# Implementing Textile Borromean Seifert Surfaces

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**Abstract**

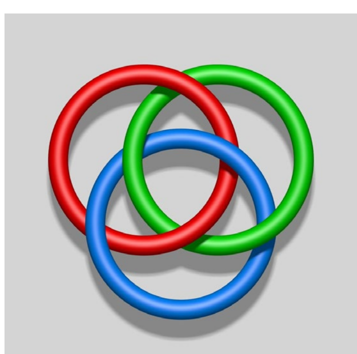
I describe how to make textile Borromean Seifert surfaces. The surfaces can be used for writing, drawing or printing. I turn to three-phase electricity, Lacanian psychology, and mathematics itself in order to fill the surfaces with meaning and aesthetic content.

## Introduction

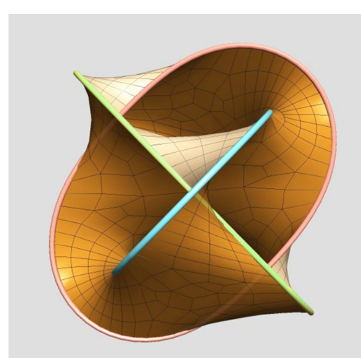
For every knot, there exists a surface such that the edges of the knot are the boundary of that surface. The same holds for each link (intertwined loops or knots). This amazing result was discovered by Herbert Seifert in 1934. Seifert surfaces, and more generally, minimal surfaces, can be turned into art works, as shown by Grossman [1], Kasuba [2], Pauletti et al. [3], and Séquin [4]. The program SeifertView by Jack van Wijk [7] makes these Seifert surfaces visible on the computer. In this paper, I focus on the Borromean link of Figure 1(a), which has three loops which hold together, although they are pairwise disconnected. The theory of knots and links is a branch of topology, also known as rubber sheet geometry. Usually that means that one does not care about the exact shape of curves and surfaces, as any folded or wrinkled version is considered the same. In this project I *do* care about the precise shape as I want the Seifert surface to be smooth, working with stretch fabric knitted from polyester and elastane (synthetic rubber). As Jack van Wijk has shown, the surface can be turned into a minimal-energy surface, like a soap-film. My first research question is how to make a Borromean Seifert surface using stretch fabric. My approach is hands-on and iterative. The second question is how to assign meaning and aesthetic content to the surface. For this I turn to three-phase electricity, Lacanian psychology, and mathematics itself.

## Starting Points and Initial Explorations

SeifertView provides a good initial estimate of the loop form, as shown in Figure 1(b). I manipulate the view to see the loops in an orthogonal manner and keep a screenshot as a template for bending metal loops. Aiming at a final assembly of about 25cm height, I fix the loops at 76cm length. The loops are not circular but resemble a superellipse ($a$ = 14.5cm, $b$ = 9.1cm, $n$ = 2.3). I use 2.5mm$^2$ single core copper house wiring electrical cable, see Figure 2(a), which is easy to deform, yet flexible (pushing back at small deformations).



(a)



(b)

**Figure 1:** *(a) Borromean rings by Jim.belk, Wikimedia, (b) Screenshot from SeifertView.*

From paper and tape, I make a first surface, shown in Figure 2(b). Removing it from the loops, I find that the surface is not developable (i.e., it cannot be flattened). I must choose to either use stiff woven fabric, cutting darts, or use stretch fabric. I choose the latter, aiming at a minimal-energy surface which finds its form in an interplay with the loops. Cutting at the six narrowest areas, I design my pattern, resembling the three (non-flat) pieces of paper obtained from cutting the model of Figure 2(b) (two "triangles", one "hexagon"), yet slightly smaller. After adding seam allowance, I use these for fabric cutting.

(a) (b) (c) (d)

**Figure 2:** *(a) Loops, (b) Paper model, (c) Prototype with 2.5mm² loops, (d) Prototype with 6mm² loops.*

### Construction Details

Initial cutting is done by laser (the outer dashed lines in Figure 3). The surface is doubled, so I need 4 triangles, pairwise mirrored, and 2 hexagons, also mirrored (provided as supplementary material). First, I assemble the six panels pairwise using pins (along the red, green, and blue long edges). Then, I sew the short seams (the arrows in Figure 3). Going from the upper (double) triangle to the hexagon involves a left-turning twist, from the hexagon to the lower triangle a right-turning twist. The short seams are sown "right sides together", so the stitching will be on the inside of the finished project (upper panels connect to upper panels, lower to lower, and colors must match). Next, the long seamlines, marked by colored pins, are topstitched with colored yarn. I use a decorative zig-zag (stitch 17 on a Brother Innovis), since stretch fabric needs zig-zag stitching. Openings are left for inserting and closing the loops (by soldering, later I switch to screw terminals). Stretch fabrics tend to curl; ironing spray helps. The openings are closed by hand-sewing, as the 3D object does not fit under the machine. The fabric is trimmed along the outer rim of the decorative stitch. I find that 6mm² Cu loops work better than 2.5mm². After six iterations[1], two of which are shown in Figures 2(c,d), the form is satisfying. For the print on the fabric, there are two alternatives, described in the next sections.

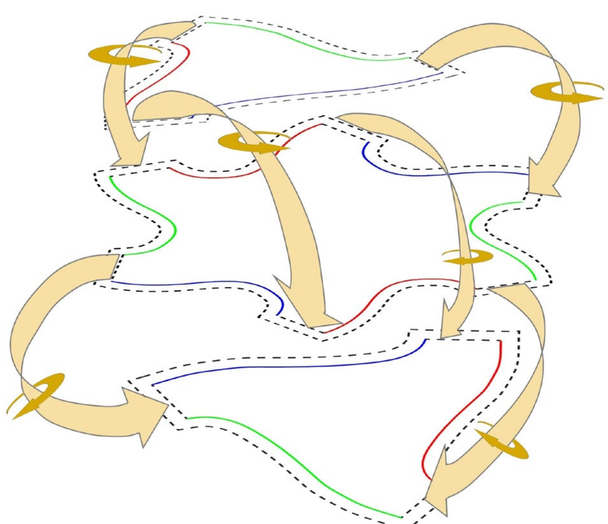


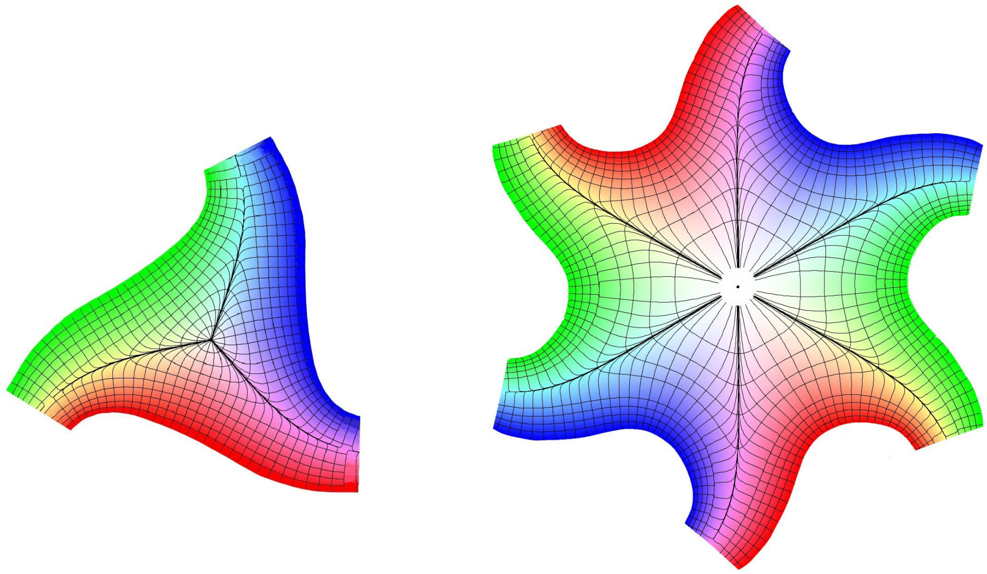
**Figure 3:** *Assembly of the Pieces.*

## Three-phase Field Lines

I design a system of lines, resembling the field lines known in physics. The lines can serve as coordinates when measuring stretch and I want them to be aesthetically pleasing too. Rather than mapping an existing coordinate system (Cartesian, polar), I let the lines originate from the three loops of the Borromean link. I take inspiration from three-phase electric systems, as pioneered by Nikola Tesla and Mikhail Dolivo-

[1] The main steps of the iterations are: from single to double surface, asymmetric pattern, digitized pattern, smaller triangles, laser-cutting instead of scissors, and thicker loops. The loop length was never changed.

Dobrovolsky. Engineers calculate AC voltages and currents by representing them as complex numbers, following a method proposed by Charles Proteus Steinmetz [5]. I imagine the three loops to carry the voltage of one phase each, and then I calculate the corresponding field in the surface.



(a) (b)

**Figure 4:** *(a) Field lines on triangle surface, (b) Field lines on hexagon surface.*

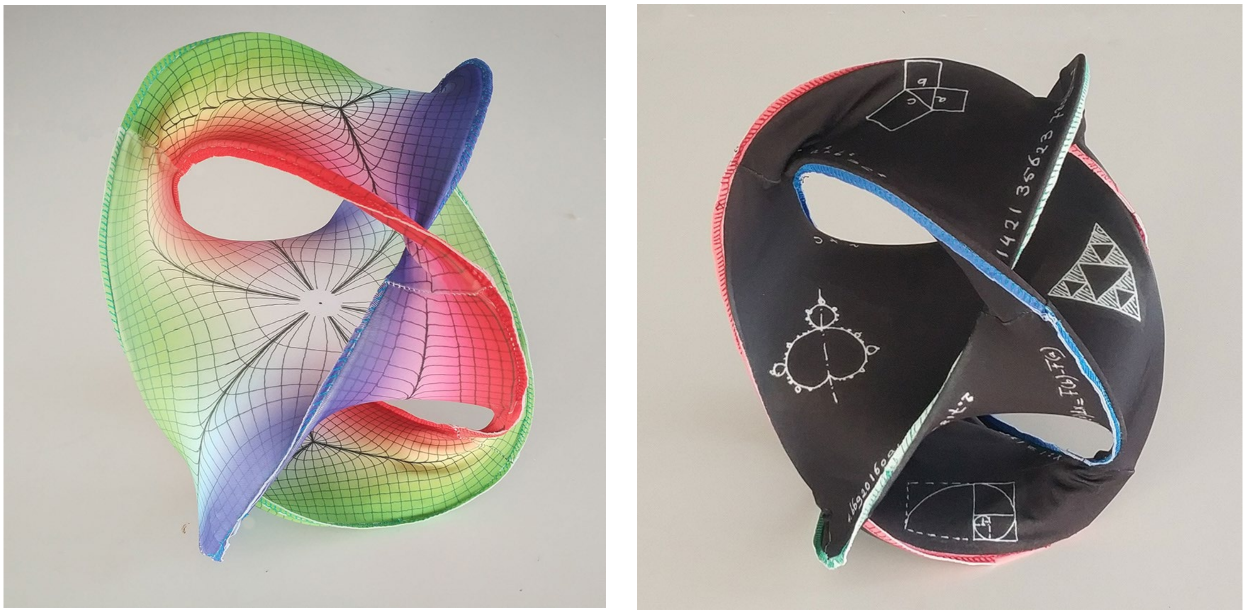
The field is calculated for the triangle and the hexagon separately (ignoring stretch). Boundary conditions are imposed on the long edges (named red, green blue), setting $V_{\text{red}} = 1$, $V_{\text{green}} = (-1 + i\sqrt{3})/2$, and $V_{\text{blue}} = (-1 - i\sqrt{3})/2$. Thus, I imagine connecting the loops to the power grid. To find the electric field, whose divergence is zero, we solve $\partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 = 0$ by a relaxation method on a $1500 \times 1500$ array of complex $V(x, y)$. In Processing, the assignment $V[x, y] \leftarrow (V[x+1, y] + V[x, y+1] + V[x-1, y] + V[x, y-1])/4$ is applied for all non-boundary $x, y$, and repeated 100,000 times to let $V$ stabilize.

Next, the equipotential lines of constant $|V|$ are found by tracing. The second set of lines consists of evenly-spread lines orthogonal to the equipotential lines. I find these by steepest descent. The field is coded in HSB color space, mapping $|V|$ to saturation and $\arg(V)$ to hue, as shown in Figures 4(a,b). The field lines are superimposed on the colors of the triangle and hexagon, seam allowance is added, and all is transferred by sublimation print onto the fabric (before sewing and assembly). The result is in Figure 5(a).

## Mathematical Tableaux

As an alternative content, I design six math tableaux (one for each of the six panels) which represent a variety of mathematical topics, from ancient to 20th century. Some are to be recognised by a wide audience, others are more specialistic. In order to structure each tableau in the same way and to respect the continuity of the edges, I deploy a schema proposed by the French psychoanalyst Jacques Lacan, who considers the Borromean link to be a possible model of the human psyche [6]. Lacan labels the loops as the Imaginary, the Symbolic, and the Real. Imaginary refers to images, which I interpret in this art work as geometrical drawings. For Lacan, the Symbolic refers to language, signifiers, and power structures, but here, for me, it means equations and formulas. For Lacan, the Real is what resists being captured in images or symbolic forms. I interpret the Real as elements of $\mathbb{R}$. Indeed, many real numbers have no defining equations, even though we can define a minority such as the integers, the rational numbers, radicals, $\pi$, and *e*.

The four small tableaux have one image, one equation, and one real number; the two large tableaux have two of each. For example, the Pythagorean tableau has the obvious triangle with two squares, the equation $a^2 + b^2 = c^2$, and one real number, viz. a decimal approximation of $\sqrt{2}$. The six tableaux are: Pythagoras' law, Golden ratio, Groups, Lambda calculus, Calculus, and Fractals. The writing, drawing, and colour scheme resemble those of a classical blackboard. The edges are again red, blue, and green.



(a) (b)

**Figure 5:** *(a) Colored Seifert surface with field lines, (b) Surface with math tableaux.*

## Summary and Conclusions

I made Borromean Seifert surfaces, overcoming practical hurdles, and combining craft and technologies such as laser cutting and sublimation printing. The term "implementing" from the title means making, but also filling (Latin *implēre*). The surface is filled by making connections to three-phase electricity, to psychoanalysis, and additionally for the surface of Figure 5(b), by the six mathematical tableaux.

## Acknowledgements

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