Hooked on Calculus: Crocheting Quadric Surfaces

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Abstract

In an effort to make the surfaces in a multivariable calculus class tangible, this paper provides crochet patterns for 6 small models of quadric surfaces. We broadly outline the mathematical concepts behind our patternmaking process, explain how this led to our research in automating crochet patterns, and provide ideas for future directions. The patterns, discourse, and resources provided are useful for educational and artistic purposes. They provide tactile examples of surfaces discretized into patterns and applications of calculus to materials and design.

Rationale for this Project

Figure 1: *"Little Quadric Lights" is a set of string lights created by Amanda Taylor Lipnicki utilizing the patterns in this paper. It was published and exhibited in art shows [4] and [5].*

Quadric surfaces are used as examples in virtually every multivariable calculus class. They build intuition about equations and their graphs in 3-dimensions, a major part of learning calculus in higher dimensions. Many students struggle to make that leap from 2- to 3-dimensions. This project was inspired by that struggle. Our work can be used to provide optional projects for students, to make tangible models for classroom use, to provide an application of calculus to design, for fiber artists learning more about pattern design, and to assemble a set of tools for segmenting mostly smooth surfaces into finite discretized patterns. We provide a calculus worksheet for use in classes at [3].

We chose to design these patterns using crochet instead of knitting for a couple of reasons. First, crocheted items create thicker fabric that better maintains structured forms. Furthermore, crocheting small models in the round is easier than knitting in the round because there are fewer live stitches to unravel. Therefore, these patterns are more approachable to everyone—including students and others with limited or no fiber arts experience. For novices, the YouTube channel *Crochet Guru* [9] is a great resource; A. Lipnicki used it in her classroom with positive student reception. We note the calculations and ideas in this paper also apply to knitting, and knitting is more easily mechanized. Therefore, those who are familiar with knitting or who work with machinery may prefer to adapt these ideas to knitting.

In the next section of this paper, we outline mathematical methods used to construct crochet patterns for the quadric surfaces. In the third section, we provide patterns for making 6 small crocheted models of quadric surfaces. In the fourth, we propose directions for future exploration and research.

Mathematics of the Patternmaking

We set out to make 6 crochet models of quadric surfaces of each of the following type: elliptic paraboloid, hyperboloid of 1 sheet, half cone, hyperboloid of 2 sheets, ellipsoid, and hyperbolic paraboloid. All of these, besides the hyperbolic paraboloid, are surfaces of revolution. Therefore, our methods for constructing the first 5 are very similar. The hyperbolic paraboloid, however, will require its own explanation.

Surfaces with Circular Cross-Sections

An elliptic paraboloid, hyperboloid of 1 sheet, half cone, hyperboloid of 2 sheets, and ellipsoid all have circular cross-sections when parameters are chosen appropriately. Therefore, they are amenable to similar treatments when constructing crochet patterns. We crochet in the round for these models. Our task was to determine how many stitches should be in each row and how many rows should be in each pattern. On a first pass, one may be tempted to take circumference measurements at equally-spaced x increments. For example, Figure 2(a) shows a placement of 8 rows at $x = -3.5, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, 3.5$. Placing rows in this manner will give us something closer to a sphere when actually crocheted because the curvature of the fabric itself is not accounted for in the row number, causing rows to accumulate toward the center where circumferences are larger. A more accurate method instead uses arclength to place rows. For example, the blue ellipses in Figure 2 show the curve $z = \pm\sqrt{\frac{x^2}{4} - 1}$ with $y = 0$. The length of the part of this curve above the xy-plane is

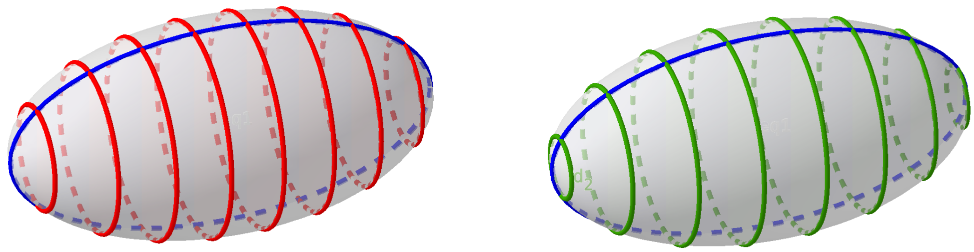
$$L = \int_{-4}^{4} \sqrt{1 + \left(\frac{x/2}{2\sqrt{\frac{x^2}{4} - 1}}\right)^2}\, dx \approx 9.688.$$

To space eight rows evenly, we can choose x-values that fall at equally-spaced arclength distances $L/16, 3L/16, 5L/16, \ldots, 15L/16$ along the blue ellipse. In this instance, we can position our rows at $x = -3.83, -2.93, -1.80, -0.61, 0.61, 1.80, 2.93, 3.83$ to achieve the spacing pictured in Figure 2(b). Note that in this spacing, the rows are pushed outward to the ends, accounting for the curvature of the fabric along the ellipse.

In addition, when determining how many rows to place in a pattern, using arclength increments will result in more rows than using x-increments of the same length. In the previous example, rounding the total arc length indicates our model will have 2 additional rows from the x-incremented model, an increase of 25 percent in the total number of rows on a tiny model.

Suppose we have determined placement of rows via arclengths and find that a row should be placed at $x = c$. Then the number of stitches in that row is based on the circumference $2\pi f(c)$ of the circle of revolution located at $x = c$. In addition, we let R = number of rows per four inches, S = number of stitches per four inches, and $scale$ = the measure of a unit of the graph in inches. Then the number of stitches in the row at $x = c$ should be $2\pi f(c) \cdot (S/4) \cdot scale$ rounded to the nearest integer.

One difficulty that arose was how to determine where to place a stitch to increase or decrease the number of stitches in a row. Within rows, we do this as evenly as possible. Another problem is if increases or decreases fall near each other on adjacent rows, then the curvature of the surface will be less smooth and the increases or decreases will be more noticeable. When creating these patterns by hand, one can "eyeball"



(a) (b)

Figure 2: *An example contrasting row placements on the ellipsoid* $\frac{x^2}{4^2} + \frac{y^2}{2^2} + \frac{z^2}{2^2} = 1$*. Figure (a) has 8 rows placed at equal* x*-increments, and figure (b) has 8 rows placed at equal arclength increments.*

the placement to avoid overlap as much as possible. However, accurately calculating and notating patterns quickly became tedious and algorithmic. Therefore, we created a program to do the bulk of the work for us. Details of how we chose to program our patternmaking are in [6], and a Sage worksheet with our program is available in [7].

Our algorithm takes in a single variable function $f(x)$, x-bounds a and b, gauges R and S (number of rows and stitches in 4 inches), and $scale$. It outputs a crochet pattern to create a model of the surface of revolution formed by rotating the graph of $y = f(x)$ around the x-axis. There are many important considerations that went into the creation of this program, including how to make optimal choices for row placement to accentuate important features of a surface and how to automate the placement of increases and decreases to minimize overlap using two distance measures.

After making patterns by hand using the methods outlined previously, we also used our computer program to output 5 out of 6 of the patterns. This required representing each quadric surface as a revolution of some function $f(x)$ with x-bounds, our gauges, and our desired scale. To enable users to adjust the gauge and scale and output new instructions, we link to a Sagemath worksheet in the patterns section of this paper. However, we expect the patterns included in this paper are forgiving enough to make this unnecessary.

The hyperbolic paraboloid pattern was created differently, so its pattern is explained below.

Hyperbolic Paraboloid

We tried a couple of different methods for achieving a satisfying pattern for the saddle, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$. In the end, we used symmetry to our advantage. We used the level curves where $z = 0$ intersects the hyperbolic paraboloid in an $\times$ shape to segment the surface into four parts. When $a = b$, these parts are identical up to some symmetry transformation and thus will require the same number of stitches and rows in our crochet pattern. When a and b are not equal, one can create the instructions for two of the parts and use symmetry for the others.

In Figure 6, we have shown the basic premise of how we took our measurements. The black lines break the shape into four identical parts (up to symmetry), the blue lines show the parabolas $5z = x^2$ and $5z = -y^2$, and the red lines show the placement of measurements for each row of one part, each spaced an equal arclength apart from each other along the blue curve. We crocheted short rows along the red lines, determining the row number by the arclength along the parabola from the center. The actual length of each row in our model was based on our gauge and choice of scale for the finished project. We then repeated the resulting pattern for a quarter of the surface 3 times, one for each of the other quarters of the surface that lay between the black lines on the base $\times$ level curves. The only difference is that the first quarter is worked

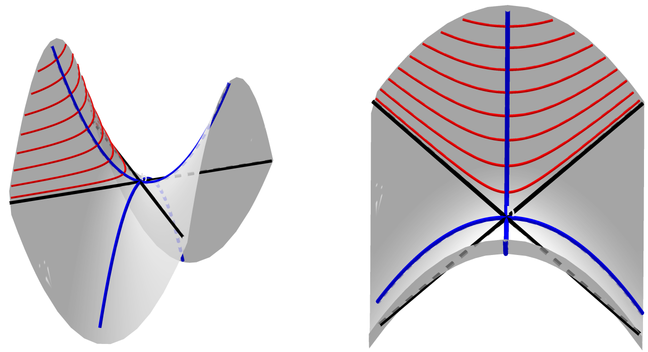


Figure 3: *The hyperbolic paraboloid defined by $x^2 - y^2 = 5z$ for $-6.8 \leq x, y \leq 6.8$. The red lines show where we measured the surface for our crochet pattern.*

backwards from the other quarters. This allows us to have one less break in yarn while crocheting the pattern. Crocheters concerned about visual distinctions between their increases and their decreases are encouraged to alter this pattern to prioritize increases and break and rejoin the yarn once more.

Note one concern with this method is that we are creating extra fabric close to the black $\times$. In practice, crochet is fairly forgiving, so this doesn't end up being a noticeable issue. Another concern is to create a figure which will stand on its own. To create more support, we truncated the entire surface at a z level curve at the bottom. For aesthetics, we also truncated the top. See Figure .

The Crochet Patterns

In Figure 1 is a picture of art created using the patterns that follow. Each piece is designed to fit within a 4"×4"×4" cube or smaller based on the artist's preferences for the relative sizes of the figures. With the recommended hook size, we expect the patterns to work up nicely. However, interested parties can change their gauges and scales for each figure except the saddle using the "crochet_quadrics.sagews" file at [3].

The first 5 pieces, our surfaces of revolution, are worked in the round. We recommend placing a stitch-marker at the beginning of each row. For joining rows, we tried a few different methods—spiraling up, slip stitch joins, and hidden slip stitch joins. We do not recommend slip stitch joins because they create visible seams. Hidden slip stitch joins invert the seam to be on the inside of the model but take substantial extra time. Therefore, our favorite method is spiraling up, where the crocheter stitches directly into the first stitch of the next row with no special moves. Blocking the final pieces is detailed in the last step. This process takes advantage of wool's enhanced malleability when wet to even out the stitches and help the sculptures keep their forms. We use American crochet terminology.

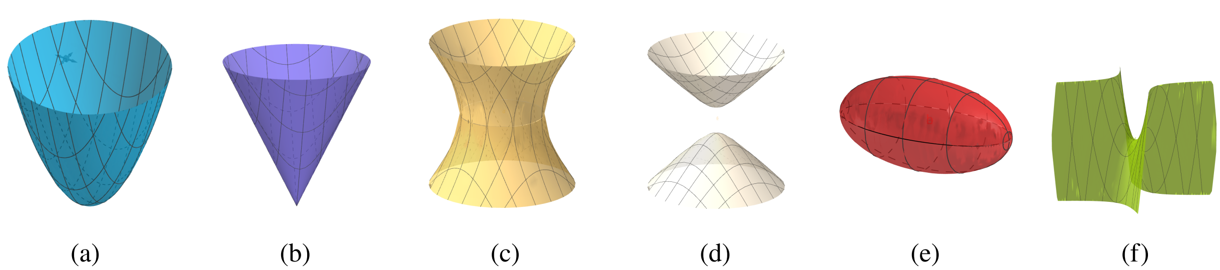


Figure 4: *Our crocheted quadric surfaces rendered by Geogebra, listed in the same order as the patterns.*

Supplies and Notions

Yarn: KnitPicks Wool of the Andes worsted weight yarn in Avocado, Persimmon Heather, Marina, Turmeric, Bamboo Heather, and Celestial.
Other materials: crochet hook in size G/6–4.25MM or size needed to obtain gauge, stitch markers, 20mm gauge copper jewelry wire, jewelry pliers and wire cutter, paper towels, water, and polyfill (if making stuffed surfaces) or string lights with bulbs of diameter less than 1 inch.
Gauge for a 4"×4" square: 17 stitches by 21 rows.
Abbreviations: Sc=single crochet, Dec=single crochet two stitches together, Inc=single crochet two stitches in one stitch, ch=chain.

Elliptic Paraboloid in color Marina

Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z, a = b = 1, 0 \le z \le 3$
Make a slip knot, leaving a 3-inch tail.
Row 1: Chain 46, join, and Sc46 (46 stitches)
Row 2: Sc44, Dec (45 stitches)
Row 3: Sc22, Dec, Sc21 (44 stitches)
Row 4: Sc10, Dec, Sc20, Dec, Sc10 (42 stitches)
Row 5: Sc6, Dec, Sc26, Dec, Sc6 (40 stitches)
Row 6: Sc38, Dec (39 stitches)
Row 7: Sc13, Dec, Sc17, Dec, Sc5(37 stitches)
Row 8: Sc8, Dec, Sc17, Dec, Sc8(35 stitches)
Row 9: Sc16, Dec, Sc17 (34 stitches)
Row 10: Sc4, Dec, Sc14, Dec, Sc12 (32 stitches)
Row 11: Sc10, Dec, Sc14, Dec, Sc4(30 stitches)
Row 12: *Sc13, Dec* 2 times (28 stitches)
Row 13: *Dec, Sc7* 3 times, Sc1 (25 stitches)
Row 14: Sc6, Dec, Sc10, Dec, Sc5 (23 stitches)
Row 15: *Sc5, Dec* 3 times, Sc2 (20 stitches)
Row 16: Sc2,*Sc4, Dec* 3 times (17 stitches)
Row 17: *Dec, Sc3* 3 times, Sc2 (14 stitches)
Row 18: *Sc1, Dec* 4 times, Sc2 (10 stitches)
Row 19: Sc2, *Dec* 4 times (6 stitches)
Tie off and weave in ends.

Half Cone in color Celestial

Equation: $z = \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}, a = b = 0.5, 0 \le z \le 3$
Make a slip knot, leaving a 3-inch tail.
Row 1: Chain 40, join, and Sc40 (40 stitches)
Row 2: *Sc18, Dec* 2 times (38 stitches)
Row 3: Sc9, Dec, Sc16, Dec, Sc9 (36 stitches)
Row 4: Sc4, Dec, Sc16, Dec, Sc12 (34 stitches)
Row 5: *Sc9, Dec* 3 times, Sc1 (31 stitches)
Row 6: Sc10, Dec, Sc14, Dec, Sc3 (29 stitches)
Row 7: *Dec, Sc12* 2 times, Sc1 (27 stitches)
Row 8: Sc6, Dec, Sc11, Dec, Sc6 (25 stitches)
Row 9: *Sc6, Dec* 3 times, Sc1 (22 stitches)
Row 10: Sc2, Dec, Sc9, Dec, Sc7 (20 stitches)
Row 11: Sc6, Dec, Sc8, Dec, Sc2 (18 stitches)
Row 12: *Dec, Sc7* 2 times (16 stitches)
Row 13: *Sc3, Dec* 3 times, Sc1 (13 stitches)
Row 14: *Sc3, Dec* 2 times, Sc3 (11 stitches)
Row 15: *Sc3, Dec* 2 times, Sc1 (9 stitches)
Row 16: *Sc1, Dec* 3 times (6 stitches)
Row 17: *Dec, Sc1* 2 times (4 stitches)
Row 18: *Dec* 2 times (2 stitches)
Tie off and weave in ends.

Hyperboloid of 1 sheet in color Turmeric

Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 2, a = b = c = 0.8,$ $-2 \le z \le 2$
Make a slip knot, leaving a 3-inch tail.
Row 1: Chain 53, join, and Sc53
Row 2: *Dec, Sc15* 3 times, Sc2 (50 stitches)
Row 3: Sc4, Dec, *Sc15, Dec* 2 times, Sc10 (47 stitches)
Row 4: *Sc21, Dec* 2 times, Sc1 (45 stitches)
Row 5: Sc8, Dec, Sc18, Dec, Sc8, Dec, Sc5 (42 stitches)
Row 6: Sc9, Dec, Sc20, Dec, Sc9 (40 stitches)
Row 7: *Dec, Sc18* 2 times (38 stitches)
Row 8: *Sc10, Dec* 3 times, Sc2 (35 stitches)
Row 9: Sc8, Dec, Sc25 (34 stitches)
Row 10: Sc3, Dec, Sc15, Dec, Sc12 (32 stitches)
Row 11: Sc23, Dec, Sc7 (31 stitches)
Row 12: Sc29, Dec (30 stitches)
Row 13: Sc30 (30 stitches)
Row 14: Sc30, Inc (31 stitches)
Row 15: Sc24, Inc, Sc6 (32 stitches)
Row 16: Sc7, Inc, Sc24 (33 stitches)
Row 17: *Sc15, Inc* 2 times, Sc1 (35 stitches)
Row 18: Sc3, Inc, Sc18, Inc, Sc12 (37 stitches)
Row 19: Sc12, Inc, Sc21, Inc, Sc4 (39 stitches)
Row 20: Inc, Sc19, Inc, Sc18 (41 stitches)
Row 21: *Sc12, Inc* 3 times, Sc2 (44 stitches)
Row 22: Sc10, Inc, Sc22, Inc, Sc10 (46 stitches)
Row 23: *Inc, Sc14* 3 times, Sc1 (49 stitches)
Row 24: *Sc23, Inc* 2 times, Sc1 (51 stitches)
Row 25: Sc12, Inc, Sc25, Inc, Sc12 (53 stitches)
Tie off and weave in ends.

Hyperboloid of 2 sheets in color Bamboo Heather

Equation: $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, a = b = c = 0.4,$
$-2 \leq z \leq 2$

Make 2 of the following and attach a distance of one inch from each other using wire. Place a spiral of the wire around the wire connecting the pieces to improve strength and aesthetics.

Make a slip knot, leaving a 3-inch tail.
Row 1: Chain 4, Sc4 (4 stitches)
Row 2: *Inc* 4 times (8 stitches)
Row 3: *Inc* 5 times, Sc3 (13 stitches)
Row 4: *Sc3, Inc* 3 times, Sc1 (16 stitches)
Row 5: *Sc3, Inc* 4 times (20 stitches)
Row 6: *Sc4, Inc* 4 times (24 stitches)
Row 7: *Sc5, Inc* 4 times (28 stitches)
Row 8: *Sc8, Inc* 3 times, Sc1 (31 stitches)
Row 9: *Sc6, Inc* 4 times, Sc3 (35 stitches)
Row 10: *Sc7, Inc* 4 times, Sc3 (39 stitches)
Row 11: *Sc12, Inc* 3 times (42 stitches)
Row 12: *Sc9, Inc* 4 times, Sc2 (46 stitches)
Row 13: *Sc10, Inc* 4 times, Sc2 (50 stitches)
Row 14: *Sc15, Inc* 3 times, Sc2 (53 stitches)
Tie off and weave in ends.

Ellipsoid in color Persimmon Heather

Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, a = 4, b = c = 2,$
$-4 \leq x \leq 4$

Make a slip knot, leaving a 3-inch tail.
Row 1: Chain 4, join, and Sc4
Row 2: *Inc* 4 times (8 stitches)
Row 3: *Sc1, Inc* 3 times, Sc2 (11 stitches)
Row 4: *Sc1, Inc* 4 times, Sc3 (15 stitches)
Row 5: *Sc6, Inc* 2 times, Sc1 (17 stitches)
Row 6: *Sc4, Inc* 3 times, Sc2 (20 stitches)
Row 7: *Sc9, Inc* 2 times (22 stitches)
Row 8: Sc21, Inc (23 stitches)
Row 9: Sc11, Inc, Sc11 (24 stitches)
Row 10: Sc6, Inc, Sc17 (25 stitches)
Row 11: Sc18, Inc, Sc6 (26 stitches)
Row 12: Sc25, Inc (27 stitches)
Row 13: Sc27 (27 stitches)
Row 14: Sc27 (27 stitches)
Row 15: Sc27 (27 stitches)
Row 16: Sc25, Dec (26 stitches)
Row 17: Sc18, Dec, Sc6 (25 stitches)
Row 18: Sc6, Dec, Sc17 (24 stitches)
Row 19: Sc11, Dec, Sc11 (23 stitches)
Row 20: Sc21, Dec (22 stitches)
Row 21: *Sc9, Dec* 2 times (20 stitches)
Row 22: *Sc4, Dec* 3 times, Sc2 (17 stitches)
Row 23: *Sc6, Dec* 2 times, Sc1 (15 stitches)
Row 24: *Sc1, Dec* 4 times, Sc3 (11 stitches)
Fill with stuffing of your choice such as polyfill. If making string lights, omit this step and tie off and weave in ends now.
Row 25: *Sc1, Dec* 3 times, Sc2 (8 stitches)
Row 26: *Dec* 4 times (4 stitches)
Tie off and weave in ends.

Hyperbolic Paraboloid in color Avocado

Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}, a = b = 1, c = \frac{1}{5},$
$-6.8 \leq x, y \leq 6.8$

Make a slip knot, leaving a 3-inch tail. Crochet this piece back and forth as normal (not in the round). This means at the end of each row, you will turn your work to crochet the other way. Use a stitchmarker at the beginning (and end) of each row, and move the stitchmarker up when you begin (end) the next row. Or you can leave them to help with counting rows.
Row 1: Ch10, turn work and Sc10 (10 stitches)
Row 2: Sc10 (10 stitches)
Row 3: Sc10 (10 stitches)
Row 4: Sc4, Inc, Sc5 (11 stitches)
Row 5: Sc11 (11 stitches)
Row 6: Sc11 (11 stitches)
Row 7: Sc5, Inc, Sc5 (12 stitches)
Row 8: Sc 4, Inc, Sc3, Inc, Sc3 (14 stitches)
Row 9: Sc7, place stitch marker in what would be the next stitch, and Ch7 (7 stitches). The start of this chain is the "center of the piece." Place a special stitchmarker to keep track of it.
Row 10: Sc14 along the chain and stitches from the previous row (14 stitches)
Row 11: Sc4, Dec, Sc3, Dec, Sc3 (12 stitches)
Row 12: Sc5, Dec, Sc5 (11 stitches)
Row 13: Sc11 (11 stitches)
Row 14: Sc11 (11 stitches)
Row 15: Sc4, Dec, Sc5 (10 stitches)
Row 16: Sc10 (10 stitches)
Row 17: Sc10, tie off, and Ch7 out from the center of the piece (10 stitches)
Row 18: Sc7 along the Ch7 from Row 17 and Sc7 in the other half of Row 9 that remained unstitched, starting where you placed the stitchmarker. (14 stitches)
Rows 19-24: Repeat rows 11-16

Row 25: Sc10, tie off
Row 26: Sc14 along the remaining internal edge which passes through the center. (14 stitches)
Rows 27-32: Repeat rows 11-16
Row 33: Repeat row 25 (10 stitches)
Tie off and weave in ends. Cut and insert wire to reinforce the shape of the hyperbolic paraboloid along the red curves in Figure 5. Reinforce the other boundaries if desired or necessary to support your piece. Use the jewelry pliers to make a small loop on the end of each piece of wire. This closes the sharp edges and helps prevent the wire from poking through the material.

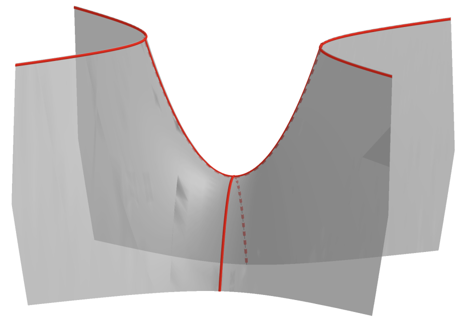


Figure 5: Curves to reinforce with wire in the hyperbolic paraboloid.

Finishings

Block your quadrics by soaking them in water, then stuffing and supporting them with balled-up dry paper towels until they have support to maintain their shapes. Allow to dry completely, then remove the paper towels. If making string lights, fill each crocheted surface with a bulb. Consider how you'll display the string lights and ensure you are satisfied with adjacency and placement of each bulb within each surface. For example, if you will hang the lights, you may want the bulb to sit on top of the surface instead of underneath it. Secure each surface to its bulb using jewelry wire, being careful to test that your attachments do not negatively impact the surface shape when the bulb is placed in your display. Use the wire to reinforce important curves on the surfaces in order to help support their shapes until you are satisfied with your display.

If you are making stuffed toys, crochet discs or cut felt to close the open ends. Sew to your models, stuffing with polyfill prior to final closure. Enjoy!

Future Directions

In this article, we outline methods for crocheting the quadric surfaces with particular equations. In many cases, we made choices to crochet shapes with particular symmetries since circular cross-sections work particularly well with crochet. This begs the question: are there methods that could be implemented to create other kinds of cross-sections?

Other interesting questions include: Can parameterized surfaces be effectively employed and allow more surfaces to be transformed into patterns via automated processes? How can we train programs to segment patterns into distinct pieces which are worked separately and seamed or which require joins using picked up stitches? How can we generalize and automate patterns featuring hyperbolic paraboloid-like iterative features such as multiple saddle points? How can we alter our results in [6] to work for flatbed knitting machines with a single seam to close and complete the circles of the surface of revolution? Can we create an additional tool which takes a function the user draws and creates a pattern for a surface of revolution based on it?

We are also experimenting with processes using different materials like crocheting with wire, using yarn hardeners, using different types of lights, and displaying lights in different ways. For example, A. Lipnicki crocheted a flexible hyperbolic paraboloid bracelet made of gossamer copper wire which can alternatively sit as a stand-alone sculpture; see Figure 6. A. Lipnicki also continues to experiment with placing lights in surfaces to highlight prominent features and spark a sense of wonder. Finally, she is experimenting with setup in environments where the lighting varies in order to maximize the impact of a displayed art piece.

Figure 6: *"Saddle Bracelet" modeled by Amanda Lipnicki*

Larger-scale work produced using our algorithms in this paper is forthcoming. Experimenting with properties of materials is crucial because the additional weight of larger yarn-based pieces make them prone to distortion under their own weight. However, hardeners can be applied to yarn-composed fabric as an alternative or in addition to wire reinforcements. On the other hand, the fabric artist may discover that appropriately controlled distortions breathe life into such pieces.

A final goal, inspired by George Hart's "sculpture barn raisings" [1], is to find ways of making crocheted model work interactive and community oriented. This is a formidable task given the time-consuming nature of crochet, but an extremely worthwhile one.

Acknowledgements

We would like to thank Daina Taimina for inspiring us to pursue mathematical modeling using crochet. Her inclusion of a pattern for crocheting hyperbolic planes in the textbook [2] sparked the authors' imaginations as undergraduate students and subsequently led to this work.

We also thank Sarah-Marie Belcastro for pointing out previous work on models for surfaces of revolution. Reference [8] lays out some ideas for creating knitting patterns for surfaces of revolution, including some mathematics that overlaps with this paper.

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