

The Mathematics and Design of a New Deck of Playing Cards

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Abstract

We present a novel deck of cards designed to have fundamentally different relationships between the cards to provide options for new types of games. We mathematically compare it to the familiar deck of 52 cards.

Why Design a New Deck of Cards? and Other Questions

The Standard deck of playing cards has been around for a long time. Depending on which expert you ask, and how much speculative inference you are willing to accept, it arrived or originated in Europe in the 1300s, or started somewhere in Asia *much* earlier than that. The current version of four suits (spades, hearts, diamonds, clubs), and 13 ranks (Ace, 2-10, Jack, Queen, King), has been around since the 1500s [3].

Instead of asking the question, “Why mess with something so popular and universal?”, let’s ask the potentially more interesting questions about a new deck: “What are the relationships between the cards?”, “How big should the deck be?”, “What information should be contained in the cards?”, “What should be the distribution of the cards?”, and “What should the cards look like?”. There are literally hundreds of games developed for the Standard deck [4]; a new deck could lead to another very interesting question: “What kind of new card games could be created with a completely different deck?”

What Ideas Have Been Tried?

To significantly move away from the Standard deck, it is not sufficient to change the number and/or color of suits, increase or decrease the range of the ranks, or add to the picture cards with unused titles from the royal court. Several variations have already done this: the *Wizard* card game has wizards and jesters; *Clubs* has 4 suits of 15 cards; *Raj* has five suits of 15 cards; *Scan* has a Standard deck, but each suit is a different color and the colors are also printed on the card backs; *Trumpet* has 6 suits of 11 cards each; *Split* has 104 cards, each of which contains half a Standard card; and *Cat in the Box* has 4 suits of 9 cards each, but the suit is not determined until the card is played. All of these decks are specific to a single game [1].

To generate a truly new deck, the relationships between the cards must be inherently different. The two games that most closely embody our ideas are *Set*[1] and *EvenQuads*[5]. Both of these decks have cards with multiple characteristics that vary independently. *Set* has four categories (color, shape, number, and shading) with three values of each (e.g. color has red, green, and purple). *EvenQuads* has three categories (color, symbol, number) with four values of each (e.g. color has green, blue, pink and yellow). Our new deck also provides multiple characteristics that vary independently, but its unique aspect is that these characteristics can be expressed as individual cards, Figure 1(bcd), called *Characteristic* cards, or combined in groups of three as *Complete* cards in Figure 1(a). Determining the number of categories and the number of options leads to questions of deck size and card distribution in the next section.

The deck that most closely aligns with our vision is *Pairs*[2]. It has 10 different cards numbered 1 to 10, and there is one copy of card 1, two copies of card 2, etc. These cards are very different from ours, but the *Pairs* deck is general enough to have the rules for 40 new games on their website. Our goal for our deck is to be this general and invite the creation of new games leveraging relationships between the new cards.

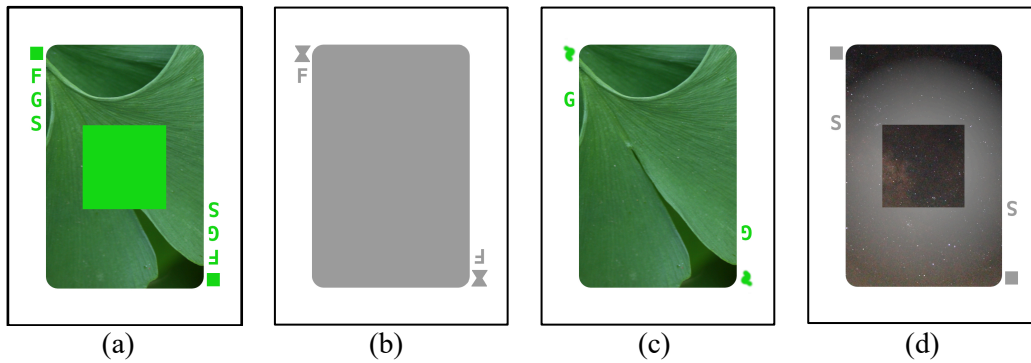


Figure 1: (a) Complete card: Filled-Green-Square, Characteristic cards: (b) a pattern: Filled, (c) a color: Green, (d):a shape: Square. (all photographs in the paper by A. Denelsbeck)

How Big Should be the Size of the Deck? What Should be the Distribution of the Cards?

A deck of this structure with fewer than three categories does not produce interesting games. A deck with six or more categories becomes unwieldy. Table 1 contains a comparison of different numbers of categories with different numbers of options. (e.g. 2 categories with 3 options might be color: red, green, blue; and size: small, regular, large. 3 categories with 2 options might be: Color: red, blue; Size: small, large; Orientation: horizontal, vertical). The number of categories (*c*) and the number of options, (*n*), determine the number of unique cards in the deck as n^c . For comparison, the Set deck has $c=4$ and $n=3$ for a total of 81 cards and the EvenQuads deck has $c=3$ and $n=4$ for a total of 64 cards. Our deck has $c=3$ and $n=3$ yielding 27 Complete cards. This is shown on the left side of Table 1 in column 3 (#Complete). The number of unique characteristics is determine by cn , and shown in column 4 (#Unique).

Table 1: Deck Size Comparison based on the number of Categories and Options.

	Categories	Options	#Complete	#Unique	Copies of Characateristic Cards				Deck Size (based on # of copies)				
					2	3	4	5	2	3	4	5	
	3	2	8	6	12	18	24	30	20	26	32	38	Too Small
Our Deck	3	3	27	9	18	27	36	45	45	54	63	72	Small
EvenQuads	3	4	64	12	24	36	48	60	88	100	112	124	Ideal
	4	2	16	8	16	24	32	40	32	40	48	56	Large
Set	4	3	81	12	24	36	48	60	105	117	129	141	Too Large
	5	2	32	10	20	30	40	50	52	62	72	82	

To define our deck, we require one of each Complete card, plus some number of copies of the Characteristic cards – shown in the middle section of Table 1. The total number of cards in the deck are the sum of these, and shown on the right side of Table 1 (labeled Deck Size). Using the Standard deck (52 cards) as an ideal target size for game creation, practical hand sizes, and ability to shuffle and hold, we specify these metrics for deck sizes: <21 is too small, 21 to 41 is small, 42 to 65 is ideal, 66 to 83 is large, and > 83 is too large. (These ranges are somewhat arbitrary and based on our card-playing experience.) Too few Complete cards impedes game design, but too many limit the number of Characteristic cards. The only practical consideration is 3 categories with 3 options. Based on play-testing and game-design, we chose 4 copies of the 9 different Characteristic cards (36 total) that when added to the 27 Complete cards make a deck of size 63.

What Information Should be in the Cards? What Should the Cards Look Like?

Playing cards are ingrained into our experience. Accordingly, new playing cards should give an immediate sense of familiarity. We have kept the white border with both single letter indicators and identifiable icons

in margin - placed this way so that the cards can be read when fanned in the hand. There are three vertical positions in the margin, one for each category, pattern first, color second, and shape third. On a Complete card, all three appear together in Figure 1(a). For Characteristic cards, a single letter appears in the correct position (Figure 1(bcd)). Color cards also have images emphasizing the color to support colorblind players.

One issue with the representation of cards in the deck is how to express a shape without any other characteristics. The deck must contain cards that represent three characteristics, a hollow red circle can be easily expressed. But how do you represent an individual shape without giving it a color and a pattern? The solution we implemented was to represent the shape as a negative space, with the background starfield image visible through the absent shape seen in Figure 2(a).

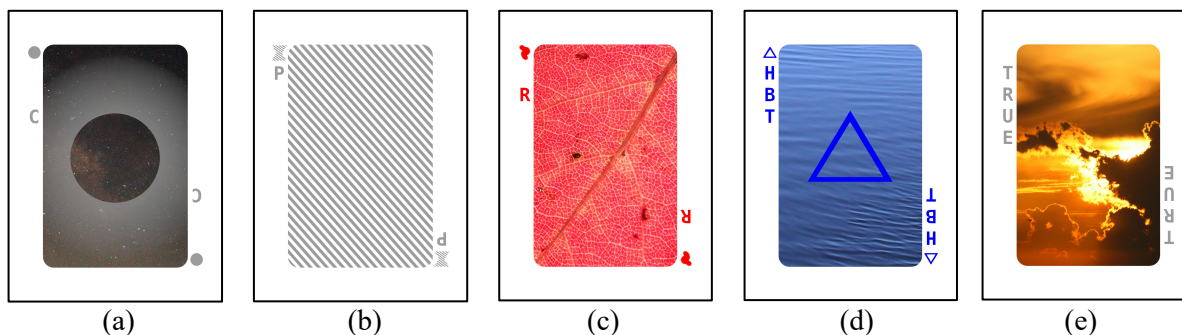


Figure 2: Some examples of cards from the deck: (a) a shape card for Circle using negative space, (b) a pattern card for Partial, (c) a Red color card, (d) a Hollow-Blue-Triangle, (e) a TRUE card.

Not unlike the Standard deck, with two extra jokers that can be used to play some games, but are not integral to the deck, the new deck has 10 True cards and 10 False cards that are used in some games but are also not integral to the deck. These cards are represented with identical images. True cards are in full color (Figure 2(e)), and False cards are the same image in black & white. They can be used to pass information between players, as a progression or score tracker, placeholders, or not at all in most games.

What Kind of New Games can be Created?

Most card games using the Standard deck leverage the relationships between cards to progress towards some goal. Goals for various games include: collecting pairs or sets of cards with matching ranks (Go Fish, Old Maid, Poker), forming runs of cards with numbers in a given suit (Gin Rummy, Poker, Spite and Malice) playing the card with the highest rank in a trick-taking game (Bridge, Spades, Hearts), playing sequences of cards in which consecutive ones must share a characteristic (Crazy Eights), and so on. Our new deck has fundamentally different relationships between the cards, so leveraging collections, sequences, tricks, and connections has a different feel, and leads to different kinds of games.

To better illustrate these differences, we will characterize both decks for comparison. The Standard deck has 52 cards that are unique. Our new deck has 27 unique cards (Complete cards) and a total of 36 cards that have duplicates (Characteristic cards). There are 9 different ones – 3 categories of 3 options each (Pattern: Filled, Partial, Hollow; Color: Red, Green, Blue; Shape: Circle, Square, Triangle). There are four copies of each, yielding 36 total. In the Standard deck, the rank and suit of the cards can vary independently. The new deck has 3 categories that can vary independently – pattern, color, and shape.

For an arbitrary card selected from the Standard deck, there are 3 other cards that match it in rank, 25 other cards that match it in color, and 12 of those that match it in suit. There are a total of 28 cards in the deck that have some connection to an arbitrary card. So, for any game that involves making connections between cards, there is a 53% chance two cards will relate. (This drops to 29% if suit is used instead of color). For our new deck, we must do this calculation for both types of cards. For an arbitrarily selected Complete card, there are 18 other Complete cards with which they share at least one characteristic (6 of

those share two). There are also 12 Characteristic cards that match, yielding a total of 30 cards out of 63 or 47%. For an arbitrary Characteristic card, there are 3 identical cards in the deck, 8 others from the same category and there are 9 Complete cards that match, yielding 20 matching cards or a 31% chance of getting one. Overall, any two cards in the deck have a 37% chance of a connection.

The Standard deck's rank values are numerically sequential, imposing an ordering. This will reduce the probability of two arbitrary cards having a connection when the ordering must be considered. For any given card, there can be 8 cards that are either successors or predecessors causing the probability to drop to 15%. Considering that the sequence does not wrap-around, there are two cards for which the probability is only half that. Overall, the probability of finding a sequential connection is about 14%. Our new deck does not have any ordering relationship between the cards, so this type of connection is not present.

Another type of relationship between two cards is the *distance* between them in characteristic space. Distance is defined as the total number of cards required to connect two cards through a chain of shared characteristics. An arbitrary card selected from the Standard deck has 15 cards that share a characteristic (e.g. 4S, has the other 12 Spades and three other 4s), these have a distance of 0. The remaining 37 cards in the deck have a distance of 1 (e.g. the 4S can connect with the 9D, through either the 9S or the 4D).

For the new deck an arbitrary Complete card has 18 other Complete cards that share at least one characteristic, giving those a distance of 0. The remaining 8 Complete cards have no characteristics in common with the arbitrary one, but can be reached with one card in between, giving them a distance of 1. (e.g. Hollow-Red-Circle can connect with a Partial-Blue-Triangle through a Hollow-Blue-Triangle or 11 other Complete cards). There are also 12 Characteristic cards that share a feature with an arbitrary Complete card. The remaining 24 Characteristic cards that do not share a feature have a distance of 1. There are 5 such Complete cards that can make this connection. (e.g. Filled-Red-Circle can connect to Square with Filled-Green-Square, Hollow-Red-Square or three other Complete cards.) The distance between two Characteristic cards is 1 if they are in two different categories (e.g. Hollow to Red can connect via Hollow-Red-Circle and two others). However, if they are in the same category, their distance is 2 because there are no Complete cards that have two characteristics in the same category. To connect Triangle to Square requires two Complete cards (e.g. Triangle to Filled-Green-Triangle to Partial-Green-Square to Square).

For designing games, a new deck is a pristine, empty playground. The deck's new features provide inspiration for developing games that feel different. The two types of cards allow the deck to be treated as two independent decks, opening new gameplay options. The unique connections between cards allow for the creation of unusual sequences, including bifurcations and trifurcations. Recognizing that cards can share more than one connection creates a new way define pairings or sequences. Three Characteristic cards from different categories uniquely determine a corresponding Complete card; this allows for collecting different kinds of sets and redefines what it means to "follow suit" in a trick-taking game. Cards with different distances can connect in different ways. The games run the gamut from cutthroat, competitive battles to cooperative, communal games, from solitaire challenges to many player games in which the cooperation or competition can be decided during game play. The rules for several of the 16+ games currently developed can be found at <https://newdeck.wading-in.net>. Coming full circle, we have answered our own first question. The reason to mess with something as universal and popular as the Standard deck is so that we can create new games, new kinds of games, and new game mechanisms inspired by the novelty of a new deck.

References

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