Puppetry, Poetry, Dance, and Sound on the Bridges of Königsberg: Embodied Work with Euler Paths and Circuits

Susan Gerofsky[1], S. Brackett Robertson[2], Karl Schaffer[3], and Ekaterina Zharinova[4]

[1]University of British Columbia, Vancouver, Canada; susan.gerofsky@ubc.ca
[2]Public Math, Saint Paul, MN, USA; sbrackettrobertson@gmail.com
[3] De Anza College, Cupertino, CA, USA; karl_schaffer@yahoo.com
[4]Independent artist-scholar, Wembley, Greater London, UK; ekazha@gmail.com

Abstract

The co-authors, from the Dance, Movement Arts and Mathematics (DMAM) group, have developed this interactive workshop based on Euler paths and circuits to explore the use of large-scale body movements, dance, puppets, storytelling, combinatoric poetry, and sound in conjunction with graph theory concepts. The aim is to create both intriguing new artistic performances and deeper public mathematical understandings in this interdisciplinary space, and to highlight the importance of innovative intersections between mathematics and performance arts.

Introduction: Who We Are

This paper and workshop have been developed in the context of the Dance, Movement Arts, and Mathematics group (DMAM), a group of mathematical choreographers, dancers, musicians, teachers, museum educators, professors, and others who have met once a month on Zoom since July 2020. The four co-authors are all members and contributors in DMAM. This group was initiated as an offshoot of the Bridges math/dance community, although it now includes many members who have not attended Bridges conferences. Our monthly Zoom meetings include presentations of our work, discussions of conceptual issues, interactive workshops, and the screening of dance, movement arts, and mathematics films. As a group, we are also in the process of compiling a bibliography and filmography of articles, books, videos and websites on our areas of interest.

Over time, we have defined our interests in three domains: (i) mathematical ideas that inspire new movement and dance creations; (ii) dance and movement ideas that inspire new mathematical or math education explorations; and (iii) interdisciplinary work that lives in a 'third space' between dance/movement and mathematics/math education, favoring neither one nor the other, but inspiring new creative work in the liminal space between movement arts and mathematics. This workshop aims to address domain (iii), an interdisciplinary space of integrated artistic creation, mathematical exploration and learning. The co-authors and leaders of this Bridges workshop, with collective experience in engaging with public mathematics and mathematical arts in public performance, offer diverse ideas to connect with and potentially contribute to all these varied experiences.

Why We Chose Graph Theory and the Bridges of Königsberg

This workshop deals with paths and circuits in the realm of graph theory, a branch of modern discrete mathematics where connectivity is salient. Briefly, and somewhat intuitively, a graph is a collection of vertices or nodes, connected by edges or arcs. Graphs are highly relevant for problem–solving in our world of connected computer and communications networks. It has been noted as a readily accessible topic that initially requires little background knowledge or notation — a great 'low floor, high ceiling' topic for engagement in mathematical thinking and artistic creation.

Graphs and networks have origins in many world cultures. For example, Marcia Ascher describes a variety of puzzles and cultural forms tracing unicursal routes through graph-like diagrams that are equivalent to finding Euler circuits [1]. The concept of Euler paths has recently been found to be of use in determining how to reassemble fragments of DNA into longer strands [11].

The first mention of graphs in the mathematical literature usually traces back to the Königsberg Bridge problem. Euler's solution to this problem in 1736 established that a graph has an Euler circuit, which is a circuit containing each edge exactly once, if it is connected and every vertex has even degree. It has an Euler path if it is connected and exactly two vertices are of odd degree. Finding the number of possible Euler paths and circuits on a graph can be a more difficult problem, with surprising results! Our workshop considers the "Eulerizations" of connected graphs that do not have an Euler circuit or path, by adding or subtracting edges. A Mathematica tabulation shows, surprisingly, that there are 1296 Euler paths of the Königsberg bridges graph given the allowance that one bridge is traversed twice (see online supplement to this paper). If two non-adjacent bridges are each traversed twice in the Königsberg graph then the "minimally Eulerized" graph that results will have an Euler circuit. The number of such distinct Euler circuits is 192. However, if we consider that each such circuit of nine edges might be traversed in either direction or started along any of the nine edges, that number should be multiplied by 18 and we get a total of 3,456 circuits. Since many of our activities involve placing distinct labels on the edges and reading them out as we traverse, that count is sensible. This also corresponds to the experience Königsbergers of old might have had if they allowed themselves to traverse two bridges twice.

We are interested in the Königsberg Bridge problem because of its colorful historical context, which lends itself to imaginative storytelling and artistic expression. We can easily imagine the citizens of Königsberg realizing, on some level, that there are many potential paths to try out to solve their bet that 'you can't cross each bridge just once'! In fact Euler himself commented that tabulating all possibilities "…is too tedious and too difficult because of the large number of possible combinations…" [3]. We note that many mathematics educators have introduced students to the Königsberg bridges, usually in the context of leading them to understand that an Euler circuit is not possible since all vertices are not of even degree, rather than having them attempt to traverse an Eulerization of the graph, as we do here [14].

Integrating Musical Sounds and Poetry with Dance, Movement and Puppetry

In our workshop, we draw special attention to performative artistic modalities (large-scale movement, dance and puppetry), and sound arts (music and performed poetry).

Go at the speed that you can
Bearing, in a flurry of anticipation --
Move without looking as you
Whirl on a bridge between two worlds.
Moving hesitantly,
Bear, on foot or otherwise --
Moving with stuttering steps, and
Going wildly whirling.

Figure 1: *Poem and dance instructions generated by an Euler path (S. Gerofsky).*

Rhythms and patterns in music, poetry, movement and story may be experienced more viscerally than those sensed primarily through vision. They have the potential to offer complementary sensory understandings of a mathematical pattern (see Figure 1). These combined understandings may be transformative in drawing our attention to features of the mathematics — see, for example, 2014 Fields Medalist Manjul Bhargava's explanation of insights he gained both through classical tabla drumming and tactile mathematical games like Rubik's Cube [16]. One co-author has been interested for some time

in the sonification of mathematical patterns and the use of gesture and whole-body movement as a way to gain new understandings [4], and to create new poetry and performance artworks [5]
.

(a) (b)

Figure 2: *(a) Plan for dragon puppet; (b) Painting of Königsberg bridge setting (S. B. Robertson).*

Puppets are an inherently story-focused medium. For a Bridges audience, we aim to explore how puppets could be used to illuminate math concepts that Bridges participants wish to share more broadly with a public audience (see Figure 2). Although Bridges papers have rarely explored puppetry – see [18] for an exception – they are popular in math education (for example see [8][9]). Puppets have been used to teach probability [6], and Javanese puppets have been used in mathematics teaching in multiple ways [12][10]. Tim and Tanya Chartier, frequent performers at Bridges, often incorporate puppetry in their shows [13]. One of this paper's co-authors created a puppet video explicating the Prisoner's Dilemma as intro to a dance piece about the arms race [15]. This workshop offers participants a hands-on introduction to the art of puppetry in mathematical context.

Bringing together these multiple artistic forms offers a multi-sensory experience for participants. At a meta-level, it can be a way to work towards an interdisciplinary performance. According to Hunter, "Interdisciplinary work is necessarily concerned with what is not present or represented in existing disciplines, but felt" [7, p. 2]. Hence, there is a potentiality of producing something not yet manifested in each of the synthesized fields. For instance, the collaboration of digital artists Marc Downie and Paul Kaiser and choreographer Merce Cunningham resulted in the visualization of Cunningham's dancing hands and fingers. The interdisciplinary work is called *Loops* and can (noteworthy for our workshop) be associated with a graph: "The motion-captured joints become nodes in a network that sets them into fluctuating relationships with one another" [2]. It could not be produced without engaging both dance and technology. This workshop's integration of music, sound, poetry, dance, and puppetry is a way to experiment in an interdisciplinary space with the integrated aims of creating new and intriguing art, engaging people (mathematics specialists and non-specialists) in experiencing mathematical patterns and processes, gaining deeper mathematical understandings, and generating new questions about Euler paths and circuits.

Workshop Description and Activities

The workshop engages participants in ways of thinking, making, composing, listening and dancing as they experience and express aspects of Eulerian circuits and paths. Participants explore these structures and their relationships through multi-sensory, collaborative problem-solving activities, using modalities as varied as puppetry, walking/ moving/ dancing, creating musical harmonies with instruments, and creating combinatoric poetry. The performative co-creation of a work of performance art, involving workshop participants, is planned for Bridges 2023 Informal Music Night. Here is the structure of the workshop:

(1) **Conceptual and historical introduction**. The workshop begins with a short talk and slides introducing the Königsberg Bridge Problem and Eulerian paths and circuits as a story and as a means of framing contemporary problems.

(2) **Split into four breakout groups**, (i)-(iv), each exploring aspects of Eulerian paths and circuits through a variety of embodied, multisensory modalities for the greater part of the workshop time.

(3) **Return from breakout groups:** Regathering as a whole group to perform and share our models, puppets, movement sequences, musical representations, and original poetry based on our embodied explorations of Eulerian paths and graphs. This is also the time to plan how to combine or otherwise perform our creations for Bridges Informal Music Night for interested participants.

(4) **Discussion about what participants take away from this experience** in terms of the affordances of embodied, large-scale exploratory activities engaging artistic modes like puppetry, dance, music, and poetry in the participants' own contexts of public and engaged mathematical art.

Description of the Four Breakout Groups

Breakout group (i): *Large-scale puppetry and story (Robertson)*: Story-telling and spatial exploration of familiar and then open-ended Eulerian paths using a model of the Königsberg Bridges, explored by participants making their own puppets and stories, and adding or subtracting bridges/ edges to help with storytelling. Inspired by the multitudes of puppetry styles, we are interested in the ways puppets can be used to approach graph theory, and revisit ways that others have found intersections with puppetry and mathematics. The workshop at Bridges presents a puppet interpretation of the Bridges of Königsberg problem involving a landscape shaped like 18th C Königsberg. Workshop participants are introduced to an example puppet, a long-tailed dragon created by Robertson, and then create their own puppets to explore the landscape. An initial story idea is exemplified here:

The dragon is passing between the seasons, over seven magical bridges. There are two bridges between spring and summer, two between summer and fall, one between summer and winter, one between fall and winter, and one between winter and spring. Each bridge the dragon passes gives them a new token to carry, whether that's a word or a leaf, but if they cross it the other direction, they have to give it back to the bridge.

In the landscape, there are multiple interactions that the workshop participants' puppets experience. Each can add to the story they write with their puppet as they travel between story stations, creating paths that do or don't turn back. There is the option to add or remove a bridge on the landscape so that participants can explore how following a complete path affects the story and the ability of their puppet to complete all the interactions. Each bridge and piece of land has a piece of the story. Participants explore what happens to the presence of Eulerian circuits when the landscape shifts.

Breakout group (ii): *Poetry, sound and dance, model 1 (Gerofsky):* Participants walking, dancing, creating poetry and musical accompaniment with Eulerian paths along the edges of a large-scale (chalk or duct-tape-on-tarp) diagram of the Königsberg Bridges on the floor. Participants alter the graph by adding or subtracting bridges to create a pleasing Euler path or circuit, and then compose an original combinatoric poem made with verbs on vertices and adverbial phrases on edges (see Figure 3). As participants walk and dance the track of the graph, in the manner suggested by the poem they are creating, bellringers at each vertex mark the parity of visits to the vertex (ringing when parity is even, stopping when it is odd). Finally, four participants, one per vertex, dance the resulting graph, each displacing the next as they reach a vertex, simultaneously creating an 'operatic' performance comprising the dance, combinatoric poem, and music composition of tuned bell harmonies.

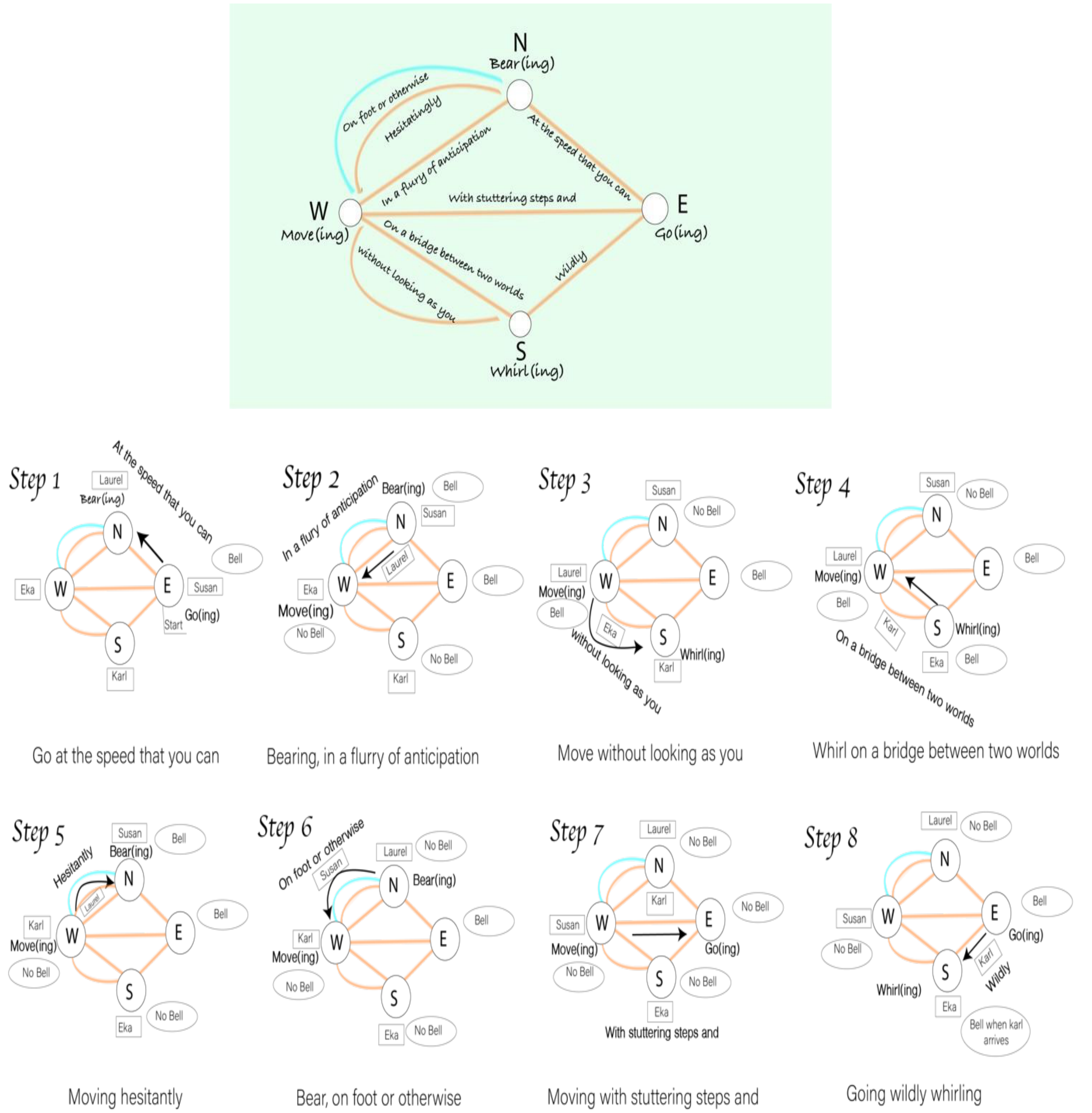
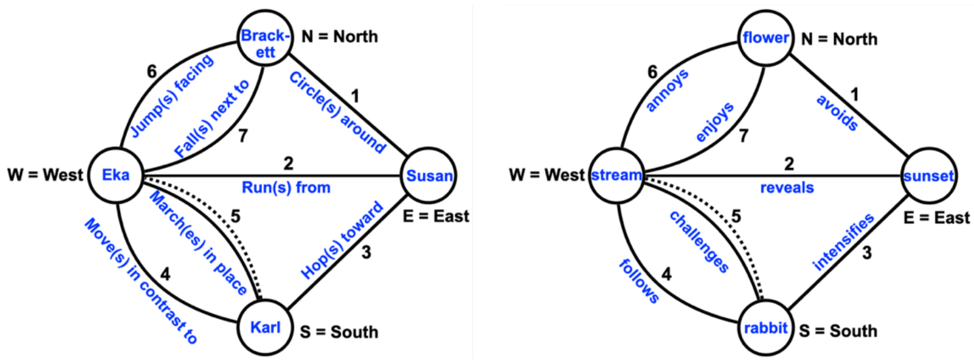


Figure 3: *Dance, poetry and sound on an augmented Königsberg graph (N. Byrne & S. Gerofsky).*

Breakout group (iii): ***Poetry and dance, model 2 (Schaffer):*** To generate a movement sequence in this model, four participants each start from a vertex and follow interaction instructions given by labels on the edges of an Euler path or circuit in an Eulerized version of the graph. The order in which the dancers move is determined by the path sequence, but the dancers are unconstrained as to where they move in space. Alternatively, combinatoric poems are created using nouns placed at the vertices and verbs placed on the edges of an Euler path or circuit that has been chosen by participants. In Figure 4(a), for example, doubling the traversal of bridge 5 results in a graph with an Euler path starting at vertex N and ending at vertex E, such as 67135452.



(a) (b)

Figure 4*: (a) Königsberg bridges graph, with vertices E, S, W, and N labeled by dancers, and edges labeled by movements and directions. (b) Labels are nouns for vertices and verbs for edges. (K. Schaffer).*

We might interpret the movement sequence suggested by this path as:

Brackett jumps facing Eka, who falls next to Brackett, who then circles around Susan. Susan then hops toward Karl who marches in place while Eka moves in contrast to his marching, whence Karl continues to march in place, at which point Eka runs away from Susan.

Following this same Euler path, and using the words in the graph in Figure 4(b), we can generate a simple word order sequence (which might then be further revised to form an actual poem):

A flower annoys the stream which is enjoying the flower as the flower avoids the sunset that intensifies a rabbit's challenge to the stream. The stream follows after the rabbit, which further challenges the stream, all of which ultimately reveals the sunset.

Breakout group (iv): ***Dance performance, model 3 (Zharinova):*** This model implements the mathematical concepts of the Euler path/circuit to create continuous movement sequences/compositions through two dance performance practices based on the graph: one generating an order of different tasks for motion assigned to the graph's edges, and the other with dancers traveling through areas of the stage corresponding to vertices of the graph. Participants choose their dance movement and physical action options, constrained by the given rules but otherwise not visibly connected to the Bridges of Königsberg.

For the first practice, used as a warm-up, we initiate movement from different body parts based on paths on a modified Königsberg diagram. For instance, there are two orders generated by two different Euler paths portrayed in Figure 5: head, elbows, hands, shoulders, legs, pelvis and legs, pelvis, elbows, hands, shoulders, head. (The second order is exemplified in the video [19].) The second practice determines locomotion in space. This time, vertices W, N, E, and S correspond to the areas marked on the floor (see Figure 6), where we perform in the areas and transition between them. A task might be assigned to an area. For instance, we might decide to dance as if there were lava in area N and dance on the floor in area S, with everything else unspecified, using a general principle similar to that of the Hub computer network music ensemble that "Anything not specified may be improvised" [17]. Different qualities of movement or particular movements or physical actions, which may include the use of voice or props/puppets, could be assigned to certain areas as well. There is an example of this practice with the spatial arrangement following the Euler circuit depicted in Figure 7(a) — see the video [20]. These two practices can be combined. All the tasks can be co-created collectively or chosen by each participant individually. Both practices can be explored solo and in groups.

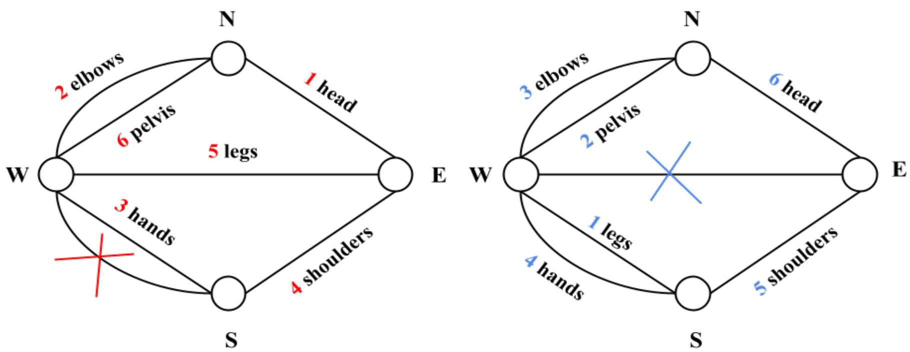


Figure 5: *Two Eulerian paths produce two different orders of body parts initiating dance, where numbers designate order: "1 head" on the left means we start from the head's movement. (E. Zharinova).*

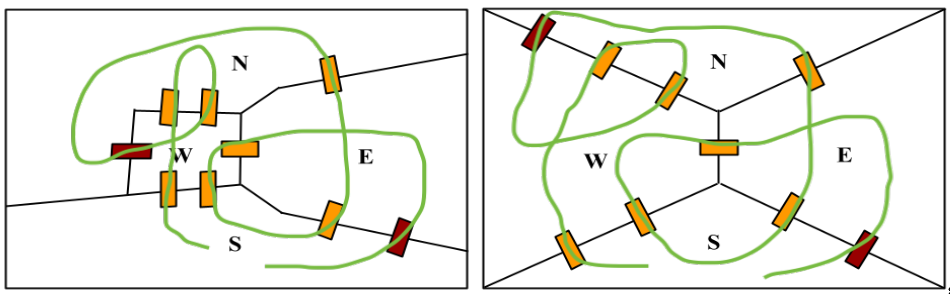
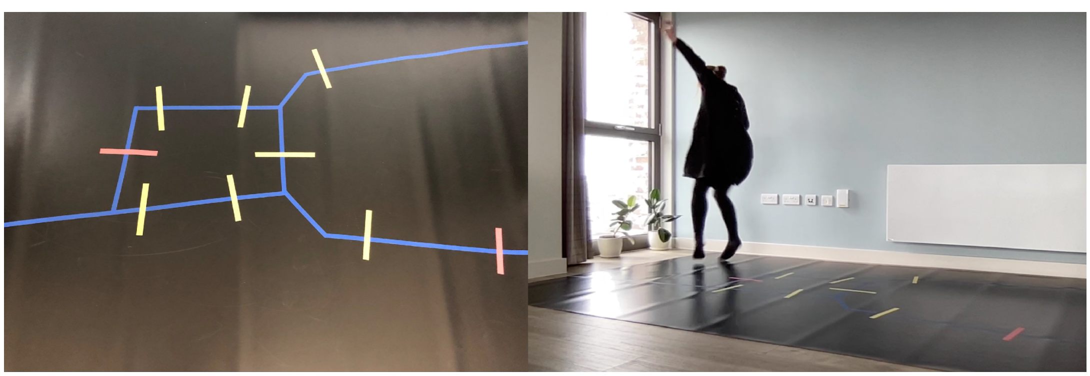


Figure 6: *One Eulerian circuit with two spatial possibilities. On the left is a replica of Königsberg's landscape, and on the right is its modification. The added bridges are red. (E. Zharinova)*



(a) (b)

Figure 7: *(a) The marks on the floor for the first spatial arrangement. (Photo: E. Zharinova)*
(b) A screenshot from videoed dancing in the lava area (N). (In photo: E. Zharinova.)

Conclusions

The four co-authors learned a great deal from one another in creating this workshop that integrates dance, puppetry, poetry, and sound in the context of an exploration of graph theoretical ideas with the Bridges of Königsberg. We encourage readers and participants to initiate their own experimental work across artistic and mathematical disciplines. Anyone interested in dance, movement arts and mathematics is invited to join the DMAM group, by contacting any of the co-authors.

References

[1] M. Ascher. "Graphs in Cultures: A Study in Ethnomathematics." *Historica Mathematica*, vol. 15, no. 3, 1988, pp. 201-227.

[2] M. Downie and P. Kaiser. "Loops." *OpenEndedGroup*, n.d., http://openendedgroup.com/artworks/loops.html.

[3] L. Euler. "The Bridges of Königsberg." In *The World of Mathematics*, vol. 1, edited by J. Newman, Simon and Schuster, 1956.

[4] S. Gerofsky. "Seeing the Graph vs. Being the Graph". In *Integrating Gestures*, edited by G. Stam, John Benjamins, 2011.

[5] S. Gerofsky. "Seven Strands of Alphabetical Braided Crows." Video, 2021. https://vimeo.com/537573341.

[6] R. E. Groth, J. W. Austin, M. Naumann, and M. Rickards. "Probability Puppets." *Teaching Statistics,* vol. 41, no. 2, 2019, pp. 54-57.

[7] L. Hunter. "Being In-Between: Performance Studies and Processes for Sustaining Interdisciplinarity." *Cogent Arts & Humanities*, vol. 2, no. 1, 2015. https://www.tandfonline.com/doi/pdf/10.1080/23311983.2015.1124481.

[8] Early Impact. *Using Puppets to Teach Maths – Top 14 Activities.* 2018. https://earlyimpactlearning.com/using-puppets-to-teach-maths-top-14-activities/.

[9] L. Kuse and H. Kuse. *Ways to Use Puppets in Mathematics*. n.d. https://sites.uni.edu/kuse/Puppets/Curriculumandpuppets/PuppetsMath/math.html.

[10] I. Perasutiyo, A. Muchyidin, A., and I. Nursuprianah. "Golden Ratio and the Meaning of the Wayang Kulit Gunungan Philosophy." *Journal of Mathematics Instruction, Social Research and Opinion*, vol. 1, no. 1, 2022, pp. 41-53.

[11] P.A. Pevzner, H. Tang, and M. Waterman. "An Eulerian Path Approach to DNA Fragment Assembly." *Proceedings of the National Academy of Sciences*, vol. 98, no. 17, 2001, pp. 9748-9753. https://doi.org/10.1073/pnas.171285098.

[12] R.C. Prahmana and A. Istiandaru. "Learning Set Theory Using Shadow Puppet: A Study of Javanese Ethnomathematics." *Mathematics*, vol. 9, no. 22, 2021, pp. 2938-2954.

[13] L. Publica. "Interview With the Mathematical Mime Tim Chartier: Proof That Math and Performance Art Add Up." *Art Publika Magazine*, 2020. https://www.artpublikamag.com/post/interview-with-the-mathematical-mime-tim-chartier-proof-that-math-and-performance-art-add-up.

[14] H. Robbins. "Walking the Seven Bridges of Königsberg." In *Tactile Learning Activities in Mathematics*, edited by J. Barnes and J. M. Libertini, MAA, 2018.

[15] K. Schaffer. "The Prisoner's Dilemma." Video, 2009. https://vimeo.com/651914032/f59135bc90.

[16] Simons Foundation and International Mathematical Union. "Fields Medal Winner 2014 Manjul Bhargava". Video, 2014. https://www.youtube.com/watch?v=WJW48dfKfUE.

[17] P. Stone. "Non-Mathematical Musings on Information Theory and Networked Musical Practice." *Organised Sound*, vol. 26, no. 3, 2021, pp. 327-332. www.cambridge.org/core/journals/organised-sound/article/nonmathematical-musings-on-information-theory-and-networked-musical-practice/F0ADA5C0EE7B7D11361F52FA29509316.

[18] B. Wells. "The Rootsellers – Retelling the Galois Group of a Quartic Polynomial." *Bridges Conference Proceedings*, Towson, USA, July 27-30, 2002, pp. 235-246. https://archive.bridgesmathart.org/2002/bridges2002-235.pdf.

[19] E. Zharinova. "Dance Sequence Generated by an Euler Path." Video, 2023. youtube.com/watch?v=g0ectrArk3s.

[20] E. Zharinova. "Traveling through Space Generated by an Euler Circuit." Video, 2023. youtu.be/WY6n5bUTrZQ.