Designing and Building a Wearable 3×3×3 Puzzle Cube

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Abstract

The goal was simple: design and build a working, wearable Rubik's-like puzzle cube. The cube was to be recognizable as a 3×3×3 puzzle cube, fully functional, and wearable by an average-size fifth grade student. Although the final product did not move quite as smoothly as was hoped, it did meet all of its design constraints including its October 31st deadline!

Inspiration

Both authors of this paper became fascinated with the Rubik's cube at about the same age, and both of them quickly learned to solve one in less than two minutes. For Jacob, that was this past year, and with Halloween fast approaching, we decided to make a cube costume for the ages. It would have been easy to make a non-functional cube, but what makes a Rubik's cube interesting is its function and all of the mathematics that it embodies. We were determined to make one that actually works.

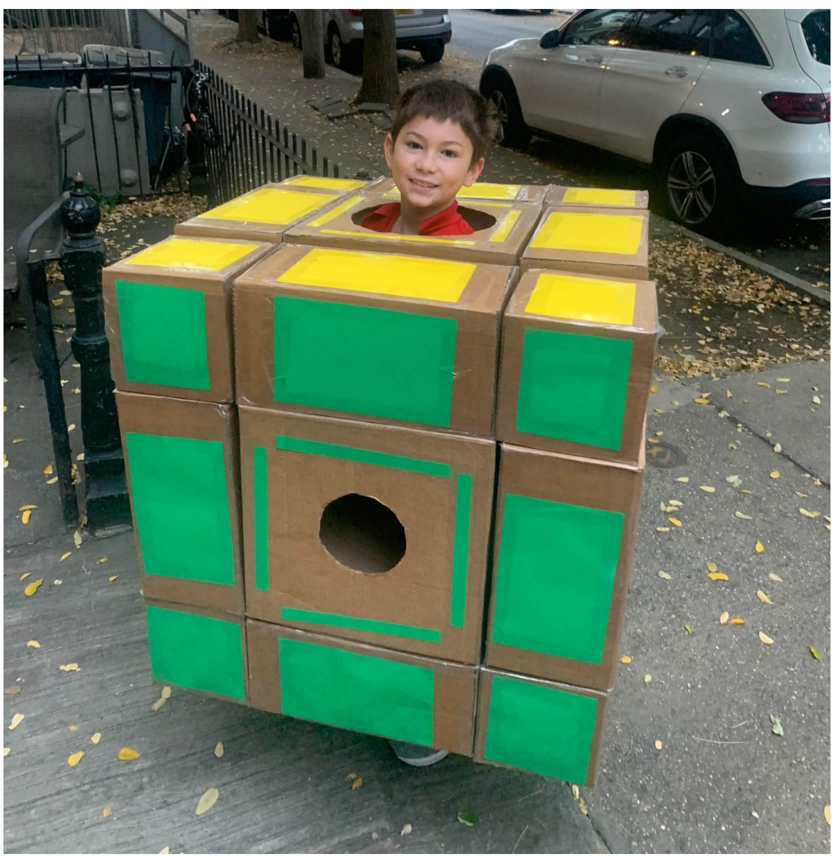
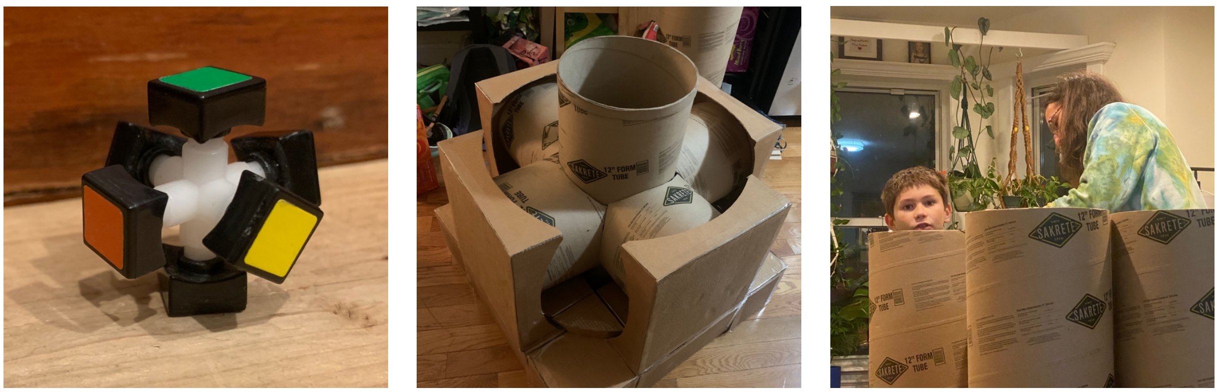


Figure 1: *Jacob wears the cube on Halloween. The cube is shown here in its solved form.*

Would It Work?

Puzzles such as the 3×3×3 Rubik's cube appear to be composed of 27 identical small cubes (sometimes called "cubies" by enthusiasts), but none of the pieces are actually cubes. The cube is composed of 3 types of surface pieces: centers, edge pieces, and corner pieces. There is also a core, which more closely resembles an octahedron than a cube, since an octahedron can be placed so its 6 vertices coincide with the centers of a cube's 6 faces [1]. The exact shape of the core can vary, with some cores including a ball over which the pieces slide, and some shaped more like the junction of 3 intersecting pipes called a "spider core," which was the design we used. The center pieces can turn on their respective arms but are otherwise fixed in place. The fact that the centers do not move relative to each other meant that we might be able to build the core in such a way that a human (at least a small one) could fit inside, with the wearer's head protruding from one center, the feet from the opposite center, and hands able to reach out from any of the remaining four centers.



(a) (b) (c)

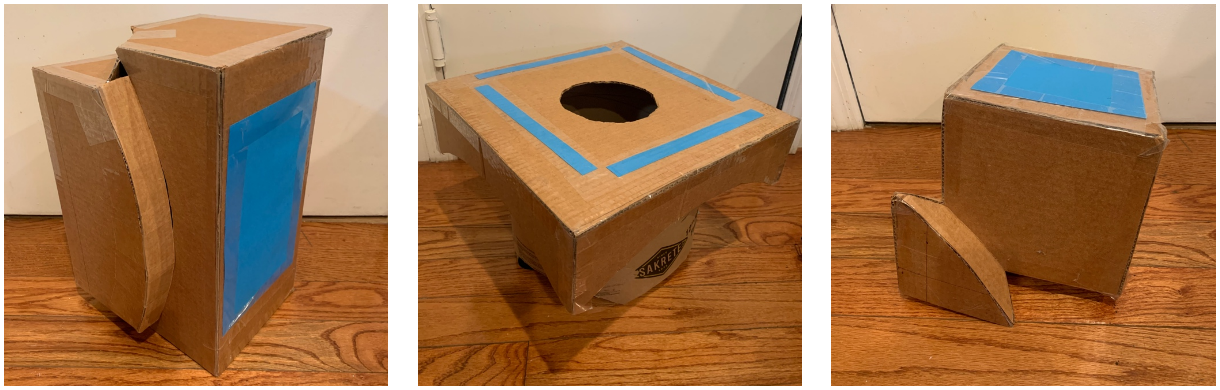
Figure 2: *Designing the core: (a) a miniature Rubik's cube core used for inspiration, (b) our core made from concrete forms, (c) concrete forms before they were cut.*

Our design was loosely based on a miniature keychain Rubik's cube which we disassembled (Figure 2a). We chose to use 12-inch diameter cardboard cylinders (meant as forms for pouring concrete) to construct our core (Figure 2b and 2c), as these were large enough to fit a small human, but small enough to allow for the cube to remain a manageable size. Even so, it was impossible to have all faces of each cubie be the same size because the centers had to be at least 15 inches square, which would have made the assembled cube 45 inches on each side! This would have been difficult for a 55-inch person to manage, let alone pass through any doors. In the end, we decided to make the centers 16 inches square, with the faces of the corners just 8 inches square, resulting in a final size of 32 inches on each side (Figure 1).

Construction

We knew from our experiences disassembling several types of cubes that the whole structure would fall apart easily until the very last piece was snapped into place. This meant constructing every last piece before we would know for sure whether the cube would crumble under the stresses of its own weight. Fortunately, we found some large pieces of lightweight cardboard, which served as our primary construction material.

The first step was to build the core. Due to symmetry, we realized that any two of our cylinders which meet at right angles had to intersect in a plane oriented at 45 degrees. Thus, each curve would be an ellipse with its major axis equal to the square root of 2 times its minor axis [2]. We used 2 nails and a piece of string to create the ellipse that served as the template for the curves that needed to be cut in the concrete forms so that they would fit together properly. Originally, the idea was to cut 6 identical pieces and fit them together, but once 4 pieces were cut and fastened together with strong tape, it was decided to use a single piece for the remaining two arms (the top and bottom) to give the structure some additional strength.

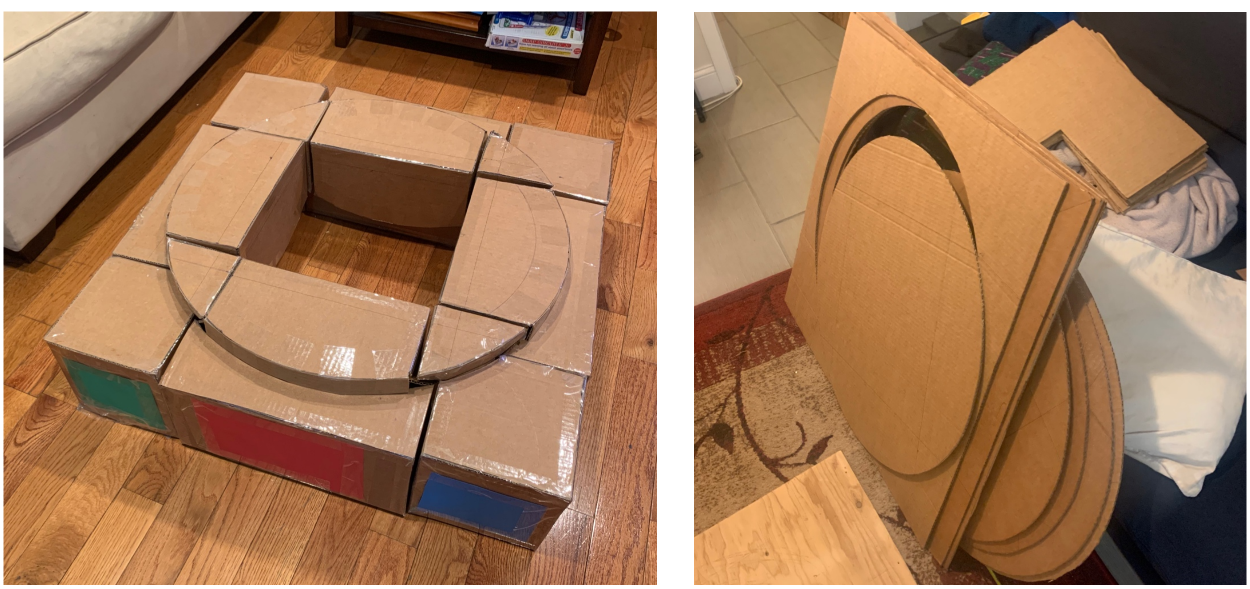


(a) (b) (c)

Figure 3: *The three types of surface pieces: (a) an edge piece, (b) a center cap and its arm, (c) a corner piece.*

The next step was to manufacture the individual pieces (Figure 3). The 20 non-center pieces, including 8 corner pieces (Figure 3c) and 12 edge pieces (Figure 3a), are held in place only by their geometry. For any given face, the 8 pieces that surround the center can be thought of as being attached by a short cylinder (ours was 2 inches high) to a raised circle (see Figure 4a). The raised circles allow the pieces to be held in place mechanically by the center caps (Figure 3b), whose edges have the same radius of curvature as the circle. This allows the entire face to turn. The square hole in Figure 4a is exactly the right size for an arm of the core to fit snugly inside.

The circles were cut out of cardboard before they were cut into individual pieces (Figure 4b). Both the cutout circles and the pieces of cardboard that they were cut from became pieces of the final construction. In the end, the cube was assembled from hundreds of individual pieces of cardboard. We assembled the pieces using tape because we anticipated that we might need to make some significant adjustments once the cube was completely assembled.

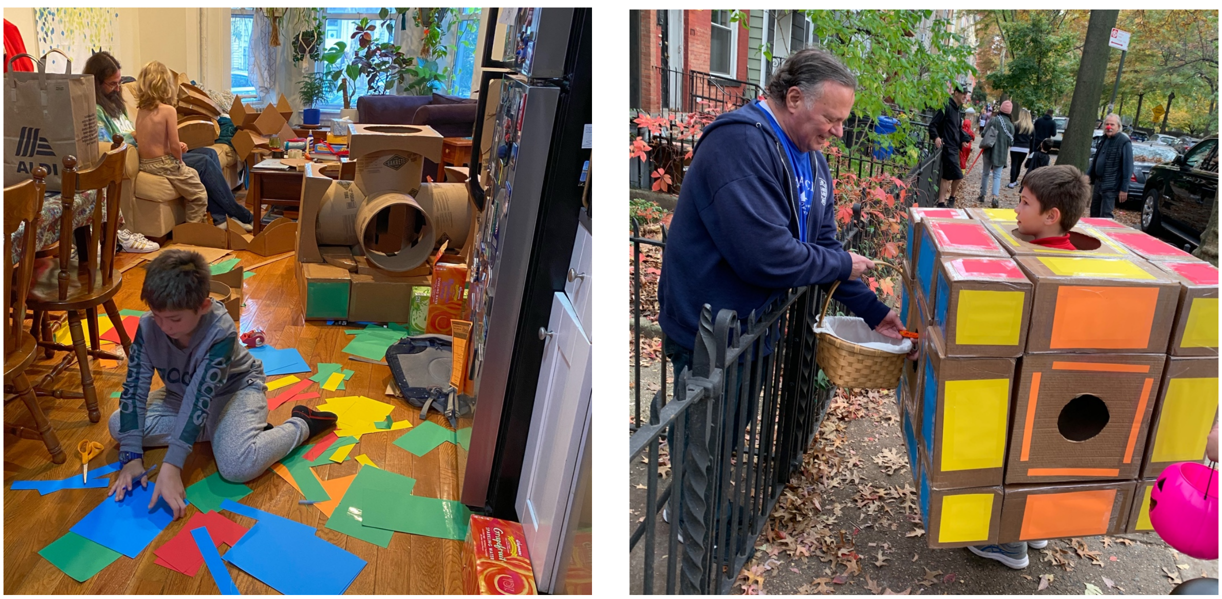


(a) (b)

Figure 4: *So many circles: (a) one layer of the cube as seen from the inside, (b) some of the many cutouts that became the pieces of the cube.*

Results

It was a great relief when the final piece was dropped in place, the cube was lifted up, and it held together! In its final form, the cube weighed 18 pounds, which made it a little hard to wear for long periods, but Jacob did manage to show it off in his school's Halloween parade. The faces do turn, but not smoothly. A full scramble and solve would take a very long time, but we can still confidently claim that this costume can be worn in at least 43 quintillion distinct configurations. After all, each corner piece has 8 possible locations and 3 different orientations, and each edge piece has 12 possible locations and 2 orientations. This give a total of $8!\cdot3^8\cdot12!\cdot2^{12} \approx 5.19 \times 10^{20}$ permutations. But of these, only 1 in 12 turn out to be solvable without disassembly [3]. In the end, this is not the largest working Rubik's cube [4], nor is it the first Rubik's cube costume, but it is, as far as we know, the first Rubik's cube costume that is fully functional. This project ended up being a tremendous amount of work (Figure 5a), but success was so very sweet (Figure 5b).



(a) (b)

Figure 5: *24 hours apart: (a) Jacob scrambles to add some color to the faces, (b) Jacob gets some candy through the blue center.*

Acknowledgements

Thank you to Ernő Rubik for introducing the world to his puzzles, and thanks to Frank Gould for starting three generations of cubing in the Gould family.

References

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