Supplement to "Large Islamic Rosettes in an Octagonal Frame"

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Abstract

Additional figures and explanations that didn't fit in the regular paper.

Rosettes and Stars with Irregular Polygons

We want to draw rosettes or stars inside our polygons. This technique is well known for regular polygons. The way that we make it apply to irregular polygons is to make triangles of each edge to the center of gravity of the vertices. We do an affine map from regular to irregular for each triangular slice individually. They will automatically meet up correctly at the edges. (Figure 1)

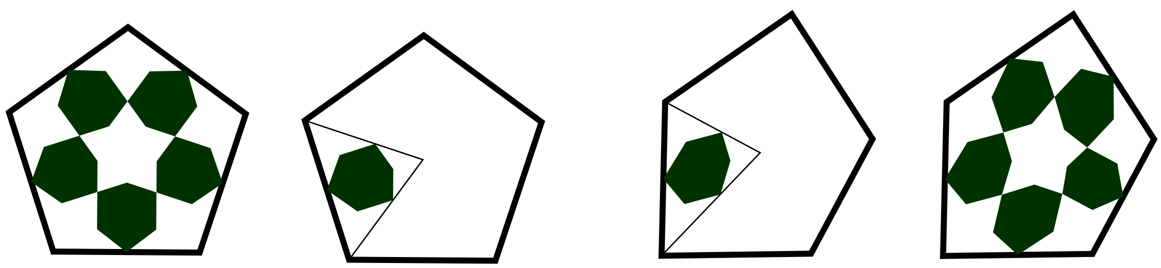

Figure 1: *Showing how a rosette from a regular polygon is mapped to an irregular polygon.*

This can also be done with stars instead of rosettes. Examples will be shown in the following pages.

One of the reviewers pointed out how similar the two types of border were and how both could be filled with triangles. Then the problem would reduce to how to find a triangulation that was as regular as possible. The one on the left of Figure 2 shows how an extra line of shaded triangles can be added for this orientation of the octagon/square tiling.

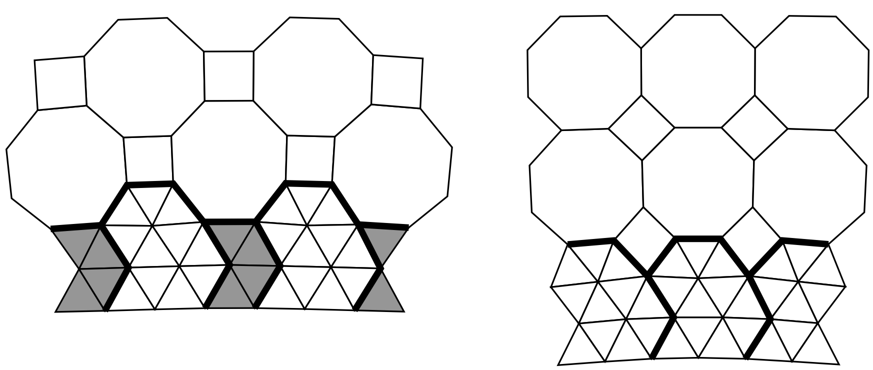

Figure 2: *Comparing the two ways to meet the octagon square tiling.*

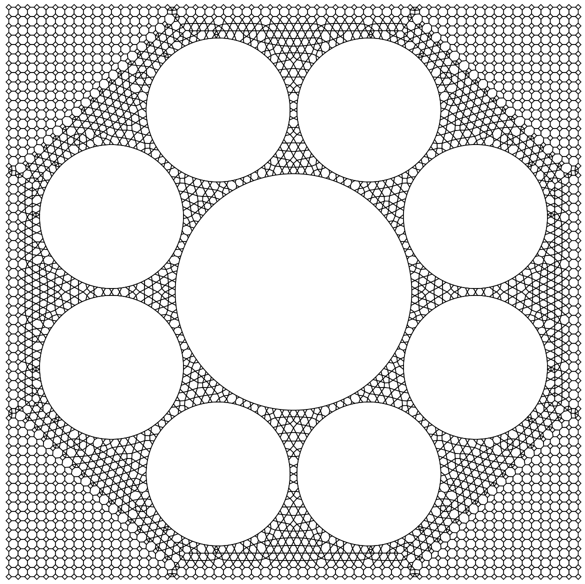

Figure 3: *Here is our finished design for the 192-gon tiling.*

We will show the different stages that we developed for this tiling. We show the entire tiling in Figure 3.

Figure 4 shows how we can use the polygons-in-contact method to change the tiling into a tiling by rosettes. Note that we cheated here. The squares and octagons have angles of 135 degrees, while everything else has an angle of 120 degrees.

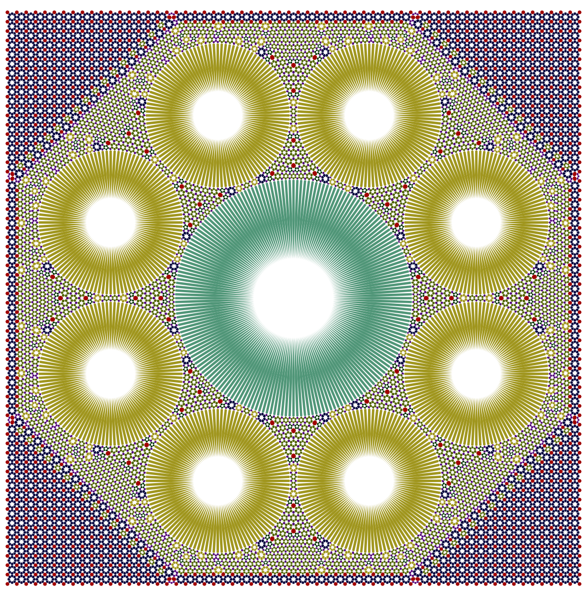

Figure 4: *Here is the 192-gon tiling decorated with rosettes of different angles.*

Just for comparison, Figure 5 is the tiling with 120 degree rosettes and Figure 6 shows 135 degree rosettes. We prefer the “cheat” where we use different contact angles. You will note that due to how we generate the rosettes that there are places where the contact angles don’t match up between rosettes. This is an area for further exploration—–how to best satisfy competing desires:

-to have all the same contact angles,

-to have the contact angles inside a polygon be ideal – where the edges line up for adjacent rosette petals within the rosette,

- to have edge lines be co-linear across the X where rosettes meet.

-to have all stars between rosettes be symmetric.

-to have all hexagons (rosette petals) be symmetric. (6 fold and/or line symmetry)

etc.

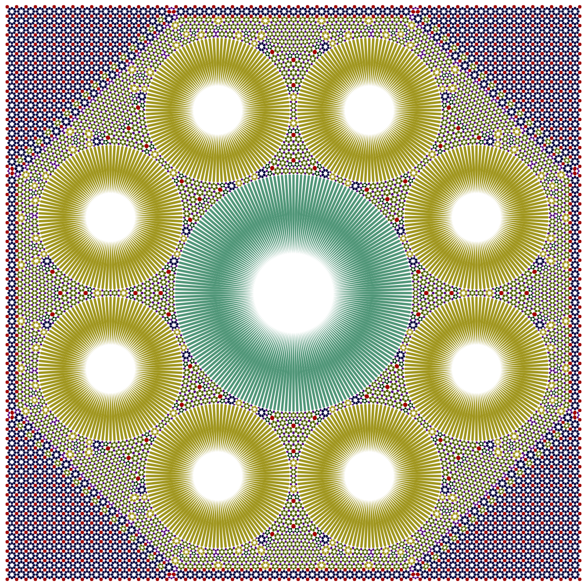

Figure 5: *Here is the 192-gon tiling decorated with 120 degree rosettes.*

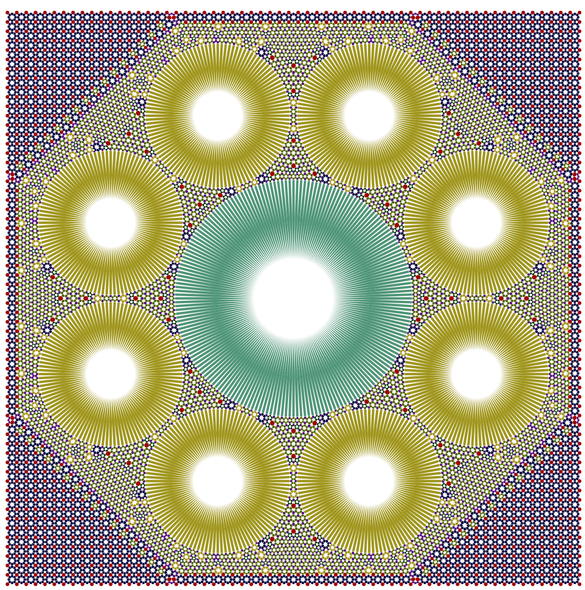

Figure 6: *Here is the 192-gon tiling decorated with 135 degrees rosettes.*

We could also put stars inside each polygon instead of rosettes. Figure 7, 8, and 9 show tilings with stars having angles of 45, 60 and 90 degrees respectively.

Now we kept certain points fixed while constructing the tiling. These include the points of the large

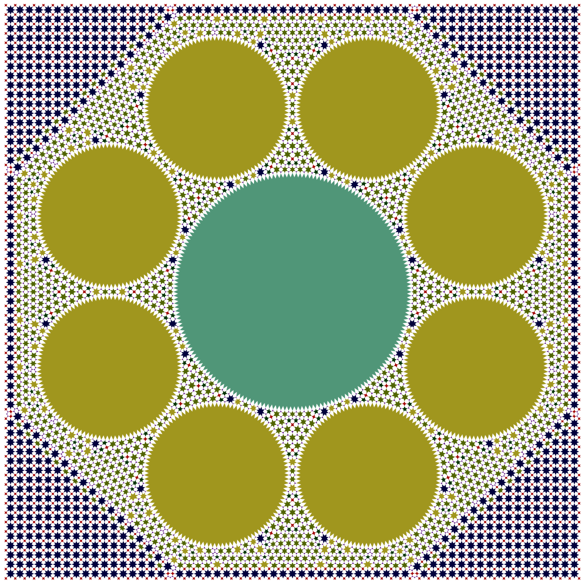

Figure 7: *Here is the 192-gon tiling decorated with 45 degree stars.*

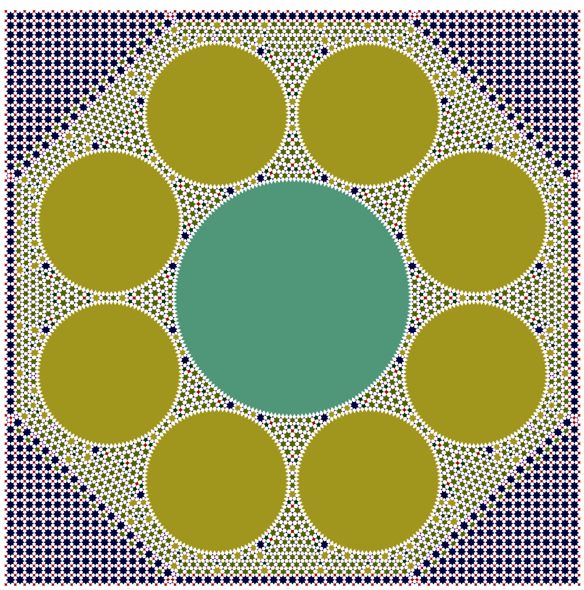

Figure 8: *Here is the 192-gon tiling decorated with 60 degree stars.*

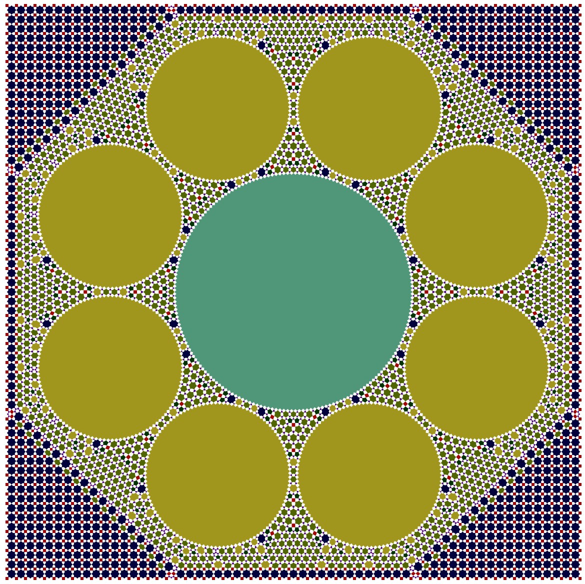

Figure 9: *Here is the 192-gon tiling decorated with 90 degree stars.*

polygons, the points on the large octagon frame, the points on the dee, and some others. We could drop all such fixed points and run our relaxation algorithm. The result looks like Figure 10. Notice how the edges of the large polygons and octagonal frame aren't perfectly aligned. In Figure 11 we fill this tiling with rosettes. There are some places where the relaxation makes the rosettes look better. There are some places where they look worse.

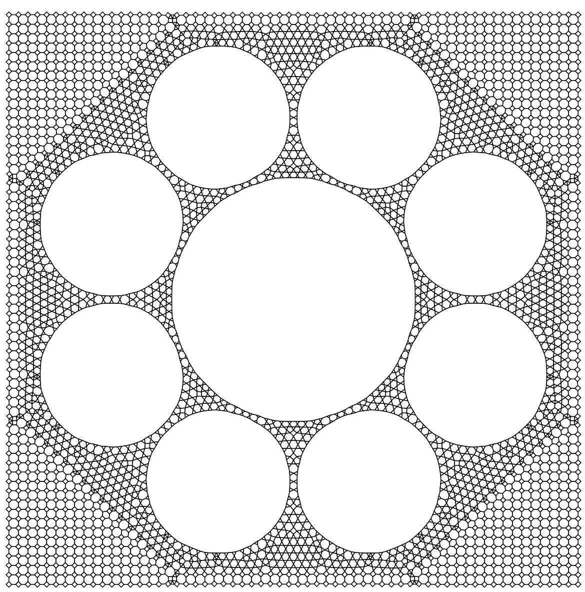

Figure 10: *Here is the 192-gon tiling with all points relaxed.*

We include a tiling with a 2024-gon decorated with rosettes in a separate file. It is very large, so we didn't want to combine the whole thing. Here are the tilings of some of the parts of it.

Figure 12 shows the deltoid. You can clearly see the recursive levels of deltoid within it.

Figure 13 shows the dee. Here the large radius matches our 2024-gon. The tips of the dee have a smaller

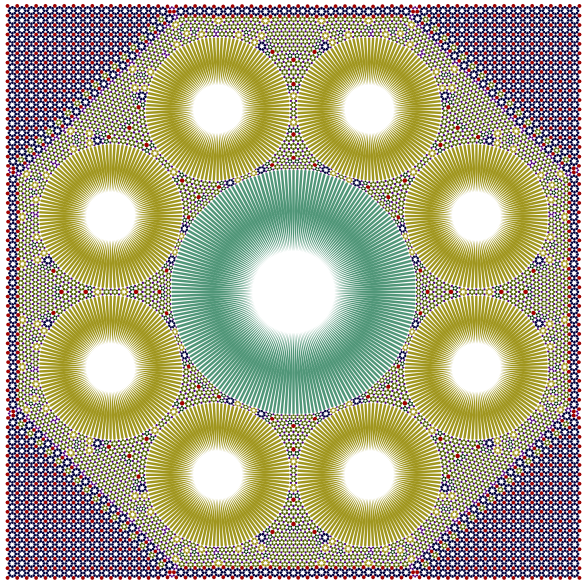

Figure 11: *Here is the 192-gon tiling with all points relaxed decorated with rosettes.*

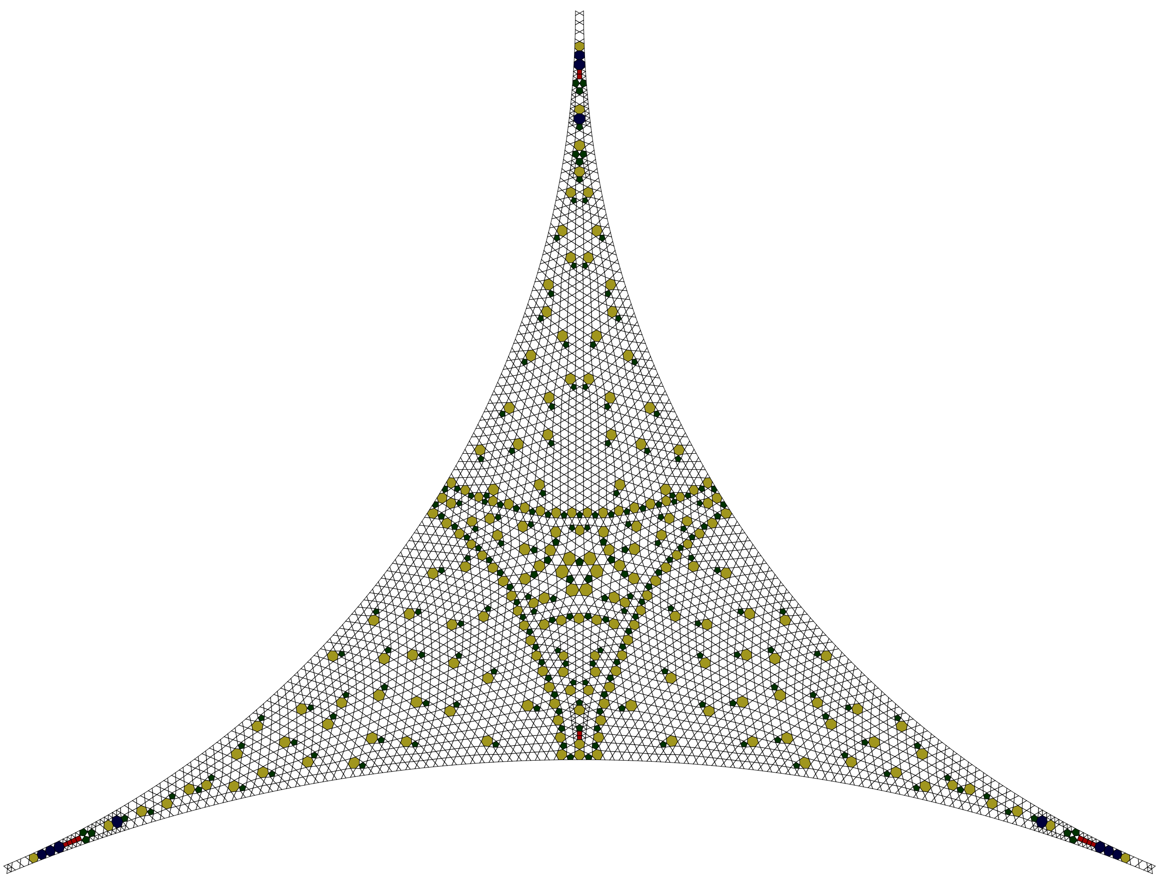

Figure 12: *The deltoid for the 2024-gon tiling.*

radius, chosen to be tangent where it meets the large radius as well as tangent to the flat line.

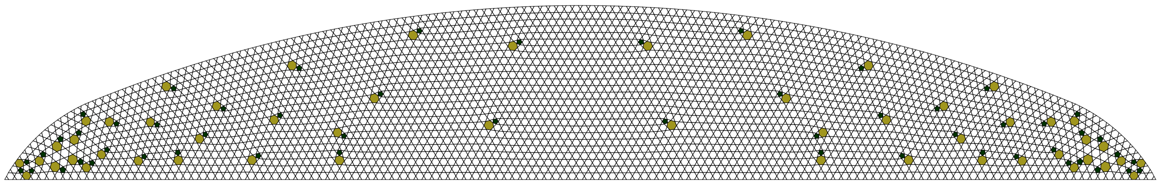

Figure 13: *The dee for the 2024-gon tiling.*

Figure 14 shows the obtuse corners. The one on the right is next to the dee. Note how one part of this obtuse corner is gently curved. This doesn't make much difference - just the fan grows faster. The obtuse corner on the left is the corner of the large octagonal frame. Here we broke some of our rules. You will notice at the narrow part between the two obtuse corners that we don't use a section of triangles. So this section is not locally symmetric. I prefer the look without worrying about making it look symmetric. If we decorate with rosettes, it will end up looking the same with or without a fault line of triangles. You will notice that the obtuse corner on the left has some weird stuff going on. We wanted this section to be locally symmetric. We needed to transition from having a vertex on the line of symmetry to having the midpoint of an edge be on the line of symmetry. You will notice that our design for the basic shapes includes cutting a fan out of the middle of the obtuse corners. We could have done that, but even with a 2024-gon in the center, the tiling isn't really big enough to warrant it. Perhaps someone could make a tiling with even larger rosettes that makes it more suitable.

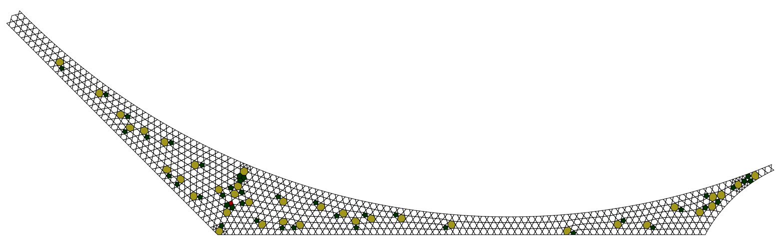

Figure 14: *The obtuse corners for the 2024-gon tiling.*