Let's explore the puzzles in Figure 2!

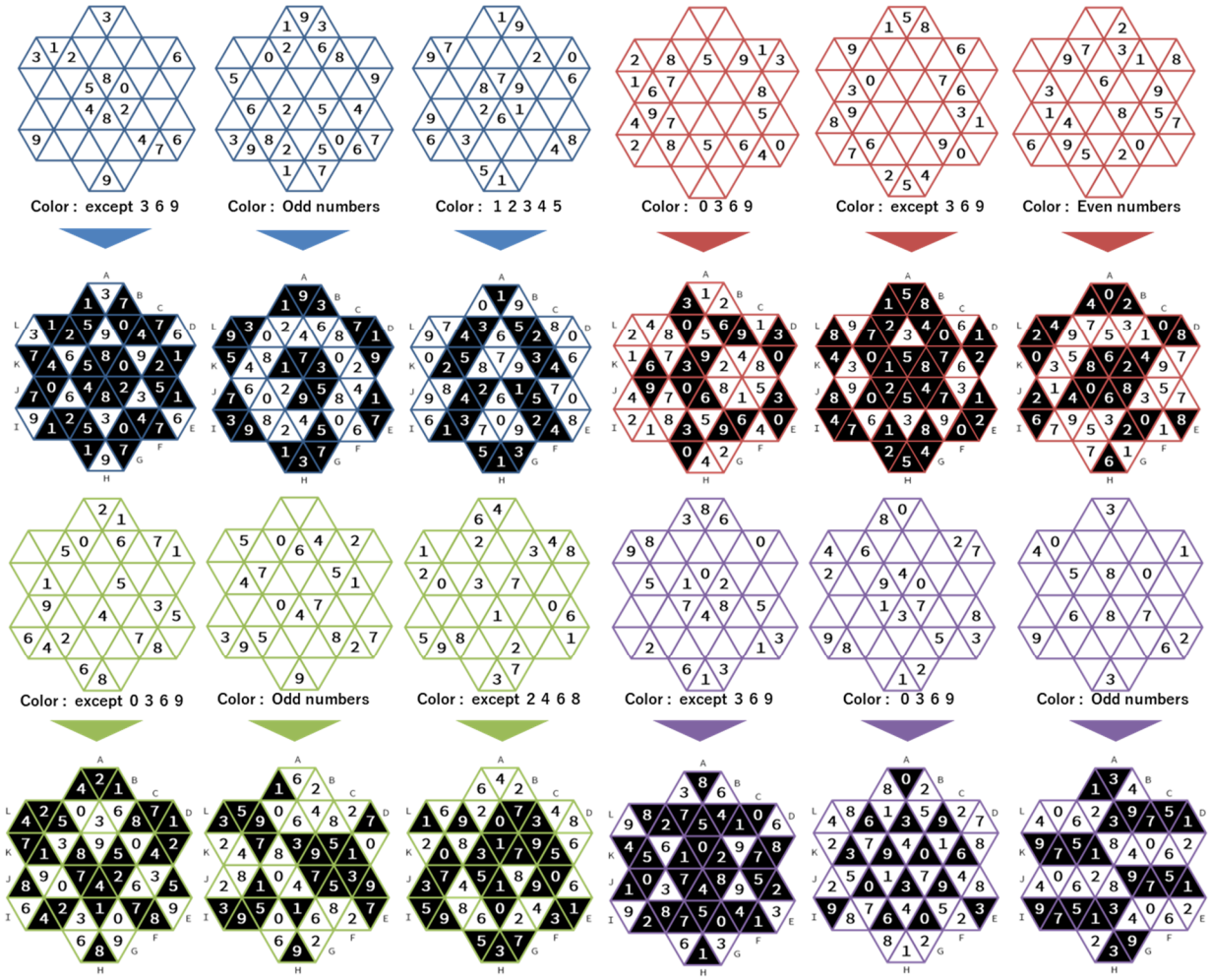


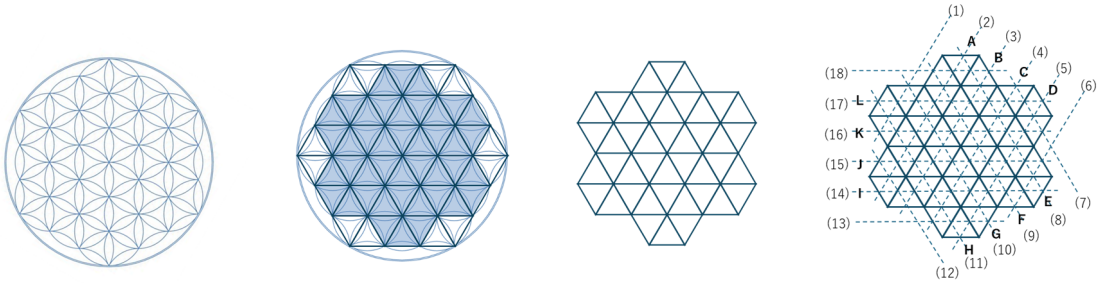
Figure 2: *Examples of Questions and Answers of HANAGRAM [2]. Let's try them!*

How HANAGRAM Was Invented

The creation of HANAGRAM was a journey of trial and error.

The Birth of Logical Thinking Puzzles: The Math Puzzle component of HANAGRAM

HANAGRAM draws inspiration from the flower shape derived from the Flower of Life. The Flower of Life, shown in Figure 3(a), is composed of circles and arcs. By connecting the intersections of these arcs, you can create a regular hexagon consisting of 54 equilateral triangles, as depicted in Figure 3(b). Initially, I aimed to fill these 54 equilateral triangles with numbers to create a Sudoku-like puzzle. However, it became evident that the longest row of the equilateral triangle sequence in this figure contains 11 equilateral triangles, which cannot be filled using only the 10 digits from 0 to 9. After much deliberation, I designed the base figure of HANAGRAM, shown in Figure 3(c). This new figure consists of 42 equilateral triangles, formed by removing the corners of the regular hexagon, with the longest row containing 9 equilateral triangles. I first attempted to solve the puzzle using the 9 digits from 1 to 9, like Sudoku, by arranging them in the three-way sequence of rows to avoid overlapping numbers. However, after exploring various approaches, the puzzle remained unsolved. As a result, I decided to use the 10 digits from 0 to 9, giving birth to the Logical Thinking Puzzle.



(a) (b) (c) (d)

Figure 3: *Extracting the Base Figure from the Flower of Life(FOL): (a) FOL, (b) Base figure in FOL rotated by 30°, (c) Base Figure, (d) 18 rows and 12 lines(A-L).*

The Birth of Art Thinking Puzzles: The Art Puzzle component of HANAGRAM

While creating various patterns, I discovered that combining symmetry and regularity in the arrangement of numbers produced more solutions than I had anticipated. When these patterns were colored, they often resembled floral designs. I named this creation HANAGRAM, with 'HANA' being the Japanese word for 'flower' and 'GRAM' derived from the English suffix meaning 'something drawn.' Inspired by the Chinese puzzle TANGRAM, HANAGRAM can also be enjoyed as a silhouette puzzle, much like TANGRAM.

Moreover, the act of coloring sparked my creativity. I realized that the 42 equilateral triangles could be used to depict not only flower patterns but also a wide range of designs, such as animals, sea creatures, geometric shapes, cultural motifs, and anything else my imagination conjured. I also found artistic value in contemplating what the colored figure resembled, evoking a sense of art appreciation. This is how the Art Thinking Puzzle was born.

A Math Puzzle of HANAGRAM

Regarding the composition of HANAGRAM, it is made up of just three elements: the base figure formed by 42 equilateral triangles, the numbers from 0 to 9, and the indications of which numbers should be colored. Like Sudoku, the puzzles are presented by filling some of the numbers into the 42 equilateral triangles. Each puzzle must be designed to have a unique solution at the final configuration. To enhance the visual appeal of the puzzle, the numbers initially provided are arranged with a degree of symmetry. The more numbers displayed at the start, the easier the puzzle becomes - not only because of the additional clues but also because fewer steps are needed to complete it.

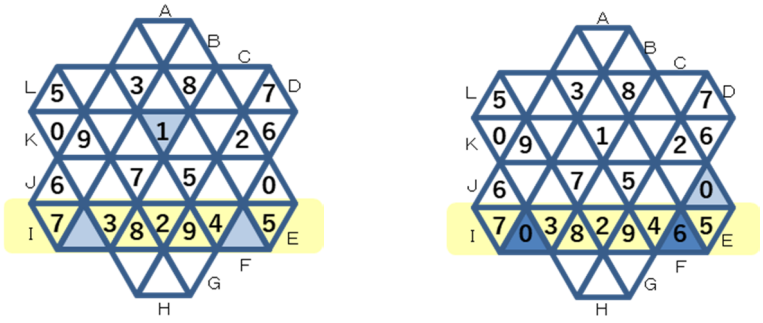
Basic instructions of HANAGRAM are as follows.

- Fill 42 triangles using numbers from 0 to 9 with no duplications in rows.
- Create images by coloring indicated numbers.

The rows mentioned here refer to the 18 rows illustrated in Figure 3(d). Additionally, Figure 3(d) defines 12 lines, labeled A through L, which are used to explain the solution of HANAGRAM. These lines are categorized as follows: ABCD lines run from top-right to bottom-left, EFGH lines run from bottom-right to top-left, and IJKL lines run horizontally from left to right. Using these lines, the 42 equilateral triangles can be identified at the intersections of two or three lines. For instance, an equilateral triangle can be located as the intersection of lines such as AE or AFL. Examples of these lines are highlighted in yellow in Figures 4 and 5. Each line consists of 9 equilateral triangles. According to the basic instructions, 'using numbers from 0 to 9 with no duplications in rows,' each line is filled with 9 unique digits. This means that one out of the 10 digits (0 to 9) cannot be used in the line. This reasoning forms the basis of Solving Technique 1.

How to solve HANAGRAM as a Math Puzzle

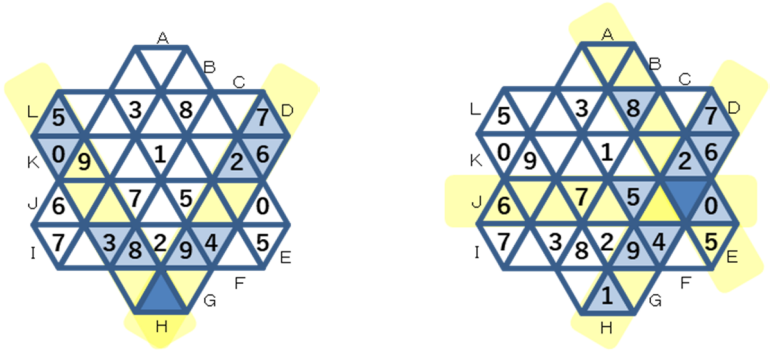
Solving Technique 1) Focus on each line from A to L and find the number that does not fit in the line. Once you've found the number that doesn't fit in the line, you can start filling the line with other numbers. This is the basic strategy of HANAGRAM solution. Figure 4 shows an example of this technique.



(a) (b)

Figure 4: *Solving Technique 1): (a)Focus on line I. From BFK1, 1 does not fit in line I. (b)Since 0 does not fit in FI from EJ0, BI0 is confirmed, and FI6 can be confirmed as 6 is the last number fixed in line I.*

Solving Technique 2) Focus on one equilateral triangle, look at the 2 or 3 lines containing that equilateral triangle, and if you find 9 types of numbers between 0 and 9, you can be sure that the remaining one number is in the equilateral triangle. Figure 5 shows two examples of this technique.



(a) (b)

Figure 5: *Solving Technique 2): (a)Focusing on lines D and H, there are 9 types of numbers other than 1, so DH1 is confirmed. (b)Focusing on lines D, E, and J, there are 9 types of numbers other than 3, so DEJ3 is confirmed.*

Notation Method of HANAGRAM is developed to describe the solution of HANAGRAM. See the solutions in Figure 8. Examples of use are as follows.

I1× 06: 1 dose not fit in line I. 0 & 6 are fixed in line I.

DH1: 1 is fixed at the intersection of line D and line H.

DH1×: 1 dose not fit in at the intersection of line D and line H.

BFL9: 9 is fixed at the intersection of line B, line F, and line L.

(BHI/EI) 9: 9 is in either BHI or EI.

An Art Puzzle of HANAGRAM

HANAGRAM is a puzzle that utilizes universal tools such as numbers and shapes. By expressing these tools through image appreciation, the puzzle fosters connections with diverse cultures and broadens the worldview of HANAGRAM - much like colorful flowers blooming across the globe. Different cultures assign unique names to the same concepts, reflecting their distinct perspectives. Exploring these cultural differences through imagination adds an intriguing dimension to HANAGRAM. The puzzle strives to create a global experience that can be enjoyed by people everywhere.

How to enjoy HANAGRAM as an Art Puzzle

Image appreciation has no right or wrong answers, allowing for free and creative thinking. As demonstrated in Figure 6, a single configuration can produce numerous images simply by altering the colored numbers.

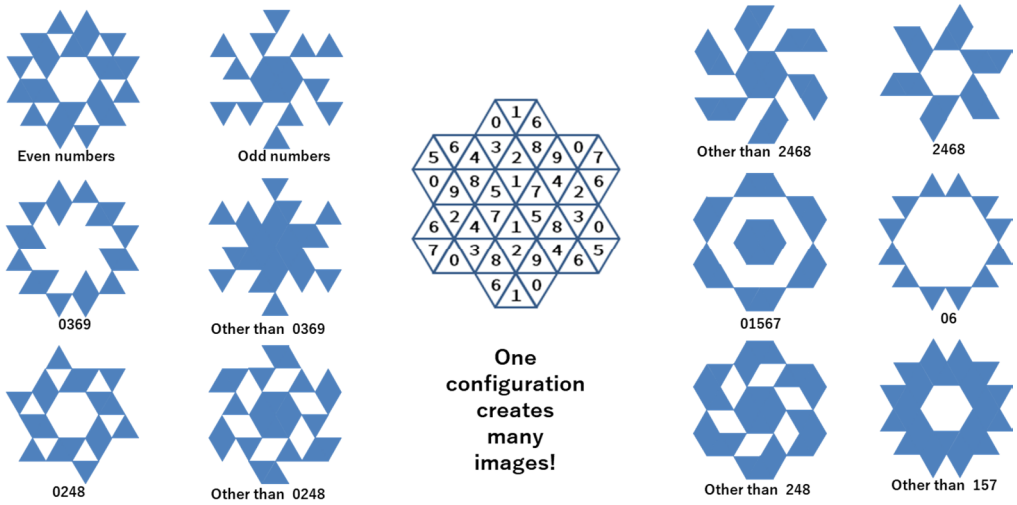


Figure 6: *Design Evolution by Colored Numbers [7].*
Just by changing the colored numbers, you can get different flower images!

Rotation reveals new images, and what you perceive will vary based on your knowledge and experiences. It would be fascinating to explore how cultural backgrounds influence interpretation worldwide. For example, without familiarity with Japanese culture, it would be difficult to identify the images in Figure 7 as a Sumo Wrestler, Omotenashi, or Hannya.

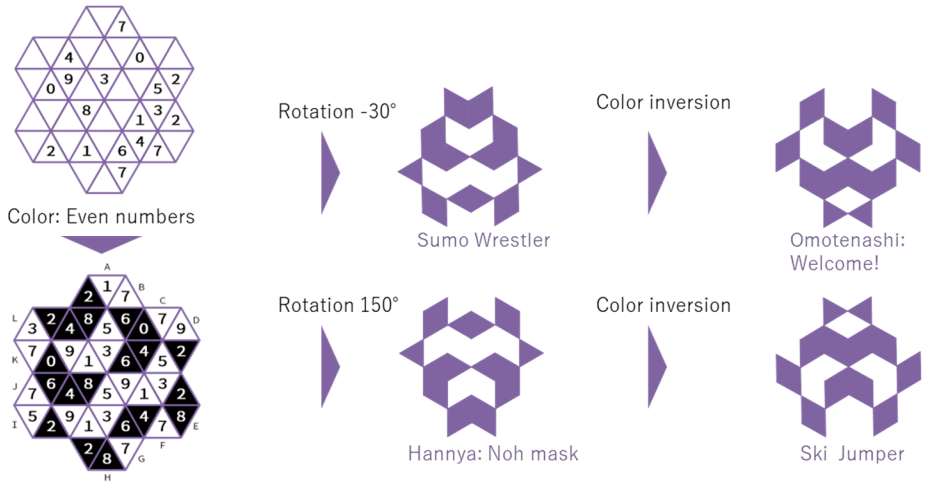


Figure 7: *Design Evolution by Image Rotation, Color Inversion, and Cultural Fusion [7].*

Image Appreciation

In image appreciation, vibrant coloring enhances the tangibility of images. Changing colors, incorporating multiple hues, or altering the color of the frame can bring new images to mind. Figure 1 showcases three distinct examples: a flower, mushrooms, and caterpillars. For a more experienced observer, these images might appear more concrete - such as one calendula flower, six shiitake mushrooms, and two swallowtail butterfly larvae. When viewing the images, let your imagination run free. Beyond simply observing, you can spark creativity by experimenting with techniques like inverting colors, adjusting color schemes, rotating, flipping, and more.

MIRU Puzzles

For those who find HANAGRAM puzzles challenging, I've created videos that allow viewers to not only solve Math Puzzles but also enjoy Art Puzzles - all in just three minutes. The word 'MIRU,' meaning 'watch' in Japanese, symbolizes the simple joy of observing and learning. While solving these puzzles typically takes around 30 minutes, many people give up halfway through. With my videos, however, you can experience the thrill of solving a puzzle seamlessly in less than three minutes. I hope these videos inspire people to try solving puzzles on their own. Figure 8 shows examples of MIRU Puzzles.

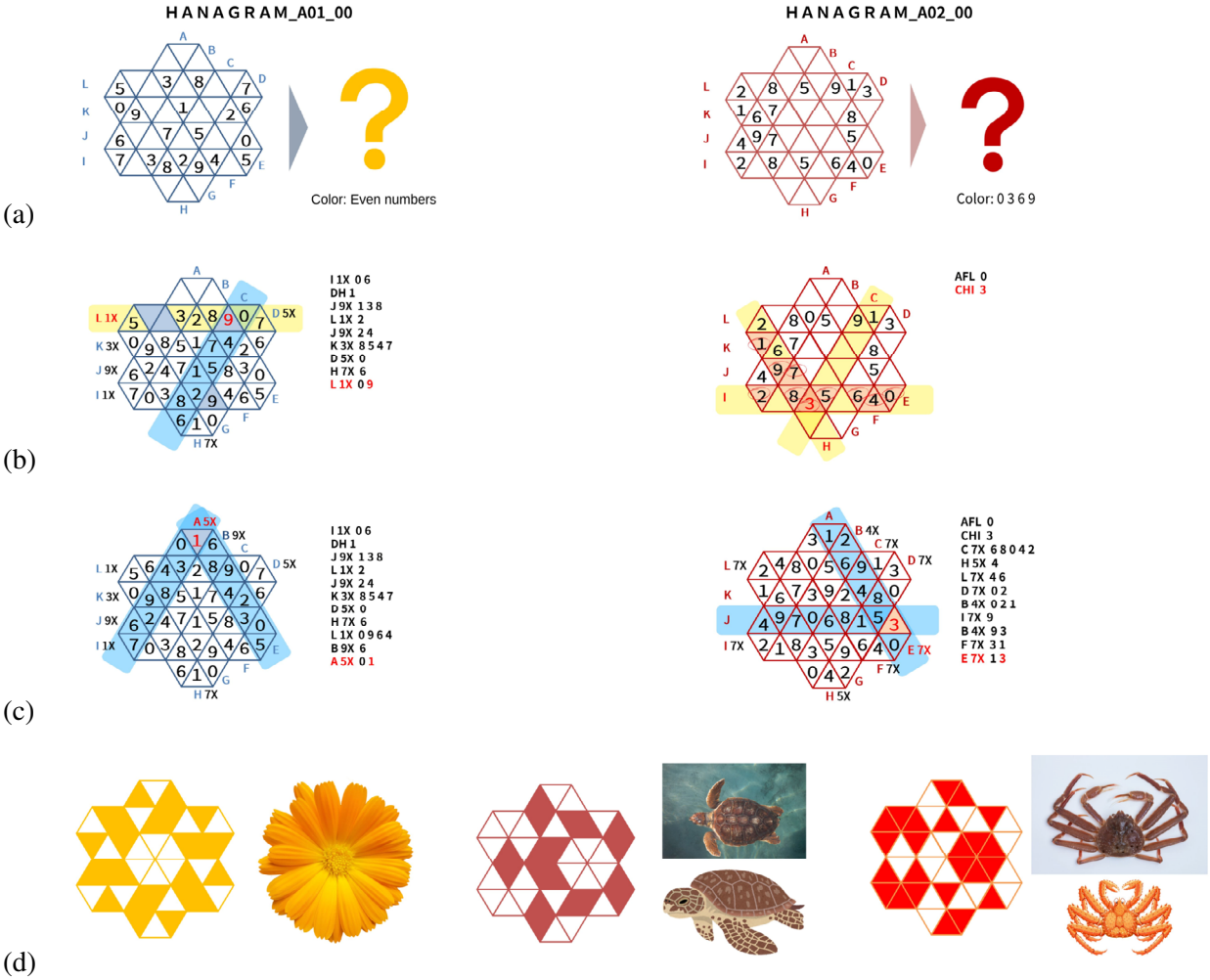


Figure 8: *MIRU Puzzles: 3-Minute HANAGRAM Solutions [6].*
(a) Question, (b)Solving Number-Fill-in Puzzle, (c)Solved, (d)Image Appreciation.

From Art Puzzle to Puzzle Art

The debate over whether puzzles can be considered art is ongoing [1]. While HANAGRAM is fundamentally a puzzle, it incorporates aspects of an art thinking puzzle. However, when HANAGRAM is fractally assembled into its component parts, as demonstrated in Figure 9, I believe it transcends its puzzle nature and becomes art. I call this form of expression 'Puzzle Art.' Figure 10 provides an example of Puzzle Art, where 42 HANAGRAM puzzles are combined as components. By altering the color scheme, the assembled puzzles can be interpreted in two distinct ways.

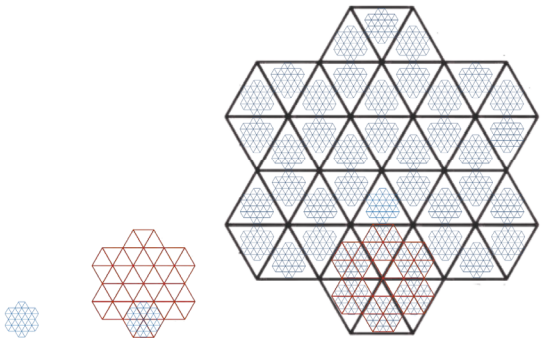


Figure 9: *Fractal Assembly of HANAGRAM Puzzles.*

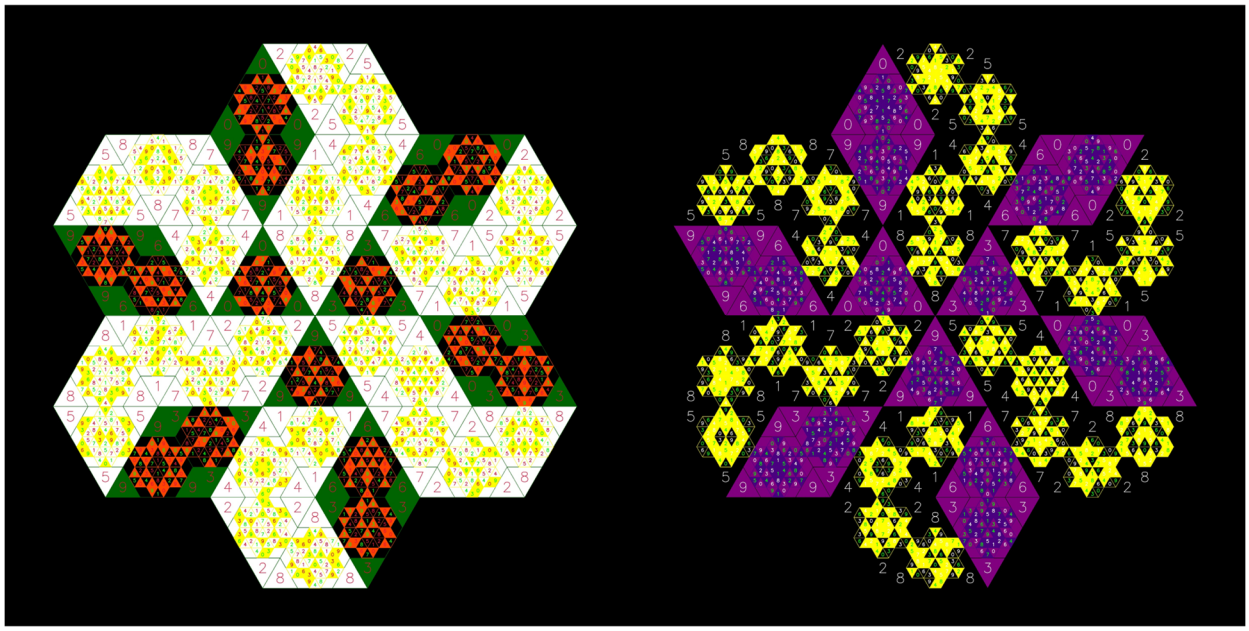


Figure 10: *Yamayuri and Clematis [5].*

The design of “Yamayuri” is characterized by six white petals with yellow stripes and red spots and six orange pollenated stamens. The three triangles in the center represent the head of a single pistil, and the dark green of the diamond shapes represent the leaves. This flower is the prefectural flower of Kanagawa Prefecture in Japan. Yamayuri is called Golden-banded Lily (Lilium auratum) in English.

The design of “Clematis” is a layered expression of time, representing the time of blooming and after blooming. The purple part represents the six petals in bloom. The yellow part represents the numerous pistils that expand and become feathery after blooming.

Japanese Zodiac Puzzles

I sought to incorporate an artistic element into HANAGRAM by creating a series of puzzles inspired by the Japanese zodiac. Figure 11 features the HANAGRAM Japanese Zodiac Puzzles showcased on TikTok. The series includes 12 puzzle movies, each representing a zodiac-sign animal, along with one summary movie.

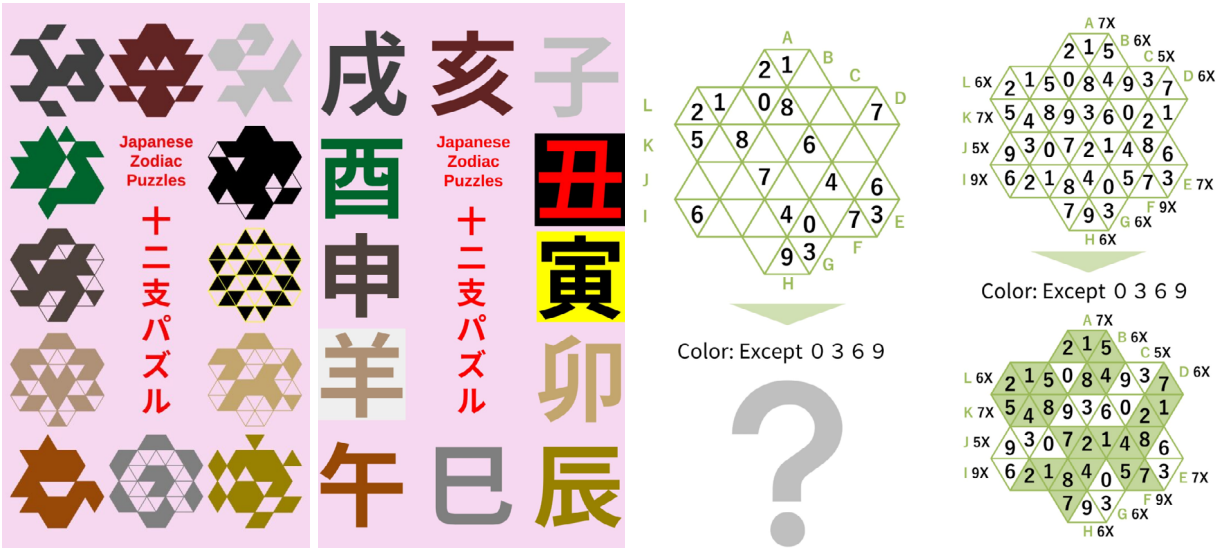


Figure 11: *HANAGRAM Japanese Zodiac Puzzles [4].*

Summary and Conclusions

This paper introduces the three facets of HANAGRAM: Math Puzzle, Art Puzzle, and Puzzle Art. It provides a glimpse into some of my HANAGRAM activities. To explore more of my work, please visit the HANAGRAM website [3], my books [2], my YouTube channel [6], and my TikTok channel [4]. I hope that, in the future, some math puzzles and art puzzles will be recognized as forms of Puzzle Art. As shown in the title of this paper, Puzzle bridges Math and Art of HANAGRAM.

References

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