# The SunRule: An Interactive Mathematical Sculpture

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## Abstract

We installed an interactive mathematical sculpture in a public park in Orono, Maine. The SunRule is a physical realization of a geometric definition of multiplication. Its novel design uses the sun's parallel rays to geometrically construct products. The sculpture consists of a bronze wall that is partially wrapped around the edge of a ruled bronze disk. The disk and orthogonal wall are affixed to a granite plinth. The SunRule is designed to manipulate beams of light that shine through one of seven narrow windows that are cut out of the wall. A bronze-cast ball-and-socket joint allows the disk to be tilted (to change the multiplier) or swiveled (to change the multiplicand), and these actions vary the apparent length of the sun beam that is projected onto the disk (the product).

McLoughlin and Droujkova elaborated a diagrammatic definition of real number multiplication that rests on a physical axiom: "The hypotenuse of the right triangle determined by an object and its shadow must be parallel to the hypotenuse of any other object and its shadow" [2, p. 2][1]. We were inspired by their definition to design a physical tool that would use the apparently parallel rays of the sun to construct products [1] (Figure 1).

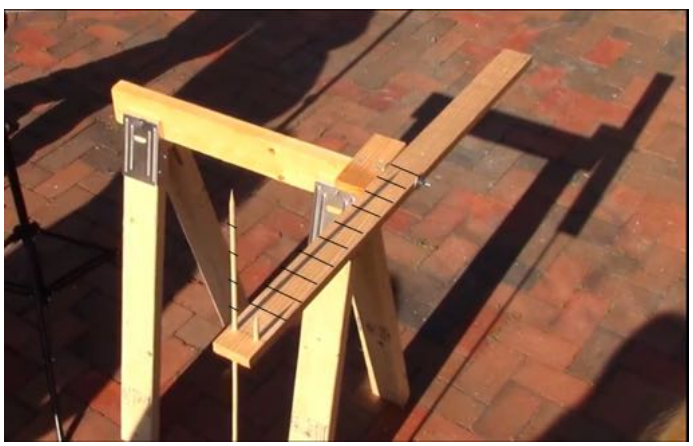


**Figure 1:** *Prototype of the* SunRule. *Rulings on the orthogonal rods are equal to those on the board. Rulings emphasized to aid visibility. In the figure, the unit shadow (shorter rod, multiplier) is 2, the multiplicand (taller rod) is 5, and the length of the multiplicand's shadow is 10—i.e., 2x5=10.*

The initial prototype of what we named the SunRule consisted of two rods (i.e., *gnomons*) orthogonal to a ruled board[2]. The height of one gnomon was fixed at one-unit; the height of the other gnomon could be adjusted by sliding it up or down. The ruled board/gnomon assembly was mounted to a portable stand.

[1] One of their stated goals was to define "multiplication, similar triangles, and the area of rectangle…independently of each other in the mathematics curriculum" [2, p.1].
[2] The gnomons do not need to be orthogonal – it is sufficient for them to be affixed at the same angle to the board. Also, the rulings on the gnomons are independent of the rulings on the board.

We designed the SunRule to solve a specific problem—what we call the *variable altitude problem.* At any moment, the shadows of two sunlit objects will be parallel, and their heights and the lengths of their shadows will be related by the same scalar, determined by the altitude of the sun (Figure 2).

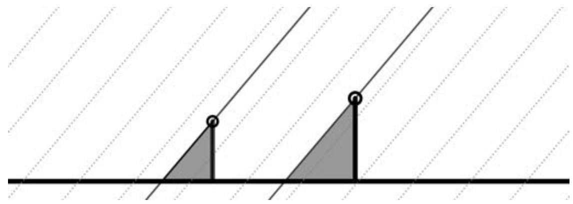


**Figure 2:** *Solar scalars: Because sun rays are locally parallel, the hypotenuses of the triangles formed by any two shadow-casting objects are parallel. Furthermore, the heights of the objects and the lengths of their shadows are related by a scalar that is determined by the altitude of the Sun.*

In the real world—as opposed to the realm of diagrams—it is possible to use the parallel rays of the sun to physically express products where the scalar (i.e., multiplier) is determined by the altitude of the sun. This is useful insofar as it allows us to measure the heights of trees, flagpoles, or other tall things by measuring and comparing shadows, but without control over the multiplier—i.e., without control over the altitude of the Sun—we are limited to one multiplier at a time. For example, when the altitude of the Sun is $\pi/4,$ the available multiplier is 1.

The innovation of the SunRule is an adjustable shadow plane—the ruled board in Figure 1. By tilting the ruled board, we can vary the *apparent* altitude of the Sun, from the frame of reference of the shadow-casting gnomons. Tilting *toward* the sun (Figure 3(a)) increases apparent altitude and shortens shadows.

(a) (b)

**Figure 3:** *(a) Tilting the shadow plane toward the Sun increases apparent altitude/shortens shadows (multiplier is less than 1), (b) tilting the shadow plane away from the Sun decreases apparent altitude/lengthens shadows (multiplier is greater than 1)*

Tilting *away* from the Sun decreases apparent altitude and lengthens shadows (Figure 3(b)). By tilting the ruled board toward or away from the sun, it is possible to use sunlight to realize any multiplier, from 0 (i.e., the Sun is directly overhead/no shadow) to arbitrarily large (i.e., the sun is on the horizon/infinite or undefined shadow).

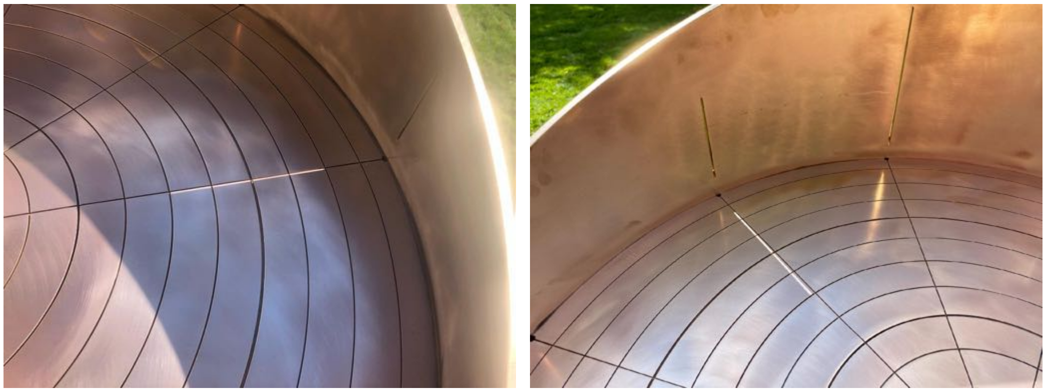
To use the SunRule to multiply: (1) adjust the tilt of the ruled board to set the multiplier—this is the length of the shadow cast by the unit gnomon (2 units in Figure 1); (2) adjust the height of the longer gnomon to set the multiplicand (5 units in Figure 1); (3) the product is read by examining the length of the longer gnomon's shadow (10 units in Figure 1). We leave it as an exercise for the reader to determine how to use a SunRule for division.

In 2019 and 2020, we constructed various prototypes of the device that were field-tested with students and teachers [1]. This prototyping led to an interdisciplinary collaboration between mathematics educators (first and second authors) and artists (third and fourth authors) that, in October of 2022, culminated in a public installation of a sculpture that realized the SunRule concept. The interdisciplinary team collaborated to translate the initial, linear design into a visual form that is both a work of art and also a mathematical tool (Figure 4).

(a) (b) (c)

**Figure 4:** *The SunRule; Webster Park, Orono, Maine, USA: (a) The granite plinth with bronze ball, (b) the authors with the installation, to show scale, (c) the inscribed disk.*

The installation is a bronze disk that sits atop a granite plinth. A bronze wall that gradually slopes up and then down again wraps around the edge of the disk. Narrow slits of various heights (7 variations, from 2 to 8 units) allow light to pass through the wall, and, when a window is aligned with the azimuth of the Sun, project a beam onto the surface of the disk (Figure 5).



(a) (b)

**Figure 5:** *Multiplication by sunlight: (a) a light beam passes through a window whose top is 3 units above the disk; the base of the window is offset from the disk by 1 unit; the disk is tilted so that the length of the shadow at the base of the window is 2 units—i.e., 2 x 3 = 6; (b) a light beam passes through a window whose top is 4 units above the disk; the disk is tilted so that the length of the shadow at the base of window is 1.5 units—i.e.,1.5 x 4 = 6.*

The disk is ruled in concentric circles spaced one-unit apart. Radial lines connect the center of the disk to the base of each window, and each of the windows is offset from the surface of the disk by one-unit. The disk/wall is attached to the plinth by a cast bronze ball-and-socket joint (Figure 6) that allows the disk to be tilted (toward/away from the sun) and swiveled (clockwise/counterclockwise). To set the multiplier, tilt the disk toward/away from the sun to stretch or shrink the shadow at the base of each window; to set the multiplicand, swivel the disk to align one of the windows with the azimuth of the Sun.

**Figure 6:** *Casting bronze for the ball-and-socket;* left: *fourth author,* right: *first author.*

The SunRule installation preserves the core mathematical function of the linear design, but there are notable differences between them. The installation, as a public art project in a northern climate, was designed to have minimal moving parts/points of failure, hence the disk/wall attached by ball-and-socket joint. Whereas the linear version allows for continuous variations of both the multiplier and the multiplicand, the installation has a fixed set of multiplicands. Finally, the multiplicative unit is the base of each window and is thus collinear with each multiplicand. In the linear version, the unit and the multiplicand each have their own gnomon. These decisions were made by consensus between the mathematical and artistic principals in the collaboration, to the end of achieving a design that would be visually striking as well as mathematically functional.

## Summary and Conclusions

The SunRule installation is a public art collaboration between the University of Maine and the Town of Orono. Our vision is that it will create opportunities for children, families, strangers, and friends to gather and consider how mathematical relationships can be embedded in the world. Specifically in the areas of mathematics education and engaging the general public with mathematics, it creates opportunities for people to consider the sun as an emitter of parallel rays that can be harnessed to provide a physical, interactive model of multiplication. This is significant because physical models of mathematical phenomenon are highly sought after by teachers but are often of low mathematical fidelity. The SunRule combines the best of both worlds.

## Acknowledgements

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## References

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