Pattern Gradients with Generalized Duotone Truchet Tiles

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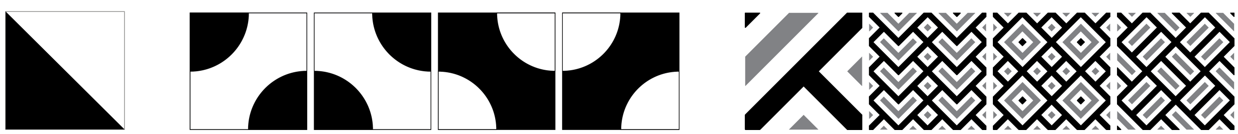
Abstract

Truchet tiles, squares diagonally divided into two colored triangles, belong to a class of tiles that inspire pattern design of surprising variability and complexity through their placement on a regular grid. Gradient structures across mosaics are constructed here using generalized Truchet tiles with motifs that align at tile boundaries, and are applied to graphic design and textiles. In one method, tile color motifs systematically evolve in shape across the mosaic, and a second method involves judicious selection of tile brightness from within defined tile families.

Introduction

Many of M. C. Escher's works of art feature tessellations in which the tiles slowly transform spatially across the artwork from one type to another, including the masterpieces *Metamorphosis I*, *II* and *III*. His works, created by hand, are extraordinary in the breadth and depth of the exploration of transformation [4] techniques utilized, and inspire the theme of this more limited study of abstract graphic design through tilings generated algorithmically and which feature spatial transformation using Truchet-like square tiles [11].



(a) **(b)** **(c)**

Figure 1: *(a) Truchet tile, (b) variant popularized by Smith with AB or BA edge pattern and (c) Nery's "Magic Tile," which has an ABCBA edge pattern. Square arrangements of four Magic Tiles with different rotational orientations form the meta-tiles repeated in the three mosaics on the right.*

Truchet's square tile (Figure 1(a)), is divided along the diagonal into two colored triangles. Employing four distinct tile orientations produced by 90° rotations, he presented numerous and diverse repeating and non-repeating patterns that emerge through tile placement on a square grid. Here we employ a generalization of Truchet's tile, following an edge classification system described by Virolainen [12]. Truchet's original tile has one of two colors, A and B, on each edge, so that a single tile can be represented, going around the square's perimeter, as A-B-B-A, where the dashes represent corners. The Truchet tile is generalized through subdivision of the tile edges in a systematic way. A popular variant, depicted in Figure 1(b), consists of a square overlaid with a pair of contrasting quarter circles centered in diagonally opposite corners [10][3], producing a single division of each tile edge and represented as AB-BA-AB-BA. This classification scheme can be extended to other polygon shapes that tile the plane, such as triangles and hexagons [7][12].

Larger shapes and patterns contiguous across multiple tiles emerge when colors on neighboring tiles align at abutting edges [1][12][6][9], as illustrated in Figure 1(b) and 1(c). For the color-aligning AB-BA-AB-BA tiles in Figure 1(b), A abuts A and B abuts B. For tiles with mirror-symmetric edges, such as some ABA [2][6][9] (described later in this paper) and ABCBA tiles, color matching at abutting tile edges occurs for any of the four 90° rotational orientations, provided that the lengths of the color segments on each edge are the same. An interesting example of an ABCBA pattern with this property is the "Magic Tile" (Figure 1(c)). Numerous intricate pattern variations with this simple motif were utilized by Nery in the construction of the Contimul train station in Oporto, Portugal [8].

Continuous Deformation of Color Boundaries via Bézier Curves

The shape of the symmetric color boundary curves in the interior of AB tiles can be controlled by representing them with Bézier curves. Using the method described by Wendt [13], the boundary evolves between the examples arranged vertically in Figure 2(a) on left (white corners on black) and right (inverted colors), respectively, by varying a single parameter *u* that establishes the curvature along the Bèzier color boundary. Application to the creation of duotone textiles is illustrated with reversible jacquard knitted samples where *u* was varied linearly from ends to center ("triangle" function, Figure 2(b)). The pattern shown in Figure 2(c) is based on a half sine wave variation of *u* between the ends, combined with a random selection of the two arrangements of the colors (left and right in Figure 2(a)). A computer program was written to construct images of individual tiles and arrange them in a grid. The resulting mosaic images were imported into design software for the Kniterate [5] computer-controlled knitting machine used to fabricate the scarves.

(a) **(b)** **(c)**

Figure 2: *(a) Representative examples of color motifs on AB Truchet tiles used in reversible jacquard knitted scarves, shown in (b) and (c). Abutting tile edges are color matched for all knitted samples except the middle one in (b), for which colors A and B abut everywhere along all tile edges.*

Mosaic Gradients Constructed with Generalized Truchet ABA Tile Family

Another approach to producing a mosaic with spatial structure on a scale length larger than individual tiles, inspired by Rohm's *Truchet Puzzle* [9], makes use of a finite set of generalized ABA Truchet tile types for which the lengths of the tile edges are divided into even thirds. Due to their mirror symmetry, the colors of abutting edges will match everywhere in the mosaic, regardless of tile type or rotational orientation. The motifs connecting to each middle edge sub-segment (color B) in the tile set has one of the following forms: i) a half circle connecting to no other edges, ii) one or two quarter circle arcs, or iii) a straight rectangle. The resulting family of seven ABA tile types is shown in Figure 3. Also shown are shaded squares with a grayscale value corresponding to the brightness of the tile above it, determined as the fraction of the tile area covered in white. We exploit the range of possible brightnesses offered by the family of tile types to construct brightness gradients through rules for the selection of tile type at each location in the mosaic grid.

Mosaics with abstract graphic design are readily constructed with the family of ABA tiles by choosing both the tile type and its rotational orientation randomly (Figure 4(a)). This type of tiling shows some

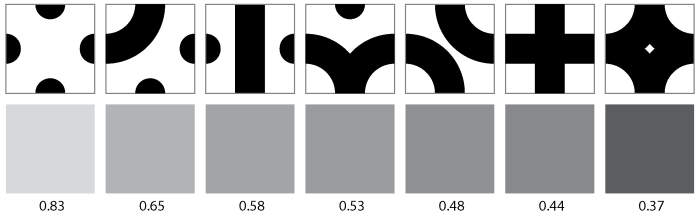
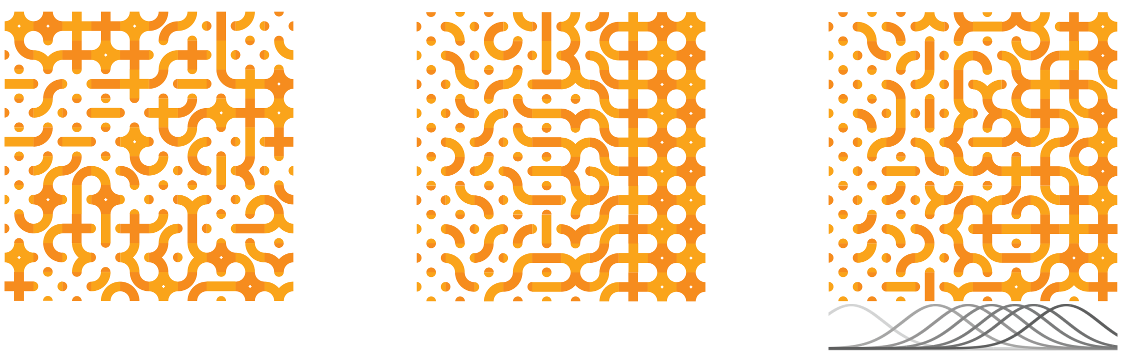


Figure 3: *Family of ABA tiles and corresponding brightness.*

structure on a spatial scale larger than a single tile, but the brightness gradients in the mosaic appear in random locations and directions. To depict a mosaic containing *systematic* grayscale gradients with the family of ABA tile types in Figure 3, a straighforward approach is to choose a tile type with brightness closest to the value specified by the gradient function at each location, as shown for a horizontal linear gradient in Figure 4(b) (highest brightness on the left). The vertical columns in this figure all contain the same tile *type* but with the rotational orientation randomly selected.

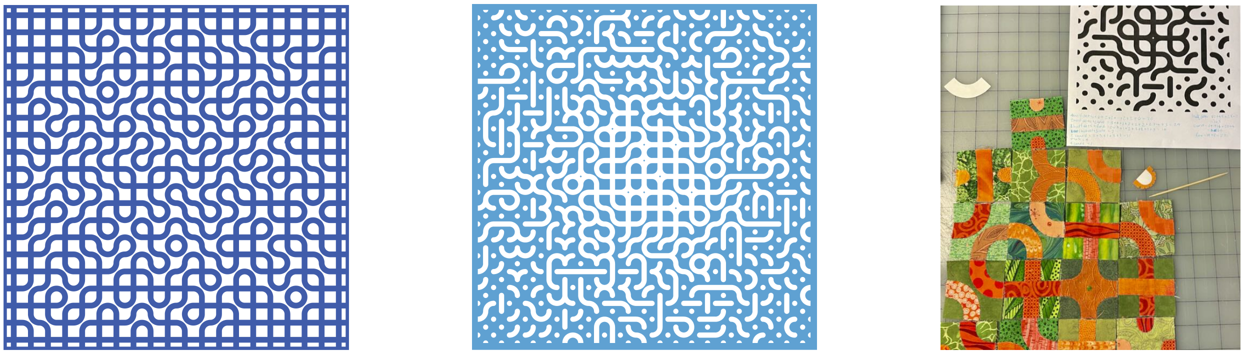
To reduce spatial clustering of tile types that occurs in mosaics utilizing "nearest brightness" tile selection, a technique using a modified random selection of tile type is introduced. The tile selection probabilities here are governed by Gaussian weighting functions that favor selection of tile types with brightness closest to that being depicted. The Gaussian weighting functions for the tile family in Figure 3 as applied to a linear brightness gradient are included in Figure 4(c). To depict a pixel of brightness b, the selection probability for a given tile type will be the ratio of its own Gaussian weighting function at b to the sum of weighting functions evaluated at b for each tile type. The selection probability weighting function for each tile type peaks at its own brightness, and the widths of the functions can be varied. In the limit of a narrow Gaussian width, the result will approach that of the "nearest brightness" method depicted in Figure 4(b).



(a) **(b)** **(c)**

Figure 4: *Arrangement of ABA tiles from Figure 3 (a) selected randomly, and a linear horizontal grayscale gradient with type selection by (b) nearest brightness and (c) Gaussian probability weighting.*

Abstract graphic designs employing spatial gradients are shown in Figure 5. Figure 5(a) uses only two tiles from the ABA family and introduces another application of probability weighting functions. The weighting function assigned to the cross shaped tile motif is spatially uniform and exceeds that for the tile with two quarter circle arcs (which varies as $\cos(x)\cos(y)$) near the edges, but is lower in the center region (by up to a factor of 5). In Figure 5(b), Gaussian weighting of tile selection probability produces brightness inversely proportional to the distance from the center. In Figure 5(c), ABA tiles are applied to quiltmaking.



(a) (b) (c)

Figure 5: *Designs resulting by varying the spatial weighting of selection probability between (a) two, (b) seven ABA tile types. (c) Quilt in progress using ABA tile 2" blocks.*

Concluding Remarks - Duotone abstract geometric patterns are of broad interest for graphic design, including textiles and quilts, and the systematic approaches using generalized Truchet tiles described herein are applicable to creating new patterns with visual interest arising from their non-periodic nature. Study of other algorithms for selecting tile type from within the ABA family to depict gradients in one spatial dimension is merited, as is generalizing the method to 2D as a means to depicting more complex images.

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