Abstract Geometry Meets Studio Art: Oil Painting the Fano Plane

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Abstract

I created a series of paintings from the geometry of projective planes. Utilizing artistic interpretations of points and lines, I set out on a journey of intellectual and emotional exploration across seven canvases. I will briefly discuss some examples of artistic uses of projective planes, then introduce some foundational concepts of incidence geometry. From there, I will describe the process of translating geometry into artistic terms and its consequences for the prospective artist. Then I will discuss the depths of my artistic process, including some powerful and unexpected personal discoveries.

Background

I used finite projective planes as a constraint to guide my creative process throughout a series of seven thematically related paintings. I will begin by discussing previous artistic interpretations of finite projective planes, then describe how I chose to utilize this geometry. Then I will give an overview of incidence geometry, including one of its subsequent geometries: projective planes. I will identify select projective plane theorems that influenced my paintings and describe how artists can interpret axioms, theorems, and proofs as a set of artistic instructions. Finally, I will describe my artistic process in detail. This includes how my personal interpretation of the Fano plane came to be, the academic and therapeutic knowledge I gained throughout, and a collection of artist statements further describing the work.

Projective planes and the arts have a rich history of connectivity. Here I will briefly introduce a few selective examples that I have encountered. My first exposure to projective planes was as a child. I was particularly interested in the game of "Spot It!®," which consists of 57 cards each containing 8 symbols. Opponents race to find the sole matching symbol between two unique cards, and whoever spots the most matches in the deck is declared the winner. At the time, I did not understand the intricate mathematics operating behind the scenes. As an adult, however, I was thrilled to learn that the game works seamlessly because it is a projective plane of order 7 [3]. Additionally, I have encountered multiple examples of artistic projective planes in the Bridges Archive itself. Artist and architect Albrecht Wintterlin constructed a stunning 3D-model of a projective plane of order 3, which explores color, lighting, and spatial dimension [5]. A second exposure to the Bridges Archive occurred during my undergraduate studies. My mathematics professor at the time, Dr. Dan May, introduced my class to the idea of using math to write poetry. Dr. May and colleague Dr. Huse Wika used a projective plane of order 2, or the Fano plane, to compose poems [2]. They chose to interpret the points and lines of the projective plane as words and stanzas respectively. Upon learning this, inspiration struck. If I can interpret abstract geometric concepts to write poetry, how else can I interpret these points and lines?

At its foundation, this painting series is based on the concept of incidence geometry. Incidence geometry is vastly different from the familiar Euclidean geometry; it does not contain distances, angles, or any measurements at all. Rather, incidence geometry pertains to points and lines and how they relate, or are incident, with each other [4]. The terms "point" and "line" are left undefined. One can build a model for incidence geometry by describing examples of these terms. In my case, I am creating a model by describing a "point" to mean "painting" and a "line" to mean "theme".

Under the vast umbrella of incidence geometry, specific geometries are defined by accepting certain axioms. An axiom is a rule that all objects in the system must adhere to. Projective planes are one such geometry, with their own set of accompanying axioms as follows [4]:

Axiom 1. For any two distinct points, there is exactly one line through both.
Axiom 2. Any two distinct lines meet in exactly one point.
Axiom 3. There exist four points such that no three are collinear.

Any collection of objects that adheres to these three axioms is considered a projective plane. The Fano plane, or projective plane of order 2, is the smallest projective plane. It consists of seven points and seven lines, which is the smallest number that can satisfy all three axioms.

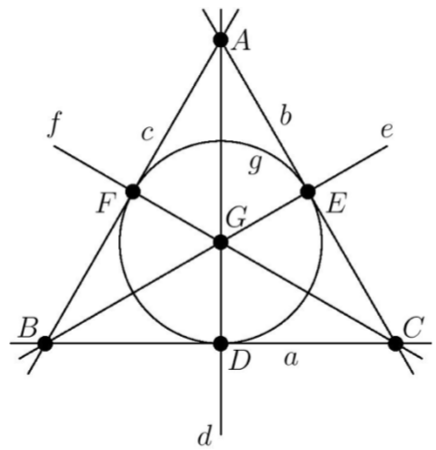

Figure 1: *Fano plane.*

The *order* of a projective plane is a whole number n defined to be one less than the number of points contained on each line [4]. As the Fano plane contains three points on each of its lines, the order of the Fano plane is 2. In examining the diagram above, notice how the Fano plane adheres to the projective plane axioms and is therefore a projective plane itself. Note that line g appears in the shape of a circle, as opposed to the straight lines of Euclidean geometry (see Figure 1).

Translating Geometry into Art

If an object satisfies all axioms of projective planes, it will also satisfy any of the subsequent theorems. From the foundations of projective planes, I have identified six theorems that inform my artwork. These theorems provide additional insight into how the points and lines, and therefore paintings and themes, interact. I will first introduce these theorems, describe how to interpret them artistically, then discuss some of their direct artistic results.

Theorem 1. Any two lines contain the same number of points.
Theorem 2. Any two points have the same number of lines passing through them.
Theorem 3. The number of points contained in each line is equal to the number of lines passing through each point.
Theorem 4. If $n + 1$ is the number of points contained in each line, then the total number of points is $n^2 + n + 1$.
Theorem 5. There is an equal number of points and lines.
Theorem 6. The Fano plane is the smallest projective plane.

To ensure no circular reasoning occurs, I listed the theorems in the order in which they are to be proven. Once a theorem has been established, its artistic implications naturally follow. By replacing the words "point" and "line" with the interpretations "painting" and "theme," the reader can translate the axioms, theorems, and their associated proofs into guidelines for aspiring projective plane artists. I will provide artistic versions of the projective plane axioms, and the remaining translations are left as an exercise for the reader:

Axiom A1. For any two distinct paintings, there is exactly one theme present in both.
Axiom A2. Any two distinct themes are both present in exactly one painting.
Axiom A3. There exist four paintings such that no three contain the same theme.

Now, I will discuss the artistic implications of a proof. I will provide a formal proof of Theorem 4, followed by a description of its impact on my artwork.

Theorem 4: If n + 1 is the number of points contained in each line, then the total number of points is $n^2 + n + 1$.
Proof: (illustrated in Figure 2)
Per Axiom 1, we have a line l. Per Theorem 1, we know each line contains the same number of points. Let that number be $n + 1$, and let's call one of those points P. Per Theorem 3, there are $n + 1$ lines passing through P, including line l. Let's consider all of the points in the plane excluding P. Per Axiom 1, each of these points is contained on exactly one line through P. This implies that every point in the plane is contained in one of the $n + 1$ lines. Per Axiom 2, we know any two of these lines intersect at exactly one point. This implies each pair of these lines only intersect at point P. Considering the points in the plane excluding P, this leaves n points per line. Because there are $n + 1$ lines through P and these lines contain every point in the plane, the total number of points excluding P is $n(n + 1) = n^2 + n$. Adding in point P, we have the total number of points in the plane, $n^2 + n + 1$. ■

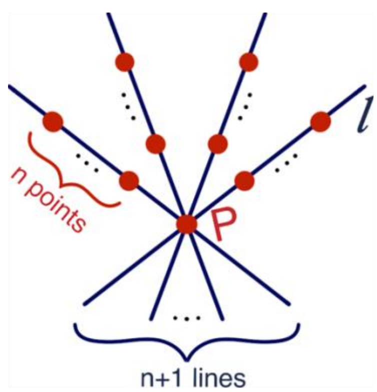

Figure 2: *Total number of points in a projective plane.*

Now that the proof of Theorem 4 is established, the reader can conduct its translation line by line. The result functions as a set of instructions describing how to determine the total number of paintings required for a projective plane series. This result is more than a tool to understand the relationship between paintings and themes; it is also necessary to understand the direct impact of Theorem 6 on my artwork. The proof of Theorem 6 invokes the order of the Fano plane, which relies on the consequences of Theorem 4. Utilizing the Fano plane, or projective plane of order 2, we have $n + 1 = 3$ paintings containing each theme. This means $n = 2$ and we have $2^2 + 2 + 1 = 7$ total paintings. It was necessary to opt for the smallest projective plane to keep the timeline of the project reasonable. If I selected the next smallest plane, I would be working with a projective plane of order 3. Invoking Theorem 4 once again, I would have $n + 1 = 4$ paintings containing each theme, and therefore need to produce $3^2 + 3 + 1 = 13$ total paintings. This would have nearly doubled the scale of the project, making the Fano plane the most suitable structure.

Using Incidence Geometry to Create Art

My series of artworks is a model for the Fano plane. Its seven "points" are represented by seven individual paintings on circular canvases which are displayed in the arrangement of Figure 1. The seven "lines" are represented by seven unique themes. Choosing the right seven themes for my artwork took time and mathematical guidance. I began with a long list of contenders and many blank templates of the Fano plane. I assigned a multitude of themes as various lines in the plane until I recognized a starting point. Somewhere on the edge of the Fano plane, I saw the themes *feminine*, *body*, and *abuse* brought together by a single point. I knew this was the right combination, and I also knew that I wanted these themes present in the very center painting. I began a fresh template with the center point labeled as such. The geometry of the Fano plane helped me pencil in more points even before I came up with another theme. If each of these themes were to be present in the center painting of the series, then these themes must also follow their respective lines and appear in two other paintings. The pattern continued. Every time I added a theme, the rules of projective planes forced new pairings into existence. This helped me group themes together in ways that would never have occurred to me otherwise, which brought on new creative challenges. For example, one such point combined the themes *healing*, *mind*, and *abuse*. I had no idea how one might go about healing their mind from an abusive situation, let alone how to represent it on canvas. Mathematics forced me to consider these questions, and finding the solutions allowed me to grow as a mathematician, as an artist, and as a human being.

Once my final draft snapped into place, I was left with the following seven themes: *feminine*, *healing*, *nature*, *energy*, *body*, *mind*, and *abuse*. I also associated a color with each of these themes to determine my palette for each painting. I utilized hints of other hues whenever I deemed it aesthetically necessary, but the following colors are the most prevalent. *Feminine* is red, *healing* is blue, *nature* is green, *energy* is purple, *body* is orange, *mind* is yellow, and *abuse* is black. Each painting was now laid out to be a combination of three themes and three colors, and these color choices repeatedly guided my paintings as well. For instance, I was having trouble conceptualizing the combination of *energy*, *abuse*, and *nature* until I made a color-coded version of my Fano plane (see Figure 3). It combined purple, black, and green as the palette for this painting, and I realized these are the main colors I see reflecting in an oil spill. It was the perfect subject matter to portray on a canvas.

As I began translating these ideas onto canvases, my interest widened to disciplines outside of math and art. My artwork became focused on learning about myself, specifically isolated and neglected parts of my personality. It was brought to my attention that this form of discovery mirrors the therapeutic model of Internal Family Systems, in which “individuals explore and confront conflict between the inner parts of themselves” [1]. Without even realizing, I was bringing characters from my own Internal Family System to life in my paintings. Having visual representations of these parts of my personality was a key catalyst in my mental and emotional growth. Rather than relying on pure introspection, I had something physical to interact with. It made these neglected parts of myself so real and tangible, which led to many breakthrough moments in resolving conflict between them. This healing process is an ongoing one, but its roots will always remain in a series of mathematical paintings.

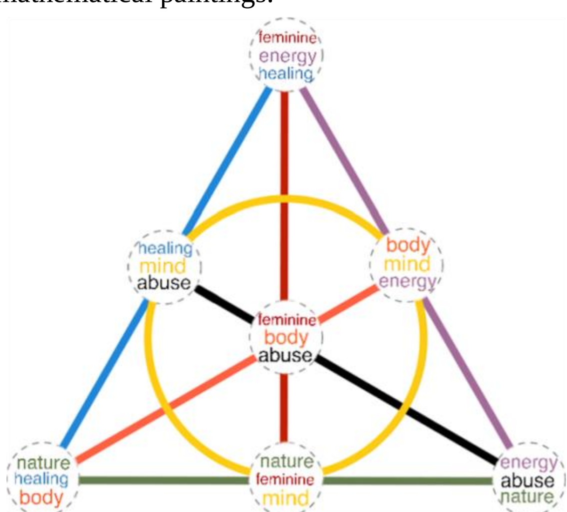

Figure 3: *Fano plane painting key.*

Figure 4: Her *(series dimensions: 183 x 213 cm; individual canvas diameter: 30 cm)*.

Artist Statements

My seven paintings come together to form a series, but they also function as individual works of art (see Figure 4). Due to the balance of their collective and individual natures, both the series and individual works have their own titles and artist statements. I will first introduce the collective series and the ideas behind it, and then move to the titles and statements associated with each of the seven paintings.

Her

Oil on canvas

Before this series was even an idea in my mind, I reached a low point in my existence. Dealing with newfound obstacles in my mental and physical health, I became overwhelmed, frightened, and entirely lost. As I reflected on my experiences, I noticed a peculiar grammatical shift. When reliving memories that I was comfortable with, I referred to myself as “me.” However, whenever I encountered something too painful, “me” changed to “her.” I realized there are parts of myself that I regard as another person entirely. I was treating her like a stranger, and it became clear that it was time to get to know her. This series represents an ongoing journey of self-exploration. It has been a beautiful space to discover the intricacies of human emotion, in which I've learned to process, to grieve, to heal, and to love. As I grow and learn about myself as a human being, I am growing and learning about myself as a mathematician and an artist. My fascination with mathematics lies in the ideas, making abstract geometry and particularly the Fano plane the perfect structure to hold my artwork. The mathematics played a direct, pivotal role in creating the paintings. The flexible yet guided structure of the Fano plane allowed me to experiment with various combinations of themes until they snapped together in the perfect formation. As an artist, my growth lies in imperfection. Healing has allowed me to move away from past perfectionism, and mistakes and experimentation have repeatedly informed my artwork. I experimented with oil painting, which gave me the depth that I was searching for. The number of paintings helped me experience my range as an artist and play with various levels of abstraction. The combination of these elements have made this series the most challenging and most rewarding project that I have worked on.

My statements for the individual paintings are much shorter than a typical artist statement. Rather than diving into the reasoning and processes behind each one, I composed a series of small poems reflecting the emotion poured into my canvases.

Hey, You (*feminine, body, abuse*): You are the most beautiful part of me. I forgive you for all of the things you never did.
Take Your Space (*feminine, energy, healing*): I will no longer make myself small for your comfort. Watch me grow.
Your Power (*body, mind, energy*): It was there all along. It's time to use it.
Big Oil (*energy, abuse, nature*): Sure looks better than it tastes. Just ask the ducks.
The Garden (*nature, feminine, mind*): So much to tend to, to nurture. It's growing beautifully.
The Return (*nature, healing, body*): This is where you came from. This is where you heal.
You Cry (*healing, mind, abuse*): "There's no way to stop a heartbreak. How do you… what do you do about that?" (D. Trussell, P. Ward, and M. L. Mayfield. "Mouse of Silver." *The Midnight Gospel*.)

This collection of artist statements functions as an intimate demonstration of how each section of this paper was united by one series of artworks. This project brings together formal geometry, the translational relationship between mathematics and studio art, and the resulting personal discoveries. This series is intended to demonstrate some of the unique connections that exist between math and art and encourage prospective artists to explore their own connections.

References

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