Ceramic Bells: Aural Performance and Geometric Design

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Abstract

I describe an exploration of the aural characteristics of ceramic structures with a variety of geometric forms, from simple test rectangles, rings, and tubes to sculptural bell-like forms, including polyhedra. Frequency spectra were compared after bisque firing and after an additional cone 6 firing. Young's modulus was determined for both cases, with a near doubling observed after cone-6 firing, from 11 to 22 × 10^9 N/m^2. This led to a near doubling of frequencies, as well as clearer tones, with reduced broad spectral features surrounding the dominant resonances. The purest tone was obtained with a truncated polar zonohedron that approximates a traditional bell form.

Introduction

Bells are common in many cultures and date from at least the 2nd millennium BCE [2,6]. Metal is the most popular choice for bells, but glass, wood, and ceramics have also been widely used. When ceramics is chosen as a bell material, that choice is often made for appearance rather than tone. Ceramics allow a wide variety of glazes, the ability to paint scenes (e.g., Delft Blue), and the relative ease of molding forms such as animal or human figures. Metal, comparatively, allows fabrication of a wide variety of forms and offers superior tone with long ring times. While everyone knows what a bell is in general terms, for the purposes of this paper it will be helpful to have a concrete definition. A bell is an object that, when directly struck, creates sound primarily by vibration of the object as a whole, without the use of strings or membranes [2][6].

In addition to the geometry of a bell, material properties are key in determining how well bells ring. Young's modulus measures the tensile or compressive stiffness of a solid material when a force is applied lengthwise [5]. Density is also an important factor. Struck bells lose energy both by sound radiation and by internal loss mechanisms [2]. Many metals ring especially long because they have low internal damping, in contrast to ceramics or wood.

Due to their expense and importance, the properties of large, metallic church bells have been studied in depth. The rich sound of church or carillon bells results from many harmonics (often referred to as "partials"), which are tied to different vibrational modes. The acoustically-important partials in the sound result from modes in which the motion is primarily normal to the bell's surface [4]. For the fundamental (prime) frequency, which is close to the strike pitch, the rim of the bell distorts from circular to elliptical, and there is also a single nodal circle.

The pitch people perceive when listening to a bell is complicated, depending on the human hearing mechanism, and can vary from person to person. It's frequently not that of a constituent partial but rather a systhesis of a number of partials [4], an effect known as *virtual pitch*. Furthermore, the different vibrational modes can decay at significantly different rates, resulting in a changing sound from the time a bell is struck to the time it drops below the listerner's hearing threshhold [6].

Vibrational modes have been calculated and measured for simple geometries such as square and circular plates [8]. These modes correspond to physical deformations, with regions that remain relatively stationary (nodes) and regions where the motion is most intense (antinodes) [6]. Vibrational modes have also been characterized in classical bell forms [6]. My primary concern in this work is how the bells sound, but since that cannot be conveyed in a paper, frequency spectra are presented. In this work I used an online sound analyzer [1] to measure the spectra of various ceramic test structures and bells. The spectra shown were captured shortly (typically a couple of tenths of a second) after the initial loud clack of the strike decayed. While the frequency resolution is relatively poor, it's adequate for the semi-quantitative analysis presented here. In this paper, the term "dominant frequency" will refer to the frequency of the largest peak in a given spectrum.

The purpose of this paper is to explore possibilities for mathematics-inspired sculptural ceramic forms that might also perform well as bells. As far as I know there is no published systematic study of the properties of ceramic bells. There are many parameters that can be varied, including type of clay, firing temperature, type of glaze, shape of the bell, and size and thickness of the bell. Since the bells examined here are relatively small, not metallic, and do not possess classical bell shapes, it is difficult to draw comparisons to the detailed studies in the literature performed on church bells.

All of the structures described here were bisque fired and some subsequently fired at cone 6 using an unsanded white stoneware clay that goes by the name B-Mix. While the higher-temperature cone 10 is commonly used for functional pottery, cone 6 is often used for sculptural work to avoid effects like drooping or other distortions that can occur when clay becomes softer at higher temperatures. The structures with simple geometries were not glazed, while those intended as sculptural bells were partially or fully glazed.

Ringing of Simple Geometries

Three series of three ceramic structures were built and bisque fired to compare simple geometries and dimensions. Figure 1 shows these, rectangles of three lengths, flat rings of three diameters, and open tubes of three lengths. The rectangles have thickness ≈ 6 mm, width of ≈ 4.8 cm, and lengths of ≈ 19.2 cm, 14.2 cm, and 9.6 cm. The ringing was measured by suspending each object with string, striking each object with a hard rubber mallet, and then examining video recordings of the transient frequency spectra. An example is shown in Figure 2. For the bisque-fired bars, the respective dominant frequencies were 400 Hz, 700 Hz, and 1590 Hz. In musical terms, these roughly correspond to the piano notes G4 (the G just above middle C), F5, and G6. The dominant frequencies are assumed to be the fundamental frequencies, which is backed up by the location of the first harmonics, as discussed below.

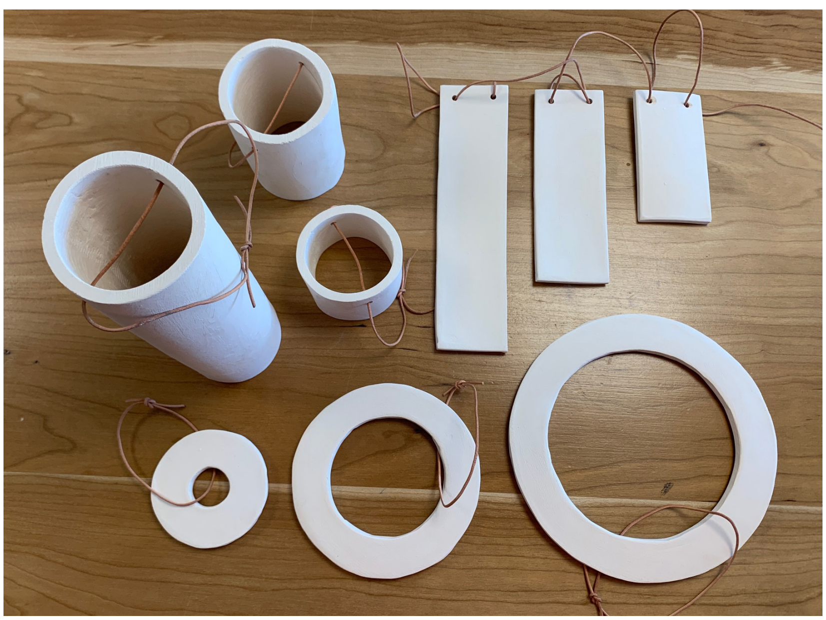

Figure 1: *Ceramic bars, rings, and tubes used in this study.*

For a free bar, the fundamental frequency (corresponding to the center of the bar deflecting in one direction) can be calculated to be approximately 1.028 $(a/L^2)/(E/\rho)^{1/2}$, where a is the thickness, L the length, E Young's modulus, and ρ the density of the material [5]. In Figure 3, the frequency is plotted vs. $1/L^2$ (leftmost line), showing good agreement to the expected linear relationship. The slope of this line can be used to calculate Young's modulus for this material, with the density calculated by measuring dimensions and weighing a bar. The result is shown in Table 1, with comparison to other common materials. Literature values for the Young's modulus of ceramics deal with commercial materials, and the wide range of compositions and production processes lead to a wide range of values for E. As a result, a good comparison to this study was not found in the literature.

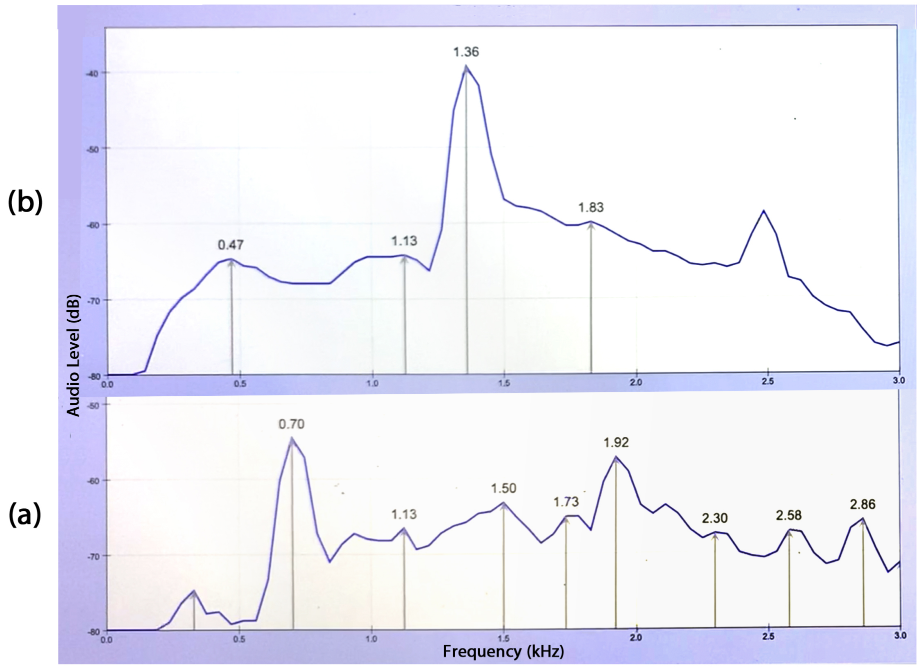

Figure 2: *Frequency spectrum for the medium-length bar, approximately 0.2 sec. after striking: a) after bisque firing, and b) after firing at cone 6.*

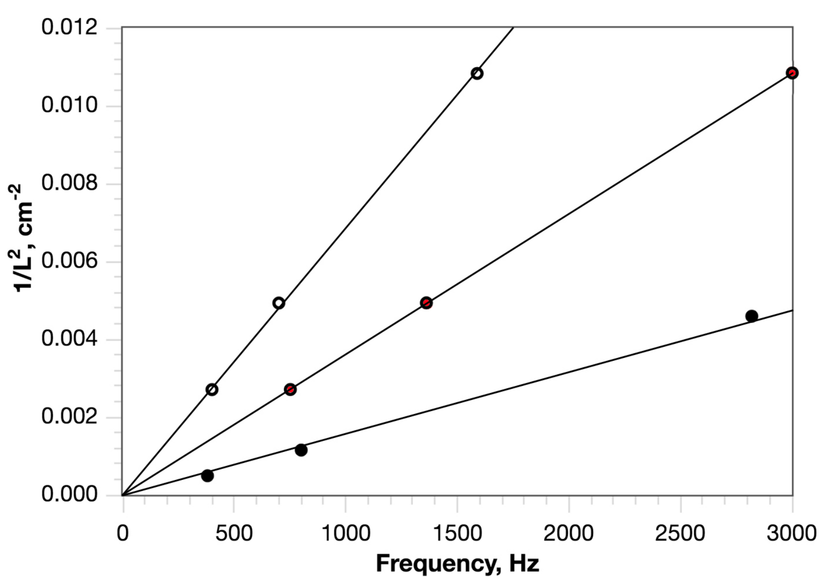

Figure 3: *Fundamental resonant frequency for bisque-fired ceramic bars (open circles), rings (black-filled circles), and cone-6-fired bars (red-filled circles; middle set). The straight lines are fits to the data with the additional requirement that they pass through the origin, since the frequencies must go to zero as the lengths becomes very large.*

Table 1: *Density and Young's modulus for some common materials (top four lines) [5] and the ceramic material used in this study, bisque fired and fired at cone 6.*

Material	Density (kg/m^3)	Young's Modulus (10^9 N/m^2)
Steel (ASTM-A36)	7860	200
Aluminum	2710	70
Glass	2190	65
Wood (Douglas fir)	525	13
Ceramic (bisque, this study)	2005	11.3
Ceramic (cone 6, this study)	2180	22.2

The unglazed bars were fired at cone 6 and retested. Pottery is typically fired twice, first at cone 04 (the bisque firing, with a maximum temperature around 1950°F or 1065°C) and then at a higher temperature for glazing. Cone 6 has a maximum temperature around 2230°F or 1220°C, and glazed pottery is often fired at the even higher-temperature cone 10 (2350°F or 1290°C). The fundamental frequencies after firing at cone 6 are shown in Figure 3 in red-filled circles (the middle line). The frequencies nearly doubled for each bar. The weight and linear dimensions decreased by approximately 3% and 4%, respectively, causing a small increase in density. The higher temperature firing was found to nearly double Young's modulus (Table 1), resulting in the near doubling of frequency. At the higher temperature, the fusing together of clay particles occurs, so the materials properties are quite different [7].

The first harmonic frequency (corresponding to half the bar deflecting in one direction and the other half in the opposite direction in a simple S shape) is expected to be approximately 2.76 times the fundamental [5]. For the bisque-fired medium-length bar (Figure 2), the expected frequency based on the fundamental is thus 2.76 × 700 Hz = 1930 Hz, and a peak is found at 1920 Hz.

For the circular rings I found dominant frequencies of 380 Hz, 800 Hz, and 2820 Hz. To compare this to the bar case, I used as the length parameter L the circumference of the circle measured in the middle of the ceramic band. The frequencies are plotted as a function of this length in Figure 3, again showing a linear relationship, though the fit is not as good as the bar case. A dominant tone was somewhat difficult to hear for the largest ring due to a broad spectrum of frequencies being present.

The ceramic tubes did not yield clear results. The dominant frequency was nearly the same for each tube, around 1900 Hz. The tone was more muddled (a broad range of frequencies) for the long tube, getting clearer as the length shortened. The fact that the dominant frequency is about the same for each tube suggests that the loudest sound is due to an excitation of the circular-ring cross-sectional shape. Shorter tubes more nearly approximate rings, which may explain the clearer tone for the short tube. The three spectra are shown in Figure 4, where the exact values for the frequencies and amplitudes are not significant. Based on these results, a tube geometry does not appear to be a good choice for a ceramic bell. Note that metal tubes do work well as, for example, wind chimes. The difference is presumably due to materials differences, though such chimes also tend to be longer relative to their diameter.

Bells with Geometric Designs

A variety of bells with strong geometric character were built and tested. The first two are relatively small (12-13 cm in height) and don't deviate too far from conventional bell forms. The bell of Figure 5 has a three-fold shape reminiscent of some flowers and possessing negative curvature near the edges. It was glazed all over and fired at cone 6, and then equipped with a cylindrical maple clapper. This bell exhibits several sharp resonances, as seen in Figure 5. The dominant peak at 2580 Hz appears to be the fundamental. For a classical bell form, the 5160 Hz peak at twice the fundamental would be the octave, while the peak at 6470 Hz, approximately 2.5 times the fundamental, would be the upper third [6]. The fifth should be at 1.5 times the fundamental; the 4030 Hz peak is approximately 1.56 times the fundamental. In a canonical bell, in terms of nodes the fundamental and fifth have a single meridian, while the octave has two. The three have, respectively, two, three, and four nodal circles [6]. The lack of circular symmetry would alter the shape of these nodes. The ring does not clearly sound like a single pitch to my ear, though I can discern a tone that corresponds to D5 on a piano keyboard (587 Hz). There is no peak at this perceived pitch, and it's not clear how the frequency relates to the spectrum.

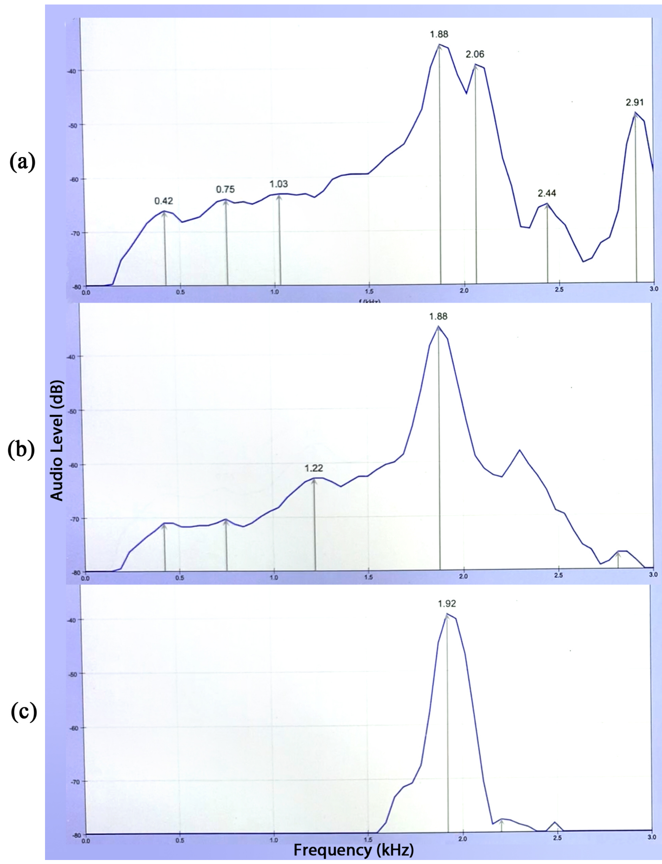

Figure 4: *Frequency spectra for 5.7cm-diameter ceramic tubes of length (a) 18.5 cm, (b) 9.0 cm, and (c) 4.5 cm.*

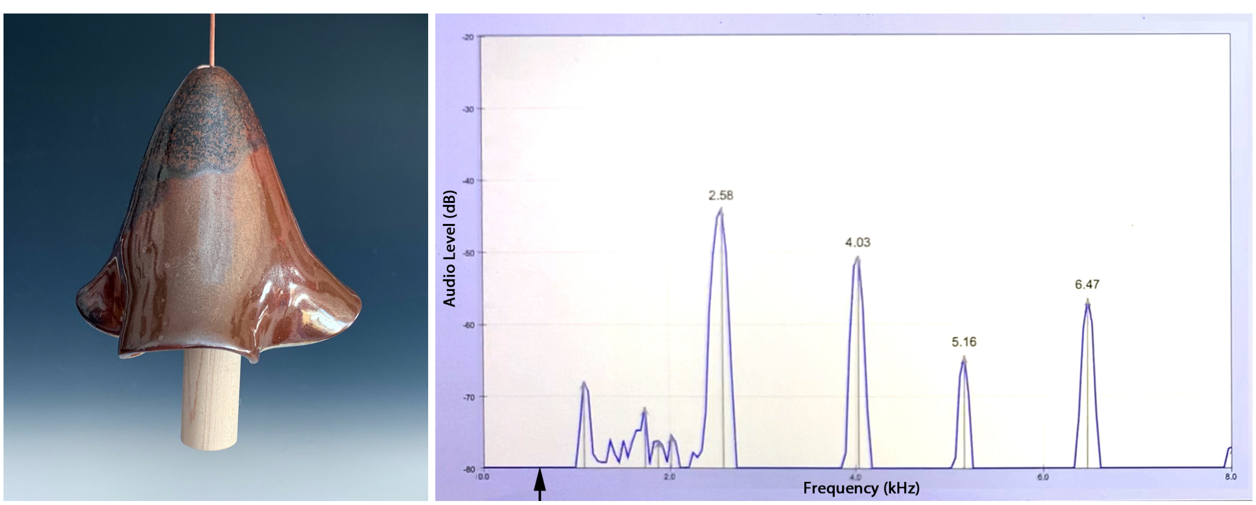

Figure 5: *Photograph of a glazed bell with a three-fold form, along with its frequency spectrum. The black arrow indicates the dominant perceived pitch after the final firing, corresponding to D_5 on a piano keyboard.*

It's interesting to note that the pitch and, to my ear, the complexity of the tone both increased after glazing and firing at cone 6. (Unfortunately, the spectrum of this bell was not recorded after bisque firing.) Glazing involves not only coating with a different material, but a higher-temperature firing. I observed an increase in frequency in all structures upon higher-temperature firing.

The second small bell is conical in shape. It was partially glazed because dipping in glaze conveniently illustrated two conic sections, the ellipse and parabola. The bell and its associated frequency spectrum are shown in Figure 6. The dominant peak could again be the fundamental, with the pair of second-highest peaks being the octave. The pronounced peak between the fundamental and octave observed in Figure 5 is absent in this case, perhaps reflecting the fact that the shape is less similar to a conventional bell. The ring of this bell sounds more like a single tone to my ear, with a perceived pitch of D5. Note this is two octaves below the dominant frequency and does not correspond to a peak in the spectrum.

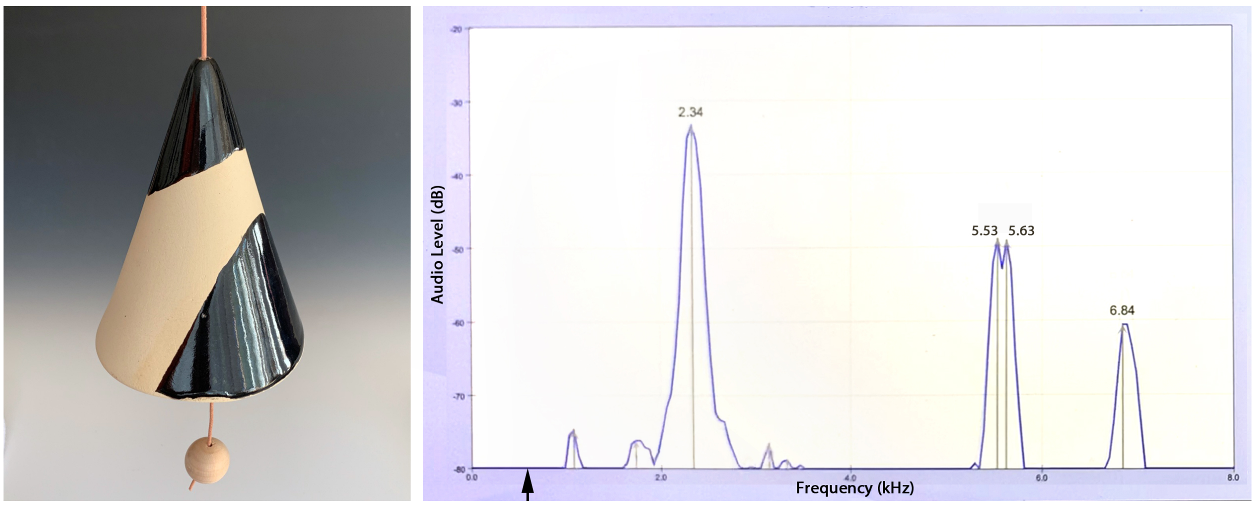

Figure 6: *Photograph of a partially-glazed conical bell, along with its frequency spectrum. The black arrow indicates the dominant perceived pitch after the final firing, corresponding to D_5 on a piano keyboard.*

With the goal of achieving a low-pitched gong-like sound in a visually-compelling mathematical design, the Hilbert curve was used as the basis for a bell. A third-order curve was filled in to create the shape shown in Figure 7, which was cut from a slab of clay. The height of this structure after bisque firing was 21 cm, and the thickness 7 mm. The gong is struck with a hard rubber mallet (not shown). The resulting frequency spectrum of the bisque-fired gong is quite broad and lacking strongly-dominant resonances, as shown in Figure 7 in blue. After firing at cone 6 (in red), the most pronounced frequencies increased by approximately 65% and became more pronounced relative to the broad overall spectrum. While the sound of the gong is complex, my ear discerned a tone at 165 Hz (E_3 on a piano keyboard), which doesn't correspond to one of the more pronounced peaks.

Two larger polyhedron-based structures were built that roughly correspond to classical bell forms. One is based on a regular dodecahedron and the other on a polar zonohedron. One face of an approximately 17-cm-diameter dodecahedron was left open, and a cylindrical purpleheart clapper was used (Figure 8). The frequency spectum of the bisque-fired bell is complex (Figure 8), with many peaks, resulting in a sound that does not evoke a single note. Firing at cone 6 increased the dominant frequency by about 80% and largely removed the broad band of frequencies, creating a clearer tone. Glazing and firing at cone 04 (a significantly lower temperature than cone 6) didn't significantly change the frequency spectrum. My ear perceives tones corresponding to F_4 and G_4 on a piano keyboard, with the tone depending on which lower pentagonal face was struck. This is approximately two octaves below the dominant frequency of 1450 Hz.

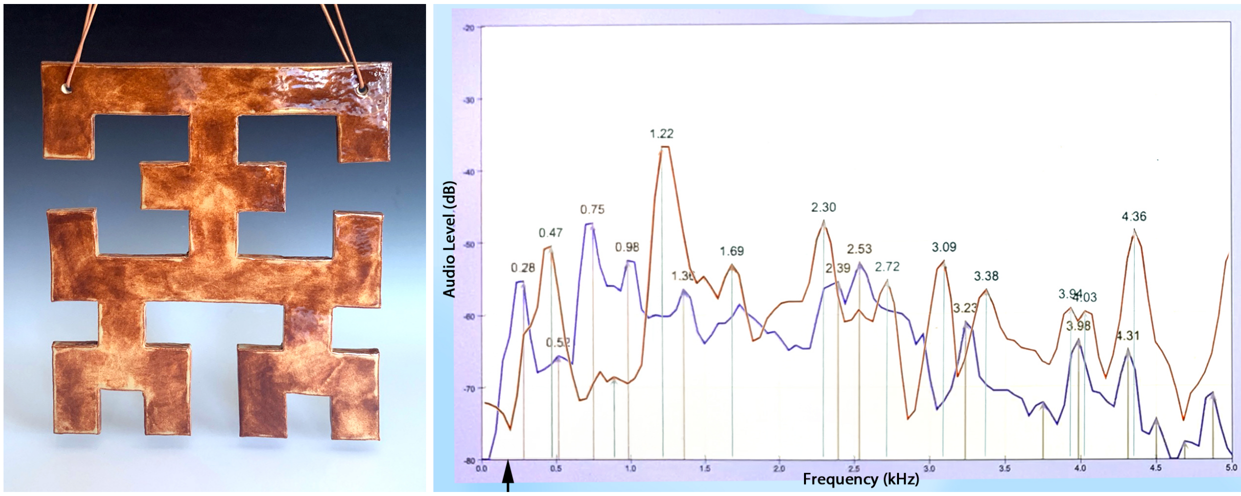

Figure 7: *Photograph of a Hilbert-curve gong, along with its frequency spectrum. The blue curve is after bisque firing and the red curve (right-shifted peaks) after firing at cone 6. The black arrow indicates the dominant perceived pitch after the final firing, corresponding to E_3 on a piano keyboard.*

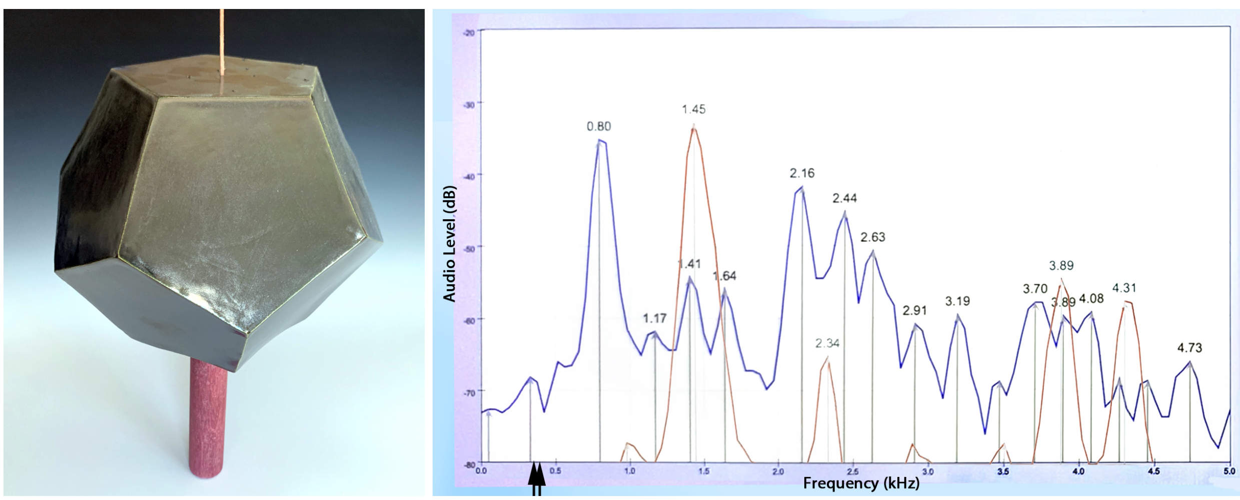

Figure 8: *Photograph of a dodecahedron-based bell, along with its frequency spectrum. The blue curve is after bisque firing and the red curve (right-shifted peaks) after firing at cone 6. The black arrows indicates the dominant perceived pitches after the final firing, corresponding to F_4 and G_4 on a piano keyboard.*

A seven-fold geometry was used for the zonohedron, with the bottom two-and-a-half rows of rhombuses omitted. Code written by George Hart [3] was used to generate a net to guide the cutting of a starting flat form from a slab of clay. The opening measures ≈ 13 cm in diameter. A relatively-large maple sphere was used as the clapper, as shown in Figure 9. The ring of this bell sounds relatively pure, with a dominant frequency after bisque firing (blue line) of 840 Hz and after cone-6 firing (red line) of 1500 Hz. The second-largest peak after cone-6 firing is approximately 2.5 times that in frequency. The perceived pitch at 784 Hz (G_5 on a piano keyboard) is not overly high, in contrast to the small bells. To my ear this was the most pleasing ring of the bells described here. The frequency spectra of Figure 9 are significantly simpler than those of the dodecahedron bell. Perhaps the sharp angles in the dodecahedron create a complicated set of relaxation modes. Cone-6 firing resulted in a purer tone, where purer is taken to mean a relatively small number of sharp peaks, without a broad shoulder of frequencies. From a materials perspective, this may indicate a removal or reduction of some loss modes as the clay densifies. Glazing and firing the zonohedron bell a second time at cone 6 had minimal effect on the overall shape of the frequency spectrum but did result in shifts of the 1.5 kHz and 3.7 kHz peaks to 1.55 kHz and 3.94 kHz.

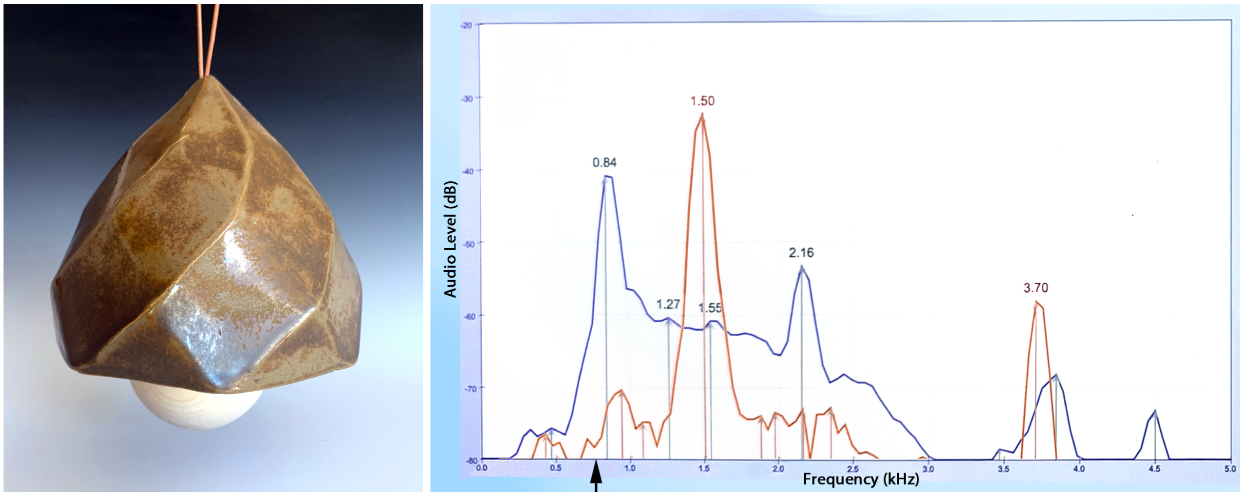

Figure 9: *Photograph of a zonohedron-based bell, along with its frequency spectrum. The blue curve is after bisque firing and the red curve (right-shifted peaks) after firing at cone 6. The black arrow indicates the dominant perceived pitch after the final firing, corresponding to G_5 on a piano keyboard.*

Summary and Conclusions

This paper presents the results of a study of geometric ceramic bells. While long ring times have not been demonstrated, relatively loud rings with pleasant tone have been achieved. In addition, I've demonstrated the ability to control the perceived pitch over a relatively wide range. The structures that perform well have more-or-less traditional bell forms, with rounded cavities open at one end. A seven-fold polar zonohedron performed better than a regular dodecahedron, perhaps due to smaller dihedral angles. I've obtained values for Young's modulus for typical stoneware and shown that it approximately doubles from bisque (cone 04) to cone 6 firing. Glazing and firing a second time appears to have minor effects on the sound of ceramic bells, at least when the glaze firing temperature does not exceed earlier firing temperatures. One area for future work would be to fill in the gap between simple geometric structures and sculptural forms with traditional bell forms. In addition, exploring the range of possibilities with different polar zonohedra would likely result in attractive bells with varying pitches.

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