The Flat Tile Set Enables Non-midpoint Tile Edge Crossings

Lars S. Eriksson

Mariefred, Sweden, lars@rixn.se

Abstract

This paper focuses on pattern creation, specifically using a technique that incorporates modular tiles within 5-fold patterns. These tiles feature a motif that crosses over each side, interacting with the motif on the adjacent tiles. The novelty of this paper lies in the introduction of crossings that are not at midpoint. Previous papers focused on non-equilateral tiles, but all tiles had midpoint crossings. Here I introduce the first non-midpoint tile set, called the Flat tile set, by presenting the core tile of the set, called the Fone tile. The Flat tile set is the missing link between two types of historical patterns, the Starry and the Floral pattern types (terms coined by Castera). When the two types are found together in existing traditional Islamic geometric patterns, one is often more dominant over the other and the other is concentrated around star formations. As Flat tiles link these two worlds together, they enable a wider spectrum of historical patterns to be tiled and gives promise to new interesting patterns. Now, both types can grow and claim equal part of the pattern.

Introduction

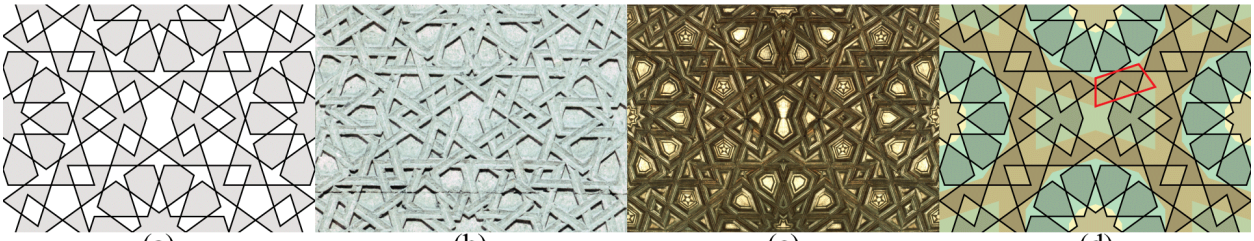

Figure 1: *The historical pattern in the Wakala al-Ghuri (b) and the Hatuniye complex (c) cannot be tiled without the Fone tile (see the red polygon in d).*

The common method of replicating Islamic geometric patterns is by hand, using compasses and a straightedge, but in recent years, the method of tiling (polygons in contact [8]) has been gaining momentum, especially after Lu and Steinhardt popularized it in 2007 and coined the name "Girih Tiles" [7], and Bonner had his extensive book on the matter published in 2017 [1].

In my previous papers, two tile sets have been defined, the evolved Girih tile set [5] and the Short tile set [6]. This paper will introduce a third tile set, the Flat tile set. In this paper new terms will be introduced. Below is a list of the new terms with a short description. Each term will be explained further in the paper.

- *Flat tile set*: The first non-midpoint tile set.
- *Fone*: The first defined tile in the Flat tile set.
- *F*: The side length that makes a tile belong to the Flat tile set.
- *Y*: The side length of the Fone tile that interacts with Floral pattern.
- *Z*: The S-side length from previous paper [6] is here renamed to Z.
- *S*: The new S-side length, which is a result of an adjustment caused by the Flat tiles.
- *Core F*: The Fone tile stands out among other Flat tiles. It is the key tile to link G with S.
- *F%*: A crossing percentage with a value of $(3 - \varphi)/2$ (here applied to the F-side length).
- *F72F%*: The full definition of the F-side's key properties.

The evolved Girih tile set

The *evolved Girih tile set* [5] consists of tiles with sides in base length (G), including the Core 5 tiles and "adapter" tiles that have a combination of different side lengths (G and P), see Figure 2. The Golden tiles are tiles scaled to fit P-sides.

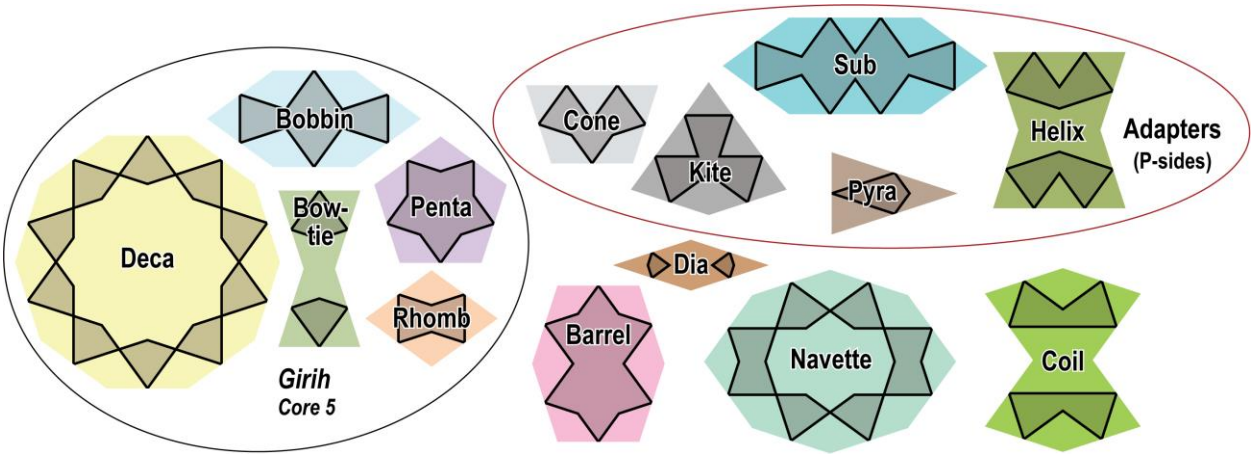

Figure 2: *The evolved Girih tile set chart, here with 72 degrees crossing angles*

The Short tile set

The Short tile set [6] includes the Core S adapter tiles and the Short Core 5 tiles, see Figure 3. The Core S tiles have two different side lengths, S and G, and the Core 5 Short tiles have all sides S. The sides of the tiles in both sets have symmetrical edge rules. All tiles have a midpoint crossing but, depending on the edge rule angle, the P-sides can also have a double crossing.

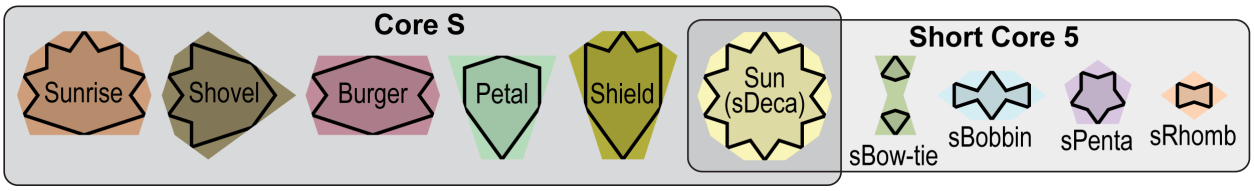

Figure 3: *The Short tile set chart. It cannot directly interact with the evolved Girih tile set.*

The Flat tile set

So far, the evolved Girih tiles and the Short tiles have only interacted with themselves, and as long as they do, it works fine, but when you want to let the Short tiles interact with the evolved Girih tiles, another tile set have to be introduced – the *Flat tile set*. As the introduction of adapters opened up for different side lengths (*non-equilateral tiles*), the Flat tiles opens up the world of *non-midpoint crossings*. There are three non-midpoint tile sets, and this is the first to be presented.

Edge Rules

In the previous papers [5, 6] I used the term "*edge rule*" to describe the way lines cross over to another tile, that is, the edge between the tiles' sides. The focus was mainly on the angle by which the lines cross the edge. This part has been covered by Hankin in his PIC method [8]. For this paper I will nuance the edge-rule concept and also extend the meaning. Besides the *crossing angle* and the *side length*, the non-midpoint concept requires yet another parameter to better define the edge rule, the *crossing percentage*.

Crossing angles

In traditional 5-fold patterns, the *crossing angles* can have values of 36, 72, 108, and 144 degrees (which is measured between the lines, not between the edge and one of the lines), see Figure 4. Other edge rule parameters, not valid for this paper, are the style of the line (like thickness or color) and how the line behave inside the tile (like if it starts to curve based on a defined radius etc.).

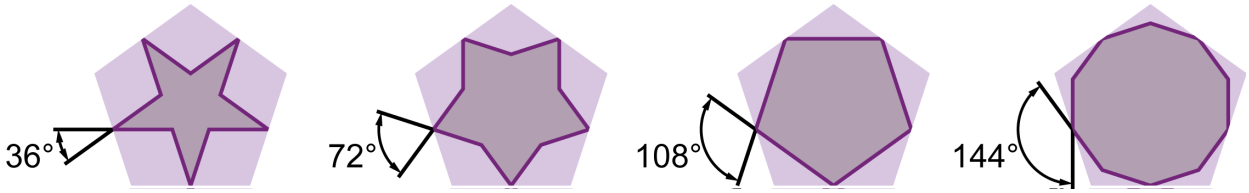

Figure 4: *Edge-rule angles for the tile crossings in 5-fold patterns.*

Side lengths

Let the standard length of the Core 5 Girih tiles' sides be 1. In the Golden tiles and adapter tiles, like the Cone tile, we have a side length of φ. The Short tile set introduced the S-side with the length of $2 - \varphi$. Let the letters G, P and S denominate these lengths.

$\mathbf{G = 1}$	= 1	
$\boldsymbol{P = \varphi}$	≈ 1.618	
$\boldsymbol{S = 2 - \varphi}$	≈ 0.382	Note that this will be renamed as Z later in this paper.

Nomenclature

The way to refer to the tiles, crossings, and angles have so far been fairly easy. With the introduction of non-midpoint crossings, we need to have a better system to clarify these parameters. Let's start with the definitions for combinations for the current side lengths and midpoint crossing angles. Figure 5 shows the edge rule combinations for the evolved Girih tile set (G72 and P72D for Starry patterns, and G36 and P108 for Floral patterns), and for the Short tile set (G144 and S72 for Floral patterns).

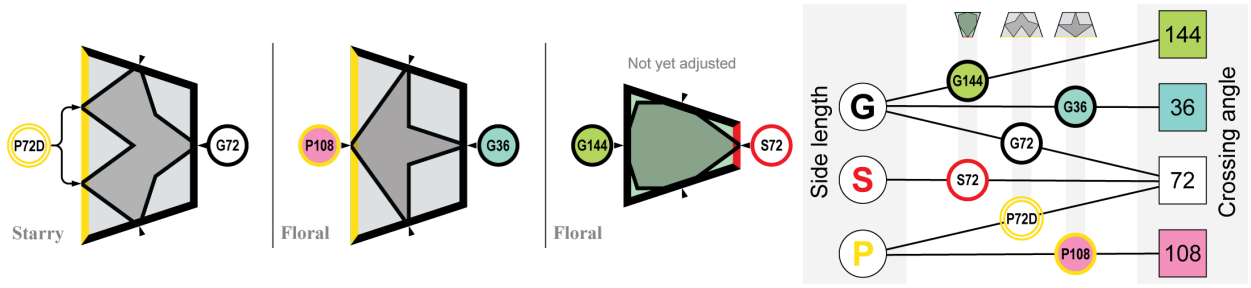

Figure 5: *Side length and crossing angle chart for G-, P-, and S-sides. The "D" in P72D stands for double, as in a double crossing. Note that the length and angles are color coded.*

In this paper three new side lengths will be introduced, and one existing will be redefined. One of the new side lengths, denoted "F", will have a non-midpoint crossing. I will present one tile to exemplify this. I call it the Fone tile, see section "The Fone Tile – The Portal to F". The Fone tile forms the core of a new tile set: the *Flat tile set*.

Crossing percentages

For a midpoint crossing the distance from the endpoint to the crossing is 50 % of the side length. For non-midpoint crossings the crossing distance are between 50 % to 100 %.

Introduction summary

Let me summarize what is going to be covered going forward. I will introduce three new things:

- The Flat tile set, populated here with its first tile, the Fone tile.
- Non-midpoint crossings
- Crossing percentage, here applied to edges with F-length.

Let's start with the Fone tile.

The Fone Tile – The Portal to F

The Cone tile is part of the evolved Girih tile set. It can carry a motif with an edge rule angle of 72° for each base sides crossing, and a double crossing of 72° each for the P-side.

In extrapolating the motif lines of the Cone tile, they meet inside the tile in an obtuse angle (144°), see the white lines in Figure 6. If we draw a horizontal line at this intersection and delete the area above, a new tile emerges that takes the form of a *flat* Cone tile. The name of this new tile is the *Fone* tile. The Fone tile is the core tile of the new set called the Flat tile set.

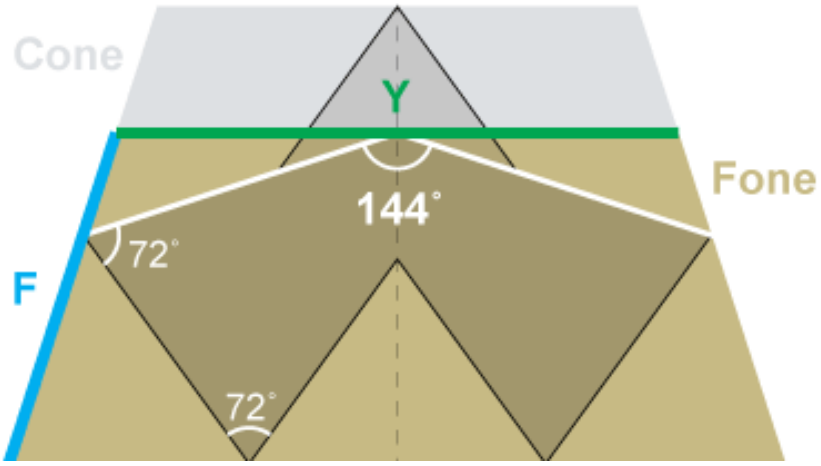

Figure 6: *Cone to Fone. Both tiles are trapezoid-shaped, but the Fone tile is flatter than the Cone tile.*

The F- and Y-side length

In transforming the Cone tile into a Fone tile, the three base sides of the Cone tile (G) have now changed into two new side lengths, Y and F, see Figure 7. The Y-side, which is opposite the P-side, have become longer: $Y = \varphi/(3-\varphi) \approx 1.171$, and the remaining two sides have equal length but shorter. This length defines the F-side length of the Flat tile (hence F): $F = 1/(3-\varphi) \approx 0.724$.

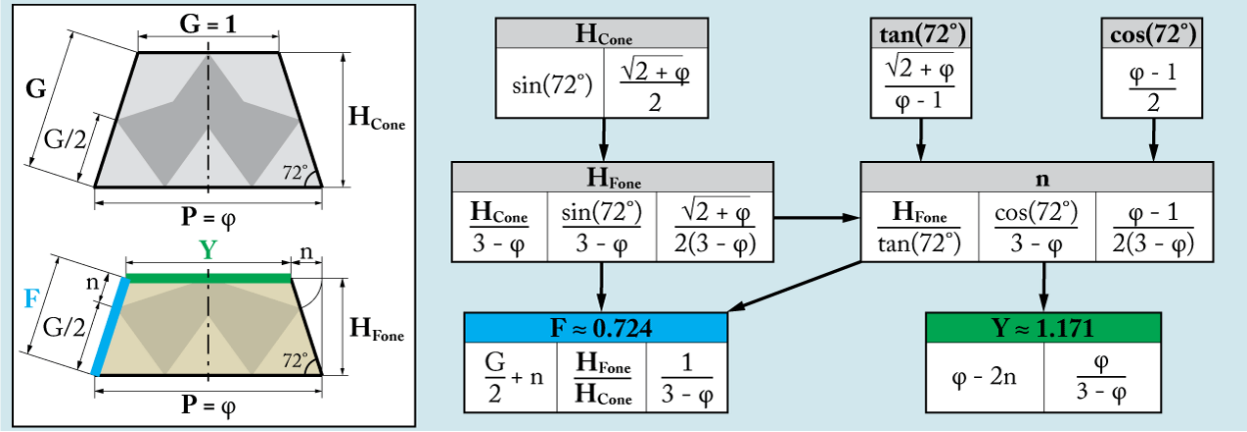

Figure 7: *Measurement connections that leads to the F and Y measurements.*

While the P-side length (P = φ) is the same for both tiles, the height has changed; from $H_{Cone} = \sqrt{2+\varphi}/2$ to $H_{Fone} = \sqrt{2+\varphi}/2(3-\varphi)$. Note, that dividing the height of the Fone tile with the height of the Cone tile gives the length of the F-side.

Starry and Floral patterns

Castera acknowledge two types of 5-fold pattern – *Starry* and *Floral* pattern [2, 3], see Figure 8. Compare with Bonner's Median and Acute pattern families [1].

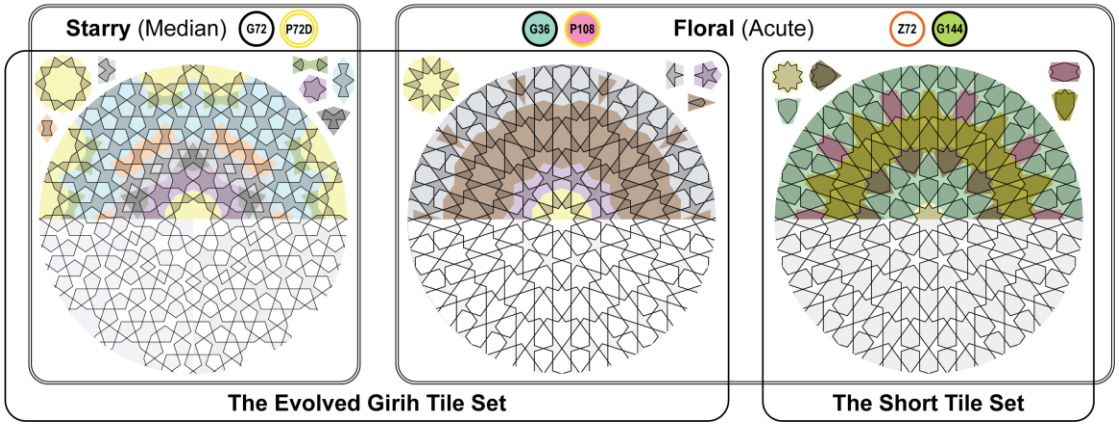

Figure 8: *Tiled Starry and Floral pattern. Note that the two instances of the Floral pattern are the same pattern. It is tiled in two ways ("invertiled").*

For 5-fold patterns, Bonner define the edge rule angle to 72º for Median, and 36º for Acute, pattern families. This is a generalization that does not take into consideration that some tiles (adapters) have multiple side lengths with different edge rule angles. This makes Castera's terms Starry and Floral a better choice when referring to these two distinct types of 5-fold patterns.

Connecting Two Worlds – Starry and Floral

When Starry and Floral pattern element are to be combined into one pattern, the existing tile sets are not enough. To solve this, we need two things; a *link between* the two, and an *adjustment of the side lengths* of the Short tiles [6]. From hereon I use the adjusted values.

The missing link

The P-side of the Fone tile adapts to the Starry pattern and the Y-side adapts to the Floral pattern, see Figure 9 and 10. This has been depicted by Castera in a workshop from 2016 [4], but the Fone tile was not specifically defined.

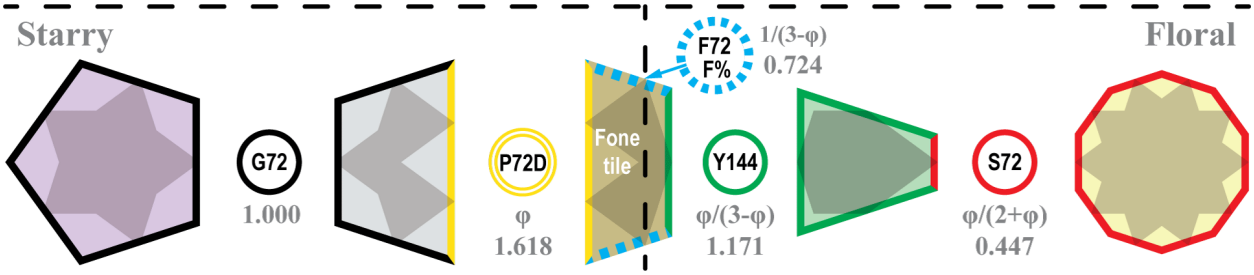

Figure 9: *From Starry to Floral – Flat makes it happen as it is the key link between the two worlds. All the pattern motif on the left of the dashed line is Starry, and Floral is to the right. Edges are color-coded by respective edge length, and edges with non-midpoint crossings have dashed lines. Note that the green and red color code refers to the adjusted side lengths, see the section "The Flat tile set Forces the Short tiles to Become Bigger" below.*

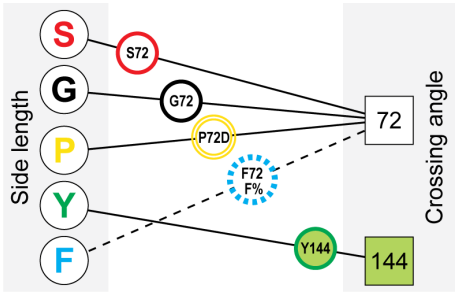

Figure 10: *The side length and crossing angle chart needed for linking Starry and Floral together.*

The 72-degree non-midpoint crossing angle of the Fone tile's F-side provide the start of the definition: *F72*. But we need to add the crossing percentage: *F%*. The full definition for the F-side then becomes *F72F%*.

The value of the *F%* crossing percentage is $(3-\varphi)/2$, which equals to approximately 69.1 % of the side length it is applied to, in this case to the F-side length. Several different combinations will be possible. The length from one side of the F-side to the crossing is $F*F\% = 0.5$. This value will appear again in future papers with other side lengths, as well as, other crossing percentages. (Hints*:* $\varphi * H\% = 0.5$ and $N * N\% = 0.5$.) As the F-side has both a *different side length* and a *non-midpoint crossing*, it does not fit the criteria for any of Bonner's four pattern families [1]. This goes for all future non-midpoint tiles too.

The Flat tile set Forces the Short tiles to Become Bigger

In the previous paper about the Short tile set [6] I defined the S-side as $2-\varphi$. This was a logic step at the time as the Petal tile derives from the Bow-tie tile of the Core 5 Girih tile set. But unfortunately, these tiles don't fit the Flat tiles, so we need to scale them up by the Y-value to fit, se Figure 11. The old G-length of 1 becomes $Y \approx 1.171$, and the old S-length becomes $\varphi/(2+\varphi) \approx 0.447$.

The question is, what are we going to call this new length. The most logic solution is to just address a letter to it. But the problem is that these tiles won't be used so much with their old side lengths; it is with the new side length they will come into play. As these tiles are referred to as the Short tiles it would be suitable to *let the S length carry this new value and denote the old side length a new letter, Z.*

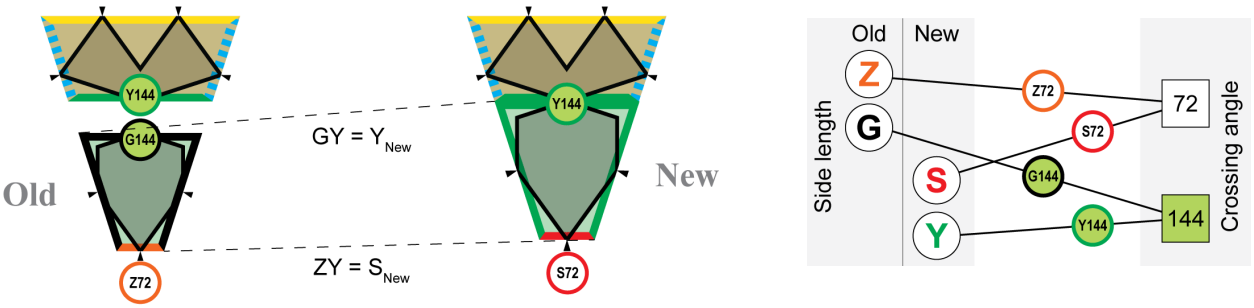

Figure 11: *The tiles in the Short tile set are scaled up by the value of the Y-length (ca.+17.1%)*

In Figure 11, the Petal tile is scaled to fit the Y-side of the Fone tile, so the corresponding sides match. This means that the Flat tile set forces all tiles in the Short tile set to scale by the Y value. This adjustment leaves us with the following side lengths:

$Y = \varphi/(3-\varphi)$	≈ 1.171	The base sides go from G-length to become Y-length.
$S = \varphi/(2+\varphi)$	≈ 0.447	A new value for S, which is the old S value times the Y value.
$Z = 2-\varphi$	≈ 0.382	The old S value have to be renamed. I've chosen the letter Z.
$F = 1/(3-\varphi)$	≈ 0.724	The F value. It can also be defined as $F = Y - S$).

The enlarged Short tiles will now be able to connect to the Flat tiles, as shown earlier in Figure 9.

Fone Tile Pattern Example

The pattern in Figure 1a is very common. For example, it can be found in the Hatuniye complex, Turkey [9], see Figure 1b, and the Wikala of Ghuri caravanserai, Egypt [10], see Figure 1c. This pattern can only be tiled with the help of the Fone tile, see Figure 1d. It adapts from the Starry Golden bow-tie tile to the Floral stars. The F-sides of the Fone tile only interacts with themselves, see the blue color in Figure 12.

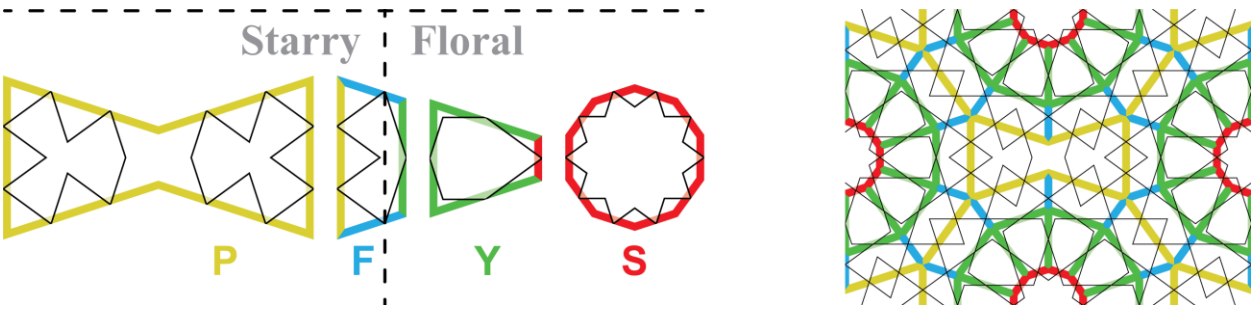

Figure 12: *Color coded side lengths.*

Summary and Conclusions

Each tile set is a collection of tiles that meet the specific criteria for that particular set. The criteria for the selection are based on a tile's usefulness. For what is a tile useful?

The criteria for a tile to qualify for a tile set are a combination of adherence to historical patterns, aesthetic appeal, mathematical simplicity, and how well it fits into the overall hierarchy. It's an assessment based on a combination of its shape, motif, and edge rule.

The selection of the *first* tile set, the evolved Girih tile set [5], has 14 tiles, while many more are possible. The initial five Girih tiles were redefined as the core subset, *Core 5*. The core of any set is a selection of the most important tiles within each set. The *second* tile set, the Short tile set [6], have six tiles as its *Core S* subset. In future papers more tiles will be presented in each tile set, but they will be non-core tiles.

The Flat tile set will contain a lot of tiles, making it hard to define a core selection. There isn't a clear selection of tiles to define as *Core F*. It is not so distinct where the tiles belong. In addition, when we combine F-sides with the other side lengths, we get a relatively complex matrix of Flat tiles, see Figure 13, which have to be made clear in future papers.

However, there is one tile in particular that elevates itself above the others. The Fone tile is here chosen as the single tile to form the Core F subset. It earns this spot due to its necessity to tile many common existing Islamic geometric patterns, and for its unique ability to link Starry and Floral pattern together. A few other F tiles can also link the two worlds together, but the Fone tile establishes the link through the P-side while other Flat tiles use the G-side instead, see Figure 13. This makes the Fone tile more useful for common patterns with Floral stars in Starry pattern. It is by far the most useful Flat tile.

The Future

Some existing Islamic geometric pattern requires other F tiles to be tiled. These will form the next level of subsets. Beyond that, there are more tiles that haven't any representation in historical patterns. They will enable the creation of unique patterns that would be considered to still look like traditional Islamic geometric pattern.

In future papers, the Flat tile set will be populated with more useful Flat tiles, based on these distinctions, see Figure 13. The future of tiling is getting more interesting, but also more complex.

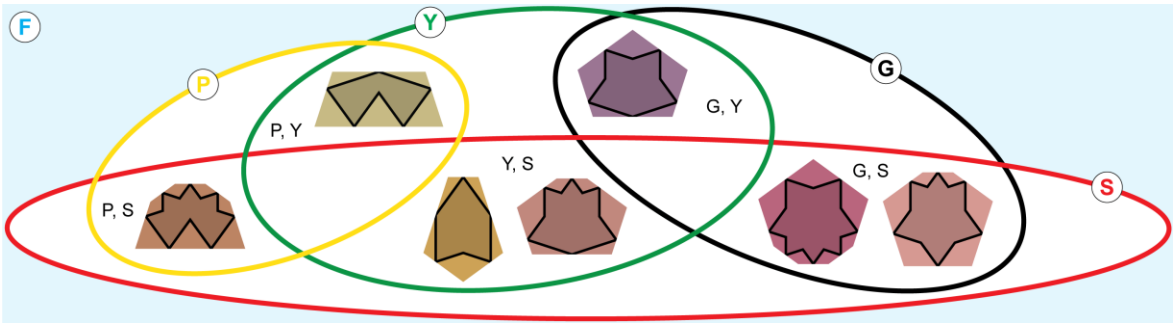

Figure 13: *A selection of future Flat tiles that adapts from one side length to another besides the F-side length. All but the Y, S combination adapts from Starry to Floral pattern.*

References

[1] J. Bonner and C. Kaplan. "Islamic Geometric Patterns: Their Historical Development and Traditional Methods of Construction." *Springer* New York, 2017.

[2] J-M. Castera. "Flying Patterns." *Proc. ISAMA/Bridges*, Coimbra, 2011.

[3] J-M. Castera. "Persian Variations." *Nexus Network Journal*, 18 (1), 2016, pp. 1–52.

[4] J-M. Castera. "Various Points of View on Geometric Patterns." *The 3rd International Workshop on Geometric Patterns in Islamic Art*, Istanbul, 2016.

[5] L. S. Eriksson. "Adapter Tiles Evolve the Girih Tile Set." *Bridges Conference*, 2020, pp. 19–26. https://www.researchgate.net/publication/351093739_Adapter_Tiles_Evolves_the_Girih_Tile_Set.

[6] L. S. Eriksson. "The Short Tiles Category." *Bridges Conference*, 2021, pp. 127–134. https://www.researchgate.net/publication/353444031_The_Short_Tiles_Category.

[7] Wikipedia. Girih Tiles. https://en.wikipedia.org/wiki/Girih_tiles.

[8] E. H. Hankin. "The Drawing of Geometric Patterns in Saracenic Art." *Memoires of the Archaeological Survey of India*. Vol 15. Government of India Central Publication Branch, 1925.

[9] Hatuniye complex, Kayseri, Turkey. Pattern in Islamic Art. https://patterninislamicart.com/s/collections/main-archive/piia_image/tur0731.

[10] Wakala al-Ghuri, Cairo, Egypt. Pattern in Islamic Art. https://patterninislamicart.com/archive/1/140.