Controlled Overlaps of Ammann Grid Based Quasi-Cells *Q*

Supplement to the Bridges 2024 paper

"Ammann Grid and Knot Structure of a Quasiperiodic Girih Pattern"

Uli Gaenshirt

Admissible and Forbidden Transformations

The quasiperiodic succession algorithm is a growth rule that combines the local effectiveness of the quasiperiodic matching and covering rules with the creation of an error-free arrangement as generated by the substitution rule. The basic building blocks of the algorithm are the quasi-cells $\boldsymbol{Q}$, which play an important role in the paper to which this supplement refers. While the paper focuses on studying the relationships between the quasi-cells and the Girih pattern, this supplement gives a small insight into the control mechanism of the quasi-cells, using only one example of transformation. The two Figures in this supplement are labeled with capital letters to distinguish them from the numbered Figures in the paper.

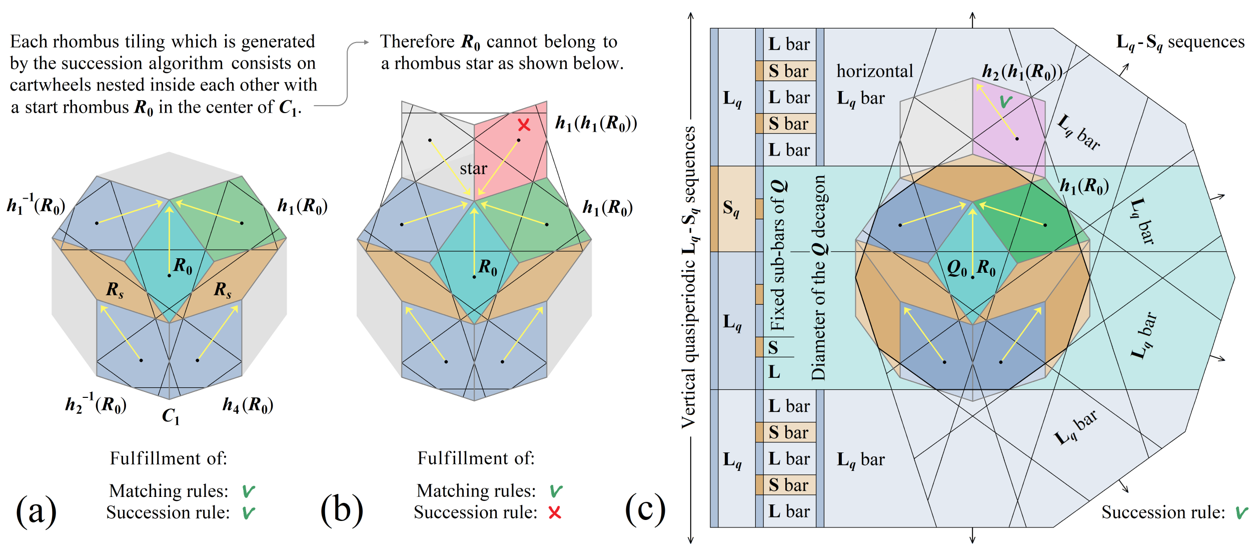

Figure A: *(a) Cartwheel C_1 with R_0. (b) Rhombus arrangement only permitted under the matching rules. (c) Rhombus $h_2(h_1(Q_0)$ corresponding to the Ammann grid generated by the succession algorithm.*

Each thick Penrose rhombus $\boldsymbol{R}$ in Figure A has an equivalence relation to a quasi-cell $\boldsymbol{Q}$ (see Figure 5(a)). For clarity, only rhombs are used in this section. Figure A(a) shows an arrangement of five thick rhombs $\boldsymbol{R}$ with yellow orientation arrows and two ochre skinny rhombs $\boldsymbol{R_s}$. The arrangement is the same as in the cartwheel $\boldsymbol{C_1}$ in Figure 3(b), but upside down. The central aquamarine rhombus $\boldsymbol{R_0}$ is the start rhombus of the succession algorithm. The green rhombus is created by the transformation $\boldsymbol{h_1}(\boldsymbol{R_0})$. Consequently, the light red rhombus in Figure A(b) is labeled $\boldsymbol{h_1}(\boldsymbol{h_1}(\boldsymbol{R_0}))$. This rhombus is a valid selection according to the matching rules, but it is not allowed by the succession algorithm due to the position of $\boldsymbol{R_0}$. The algorithm requires the rhombus $\boldsymbol{h_2}(\boldsymbol{h_1}(\boldsymbol{R_0}))$ from Figure A(c)), which matches the cartwheel $\boldsymbol{C_2}$ (see Figure 3(c)).

The right side of Figure A(c) shows an Ammann grid generated by the succession algorithm. Outside the darkly emphasized $\boldsymbol{Q_0}$ decagon, there is an $\mathbf{L}_q$ bar at each of its edges. The continued sequences of $\mathbf{L}_q$ and $\mathbf{S}_q$ bars are identical in all ten orientations. They can be calculated one after the other by transferring and converting suitable values. The locally acting quasi-cells $\boldsymbol{Q}$ work on the same basis.

Controlled Overlaps of Quasi-Cells Q by the Example of the Transformation $h_1(Q)$

The five twin-scales **I** of a quasi-cell $\boldsymbol{Q}$ are enclosed in the decagonal boundary of Q. Inside Q, only the elementary trapezoid $\boldsymbol{T_{el}}$ is a quasi unit-cell. The three grey areas outside T_{el} are variable, as they each contain an undefined red Ammann line (parallel dashed lines indicate the alternative positions). Figure B(a) shows the overlap of a cell $\boldsymbol{Q}$ by a transformed cell $\boldsymbol{h_1}(\boldsymbol{Q})$, where Q refers to to the predecessor cell $\boldsymbol{Q_{pred}}$ and $h_1(Q)$ to the successor cell $\boldsymbol{Q_{succ}}$. The transformation $\boldsymbol{h_1}$ corresponds to a rotation of the rhombus $\boldsymbol{R}$ around its top point $\boldsymbol{T}$ by 72 degrees counterclockwise (the transformed rhombus is refered to $\boldsymbol{R'}$ in this Figure). This covering defines the variable area on the right side of Q as an **SL** flip type.

The vertical yellow sliding ruler $\mathbf{L}_{av}$ in the enlarged detail in Figure B(b) synchronizes the two single scales of the twin-scale $\mathbf{I_a}$. Its length (3**L**+ **S)** is the average length of the intervals $\mathbf{L}_q$ and $\mathbf{S}_q$ (see Figure 3(a) and 3(e)) in a quasiperiodic $\mathbf{L}_q$-$\mathbf{S}_q$ sequence that approaches infinity. The value transfer of a twin-scale **I**, which belongs to a quasi-cell Q_{pred}, onto a parallel twin-scale **I** of an overlapping quasi-cell Q_{succ} takes place perpendicular to $\mathbf{L}_{av}$. The transfer of the values of the vertical red twin-scale $\mathbf{I_a}$ to the vertical green twin-scale $\mathbf{I_d}$ is shown by the horizontal yellow correlation arrows.

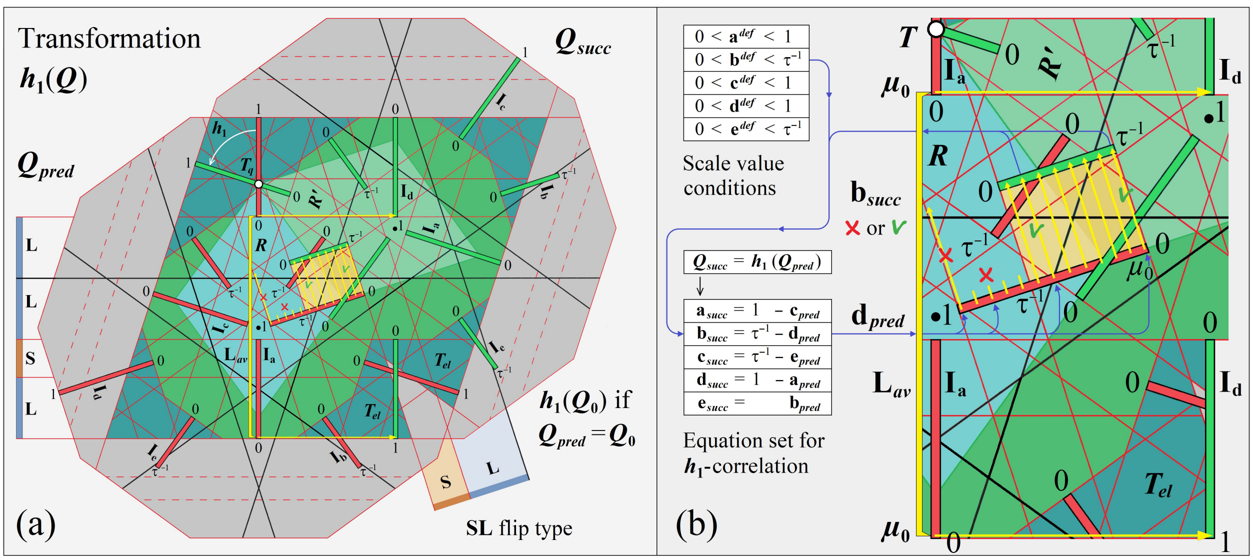

Figure B: *(a) Transformation $h_1(Q)$. (b) Detail of $h_1(Q)$ with value conditions and h_1 equation set.*

The scale value of the vertical twin-scale $\mathbf{I_a}$, which is defined by $\mathbf{L}_{av}$ in Figure B(b), is an infinitesimal small value $\boldsymbol{\mu_0}$, i.e. the correlation arrows are very close to the **(**forbidden) values **0**, which lie on the red Ammann lines directly below the yellow arrows. Figure A(a) shows that the five start values $\boldsymbol{\mu_0}$ of a start cell $\boldsymbol{Q_0}$ can be transferred to the green twin-scales **I** of the cell $\boldsymbol{h_1}(\boldsymbol{Q_0})$. The yellow arrow next to the right border of the yellow correlation area in Figure B(b) shows the transfer of the value $\boldsymbol{\mu_0}$ from $\mathbf{d}_{pred}$ to $\mathbf{b}_{succ}$. The equation set confirms that the cell $\boldsymbol{h_1}(\boldsymbol{Q_0})$ is an admissible cell of the succession algorithm.

In the previous section, it was asserted that the cell $\boldsymbol{h_1}(\boldsymbol{h_1}(\boldsymbol{Q_0}))$ must be forbidden by the succession algorithm. The correctness of this statement can also be checked with the five $\boldsymbol{h_1}$ correlation equations by denoting the successor values of $h_1(Q_0)$ as the predecessor values of the transformation $h_1(h_1(Q_0))$.

The value d_{pred} of $h_1(h_1(Q_0))$ is calculated as: $d_{pred} = 1 - \mu_0$ (= successor value of $h_1(Q_0)$)

The value b_{succ} of $h_1(h_1(Q_0))$ is calculated as: $b_{succ} = \tau^{-1} - d_{pred}$ i.e.: $b_{succ} = \tau^{-1} - (1 - \mu_0)$

Thus, b_{succ} of $h_1(h_1(Q_0))$ is a negative value which contradicts the scale value condition $0 < b^{def} < \tau^{-1}$ in Figure B(b) above. Consequently, the transformation $h_1(h_1(Q_0))$ is forbidden by the succession algorithm! In this case, the alternative transformation $\boldsymbol{h_2}(\boldsymbol{h_1}(\boldsymbol{Q_0}))$ leads to an admissible cell with five valid values.