Dynamical Tilings in Industrial Design

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Abstract

This manuscript shows how tiling and its respective groups in two-dimensional space can be applied in design. These kinds of mathematical structures can help designers in the planning steps to know if a dynamical transformation of the components leads to different patterns/arrangements, each of which accomplishes a different goal. Our construction can be applied for folding tables (reduction and expansions), lightning systems (exposure and hiding), and more.

Introduction

One of the essential aims of industrial design is to innovate new ideas and details in products. Designers are driven by this objective to push the boundaries of other scientific domains, including materials engineering (especially mockups in $3D$ printing) [3], artificial intelligence, mechanics [6], and other fields, all of which are utilized in the realm of design. When designers deal with mathematics, it usually relates to the geometric properties of the product due to a lack of mathematical education in the general art and design curriculum, while mathematics has a variety of tools; algebra, topology, etc.

This collaboration was an accidental encounter with unique folding tables[1] [1][2], with a respective dynamical system which led us to the artwork of Khushbu Kshirsagar [5], presented in the Bridges 2022 art gallery and define a dynamical system based on a hexagon. While the designer was impressed by the performance of the moving objects and the mechanism in these cases, the mathematician raised the thought, “Can the product designer use the algebraic structure properties of patterns to determine when a dynamical transition can be obtained between the desired two stages (in the plane)?”

In product design, repetitive patterns are usually applied in the finishing process and relate to the texture or aesthetic the product is trying to transmit. Our approach will also show how to use repetitive patterns in the initial steps of planning a product based on mathematical structures. The group exploration (see Background section) can give a justification when a design can define the transition between patterns with the help of a dynamical mechanism (see Figure 4). From the industrial designer’s point of view, these two patterns represent two opposite states, which can be applied to different goals, such as reductions and expansions or exposure and hiding. With further development and the respective budget, this could be used for lightning, tables and more.

Background on Wallpaper Groups

Symmetry in the plane is defined by a combination of three transformations: translation, rotation, and reflection, which leads to 17 different tilings - the wallpaper groups. If $H \leq G$, then H is a subgroup of G, i.e., H contains the transformations of G. For this particular case of this tiling, Conway [4] defines a signature for each pattern where the transformation is defined by symmetries. The signature is determined

[1]The complexity of these tables leads to a price tag of more than $\$100k$.

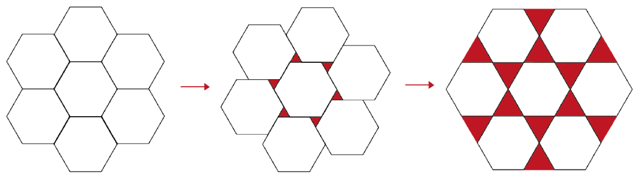


Figure 1: *First sketch examining the feasibility of our idea. Given a pattern with signature* $*632$*, which 'transformed' to* 632 *pattern, and last in the open stage define a new pattern with the initial signature.*

by reflection lines and points of rotations in the pattern; these symmetries need to shift in an equidistance in the plane to obtain a repetitive pattern. In the signature, an asterisk before a number notates that a reflection occurred. When we write $*\alpha$, we mean that there are α lines of reflection that all meet at one point. If it's only α the meaning is that there are α rotations in a given point. Since the pattern is repetitive, this number of different types of symmetries points is finite. As an example, when we write $*\alpha\beta\gamma$, we mean that there are three different points satisfying reflection symmetry of order α, β, and γ. If we omit the asterisk as $\alpha\beta\gamma$, we mean the pattern is defined by three rotations points. If we write $\alpha * \beta$, there are α rotations respective to a point and β reflections at another point in the pattern. Lastly, $\times$ denotes glide reflection, and $\circ$ indicates a pattern defined only by translations, i.e., $\circ$ is a subgroup of the other wallpaper pattern groups. This is a partial list of the relations between the 17 different wallpaper groups:

- $632 \leq *632$
- $442 \leq 4*2 \leq *442$
- $333 \leq 3*3 \leq *333$
- $2222 \leq 22\times \leq 22* \leq 2*22 \leq *2222$
- $\circ \leq \times\times \leq *\times \leq **$

Each relation indicates that there exists a transition between the respective tilings by adding or omitting elements (reflection, rotation). Now we are ready to apply it to industrial design through a dynamical system, which defines a transition between two stages.

Figure 1 is our first step in shifting our mathematical idea to design. Given seven regular hexagons, one in the center and six neighbors define a reflection symmetry. Sliding these six hexagon neighbors simultaneously along the edges of the center hexagon leads to gaps between them which breaks the reflection symmetry and leads us to a new pattern, which is defined by rotation. Generalizing this idea indicates that a transition between patterns with signatures $*632$ and 632 ($632 \leq *632$) can be obtained.

Our Results

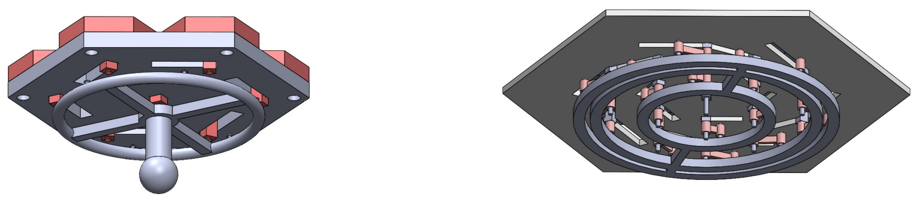
The design thinking began with understanding the basic geometrical form that will be comfortable working with moving objects. After exploring a number of polygons that stood out in the $*632$ patterns, the hexagon shape was chosen since it is limited to reflection, so it has a subgroup that is defined only by rotations.

To examine the possible structures to enable the simulation motion of the tiles, we tested several models. The first realized model movement of the hexagons (see Figure 2), is based on the sliding of six hexagons by the sides of the hexagon they surround. The hexagons slide at the same time and keep the pattern symmetrical. In order to keep the distance symmetrical between the six hexagons and simultaneously slide them, a mechanism was required. The constructed mechanism for Figure 2 is Figure 3a, which works in this way: there are seven hexagons arranged on a surface in a way that the middle hexagon remains static. The six hexagons surrounding the middle hexagon are able to move on the surface. Each of the six surrounding hexagons has

(a) *Hexagons 'tiling'* **(b)** *Side view of the mechanism*

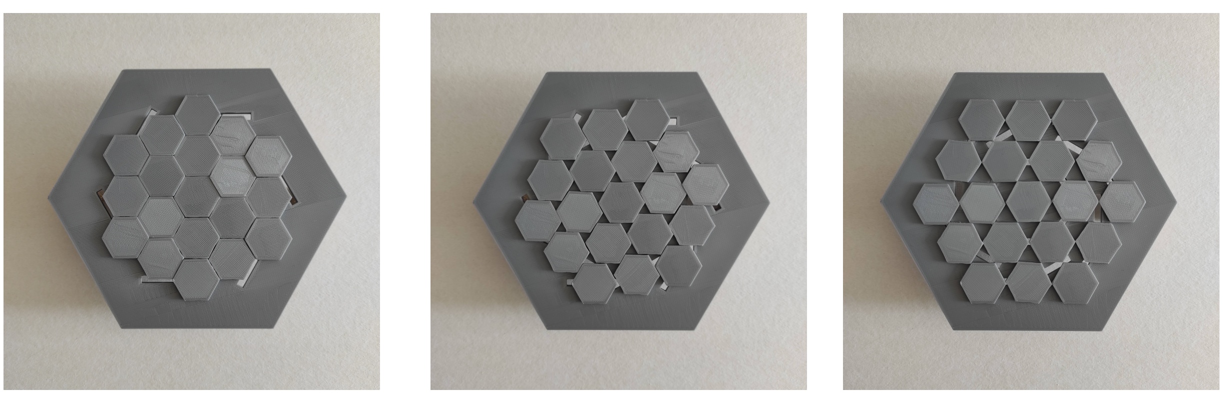
Figure 2: *The initial functional prototype provides us with the validation that a transition guided by a mathematical concept can be implemented through an appropriate mechanism.*

a square-shaped pin in the middle of the shape that comes out from it downwards under the surface. Each square pin has a track on which it can move back and forth. The track is designed so that when all the



(a) *Feasibility model mechanism.* **(b)** *Final model mechanism.*

Figure 3: *Both mechanisms are activated by a bottom ring as in Figure (a). The left side (the feasibility model is activated by pulling strings), while the model on the right side is activated by the connectors between the bottom ring and the hexagons.*



(a) *Initial state* $*632$ **(b)** *Middle state leads to* 632 **(c)** *The model open stage* $*632$

Figure 4: *Our model, with a respective mechanism, leads to dynamical movement between tilings (made from PLA). The first figure, Fig. (a), represents the original pattern designated by the signature* $*632$*. When the mechanism is activated, the pattern's signature changes to* 632*, which is depicted in Fig. (b). Finally, as shown in Fig. (c), the pattern reaches the end of the rail, resulting in an extended pattern that retains the original signature* $*632$*.*

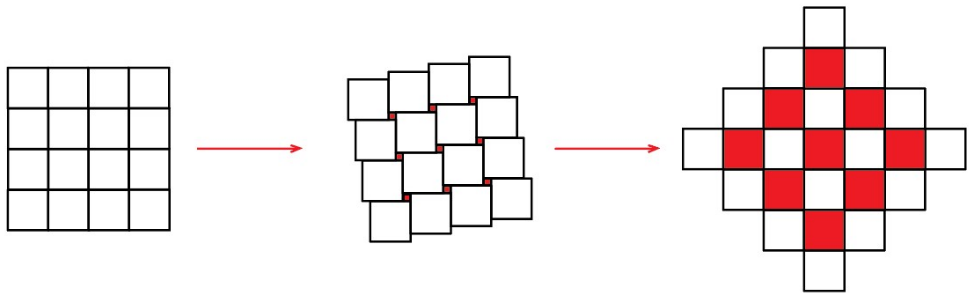


Figure 5: *Given a pattern with signature* $*442$*, which 'transformed' to* 442 *pattern, and last in the open stage defines a signature* $*442$*.*

hexagons are at the beginning of the track, there is one (closed) pattern of $*632$ signature. The second stage is obtained by the dynamical movement along the tracks, which leads to a 632 signature. Lastly, when all the hexagons are at the end of the tracks, there is a third pattern (open) of $*632$.

In Figure 3b, we illustrate the mechanism of our final model, which is derived from the feasibility model mechanism (depicted in Figure 4). Compared to the feasibility model, the final model incorporates an additional layer and a greater number of hexagons, resulting in a more intricate design.

This idea can be modified to different patterns. Figure 5 is another example, which presents a feasibility model for dynamical tiling defined by $442 \leq *442$. The same mechanism as in Figure 3b can be applied with slight changes.

Conclusions and Future Work

The present paper demonstrates that group theory can be a valuable resource for innovation, particularly in the context of defining dynamic tilings in the plane. This development has important implications for designers seeking to expand their skillset. In our ongoing research, we aim to explore additional signature relations within the framework of dynamic tiling, and may even consider altering the underlying mechanisms in some cases. Looking ahead, our most ambitious goal is to extend the concept of dynamic tilings to the sphere.

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