An Embroidered (3,7) Hyperbolic Butterfly Pattern

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Abstract

We have used embroidery to render a butterfly pattern in the Poincaré disk model of hyperbolic geometry. It is one of a family of such patterns. We will describe the embroidery process and show a picture of the resulting fiber work.

Introduction

Our objective was to use embroidery to render a pattern of butterflies in the Poincaré disk model of hyperbolic geometry. The butterflies had filled areas, curved lines, and round spots that are amenable to production by a computer-controlled embroidery machine. Figure 1 shows a preview of the embroidered artwork.
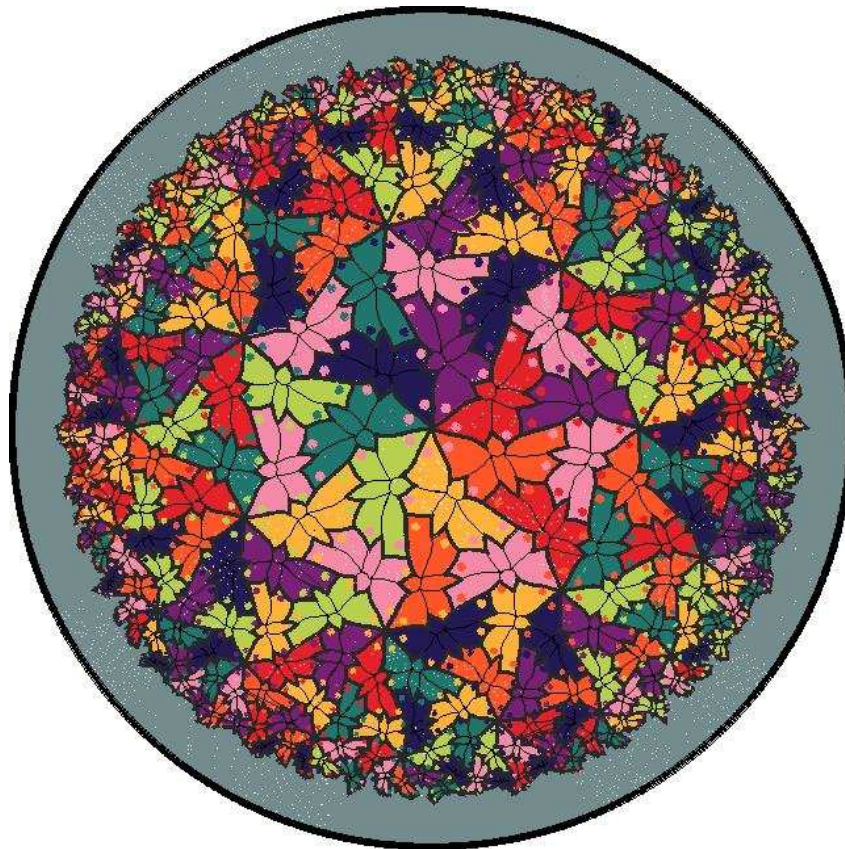
Figure 1: *An embroidered (3,7) hyperbolic butterfly pattern.*

The Dutch artist M.C. Escher provided two inspirations for this project. The first was his planar repeating Regular Division Drawing 70 of butterflies. The second was his set of four hyperbolic "Circle Limit" patterns.

In the early 1980s these patterns inspired Dunham and his students to write a computer program that could replicate those four and other hyperbolic patterns [1]. We will discuss these inspirations in the next section. Then, we will explain how this pattern fits into a whole family of butterfly patterns. Next, we examine the embroidery process and show the final result. Finally, we summarize and indicate possible directions of future work.

M.C. Escher Inspirations

In 1948 Escher created Regular Division Drawing 70 of butterflies ([4] p.172, [3]). It has $p6$ symmetry or 632 in orbifold notation. We noticed that this pattern could be considered to be one among an entire family of butterfly patterns, and thus the hyperbolic ones could be rendered by Dunham's program.

In the late 1950s the Dutch artist Escher created four circular patterns *Circle Limit I, Circle Limit II, Circle Limit III*, and *Circle Limit IV*, which can be considered to be patterns in the Poincaré disk model of hyperbolic geometry. In that model, the points are interior points of a bounding circle and the hyperbolic lines are represented by circular arcs perpendicular to that bounding circle (including diameters). See Figure 2 in which some hyperbolic lines are superimposed on *Circle Limit I*, a pattern of angular fish. Escher also implied the concept of a family of angel and devil patterns with his Regular Division Drawing 45, a carved wooden sphere called *Heaven and Hell*, and *Circle Limit IV* (see Figure 3). These are shown in [4] on pages 150, 244, and 296 respectively.
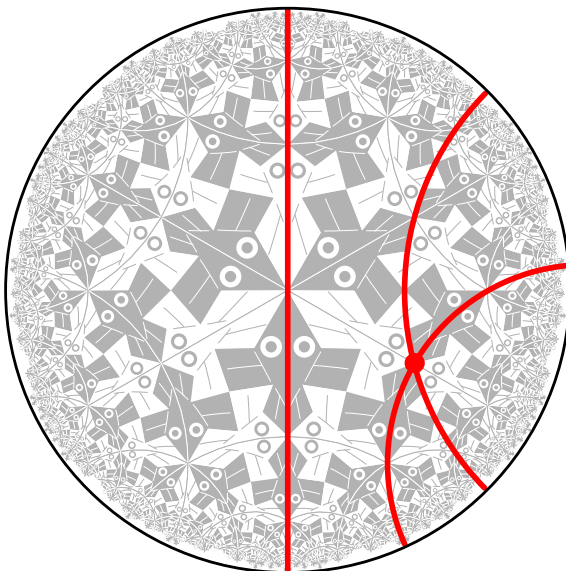

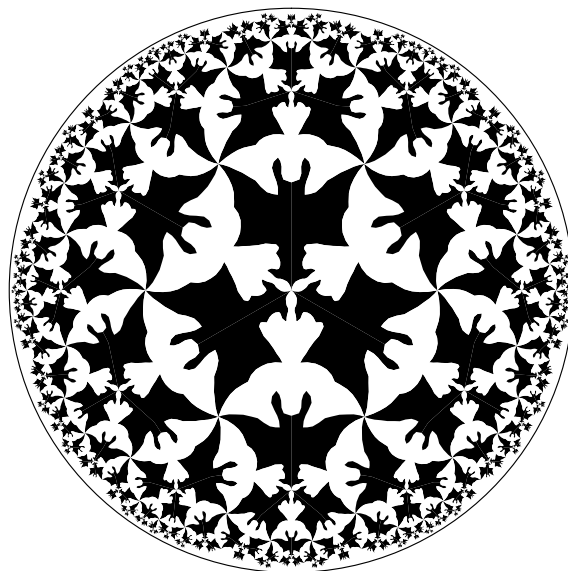**Figure 2:** *The Poincaré disk model.*

**Figure 3:** *Our* Circle Limit IV *rendition.*

Families of Patterns

For integers p and q greater than 2, there is a regular tessellation $\{p, q\}$ of the sphere, Euclidean plane, or hyperbolic plane by regular p-sided polygons meeting q at a vertex depending on whether $(p-2)(q-2)$ is less than, equal to, or greater than 4, respectively. Many of Escher's patterns are based on regular tessellations. Regular Division Drawing 70 is based on the $\{3, 6\}$ tessellation, and both *Circle Limit I* and *Circle Limit IV* are based on the $\{6, 4\}$ tessellation.

Thus we will define a (p, q) butterfly pattern to be one that is based on the $\{p, q\}$ tessellation. Figures 4 and 5 below show the $(3, 8)$ and $(4, 5)$ butterfly patterns, respectively. These have regular color symmetry.

And, following Escher's Regular Division Drawing 70, the round spots on the left front wing tips where *q* butterflies meet are all the same color (and different than the butterfly bodies).
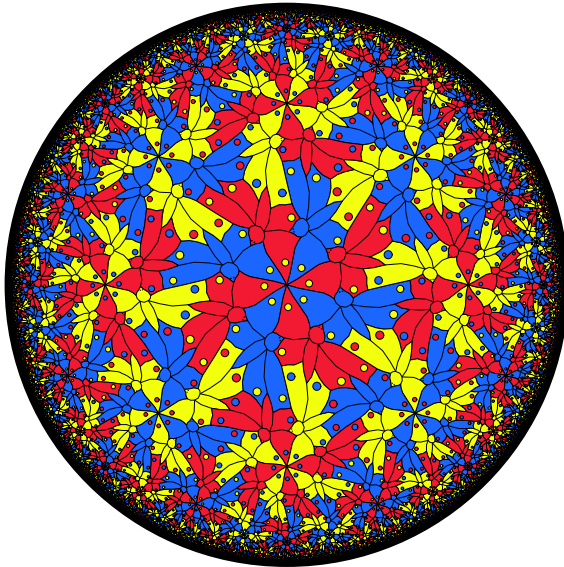
Figure 4: *The (3,8) butterfly pattern.*
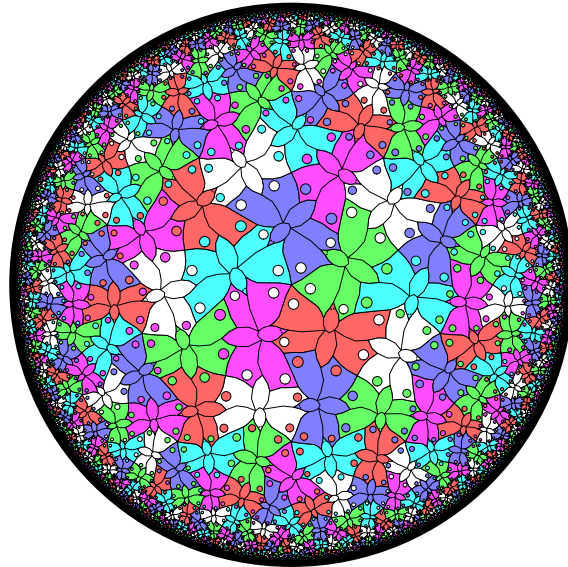
Figure 5: *The (4,5) butterfly pattern.*

The Embroidery Process

We previously embroidered a hyperbolic fish pattern [2] and thought that process could also be applied to our (3,7) butterfly pattern. The embroidery process involves two major steps: #1 the creation of a digital file of the pattern as input to the computer that controls the embroidery machine, and #2 the actual stitching process, which involves several sub-steps. For step #1, the starting point was a high-resolution image of the (3,7) butterfly pattern. Coordinates were measured from this image in order to define the sewing objects of the pattern, creating a .waf file, a file defining those objects, using Floriani Total Control embroidery software. This file contained 2195 objects including 266 butterflies with 1064 spots on the wings, the outlines and wing details of the butterflies, and 21 outer regions to fill out the Poincaré disk. This was done by hand and required two weeks. Then, the Floriani software is used to convert the .waf file to a .vip file, a list of the hoop movements for the individual stitches that the embroidery machine makes to create the final pattern. The software allows for previewing the image as shown in Figure 1.

For step #2, the pattern was stitched out on a Husqvarna Epic embroidery machine, which can use a 2-position Husqvarna Majestic hoop (the largest available) to maximize the diameter of the circular image (about 35cm). There are some considerations that must be taken into account when using a 2-position hoop. The hoop can do one side, but then must be manually remounted to do the other side. The hoop must be switched frequently to avoid differential stretching (stretching by different amounts in different directions) and the machine must be left on so that the coordinates of the mounting point are retained. Also in the design step, a few butterflies on the dividing line between the two halves must be split into separate objects. Of course the butterfly bodies+wings and outer regions are stitched first, and the wing spots and black butterfly outlines are stitched on top. The whole stitching process took two days, with the thread changes (the Epic is a single-needle machine) and hoop rotations. There are over 380,000 stitches in all. Figure 6 shows a picture of the Husqvarna machine just as it is starting to stitch.

In order that parts of the pattern are not differentially stretched, several steps were taken. We started

with black cotton duck canvas as a sturdy foundation material. First, Floriani Heat 'N Sta stabilizer was ironed onto the back of the canvas. Then, two layers of Floriani Medium Weight Tear Away Stabilizer were floated under the design and stitched in place by a preliminary spiderweb design of black thread, binding the four layers together. Also, to avoid visual artifacts caused by all the fill stitching running the same direction, that direction was determined by each individual butterfly. The direction of the wing spot stitching is perpendicular to that of the underlying butterfly.

We used Floriani Polyester Embroidery Thread for the pattern (after trying Superior Fantastico, which produced disappointing results). That thread comes in 360 colors of which we chose 8 for the butterflies, gray for the outer regions of the Poincaré disk, and black for the outlines and butterfly details. Figure 7 shows the resulting embroidered butterfly pattern.

Figure 6: *The Husqvarna Epic Embroidery Machine.*

Figure 7: *The Embroidered (3,7) Butterfly Pattern.*

Summary and Future Work

We have shown an embroidered butterfly pattern and previously we showed a fish pattern in the Poincaré disk model of hyperbolic geometry. We could also embroider other members of the family of butterfly patterns, and even hyperbolic patterns based on other animals.

References

[1] D. Dunham, J. Lindgren, and D. Witte (now D. Witte Morris). "Creating repeating hyperbolic patterns." *SIGGRAPH Conference Proceedings*, 1981, pp. 215–223.

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[3] M.C. Escher official website. https://mcescher.com/gallery/symmetry/#iLightbox[gallery_image_1]/55 (accessed Feb. 13, 2024).

[4] D. Schattschneider. *M.C. Escher: Visions of Symmetry* New Edition. Harry N. Abrams, 2004.