Parametric Design of a Ceiling with the Ammann–Beenker Tiling

Tatsuki Hayama[1], Sota Ozu[2], and Daisuke Tofuku[3]

[1]Senshu University, Tokyo, Japan; hayama@isc.senshu-u.ac.jp
[2]Ozu Kanban, Gifu, Japan; ozu.s.312@gmail.com
[3]worklounge 03-, Nagoya, Japan; tofuku@03-x.com

Abstract

This paper describes the ceiling design for the renovation of the Ryufuku-ji Temple, a Buddhist temple in Nagoya, Japan. We applied the Ammann–Beenker tiling to this ceiling design as a modern Buddhist architecture. Moreover, the 3D shape of this ceiling is optimized for acoustics, adopting a computational parametric design method.

Introduction

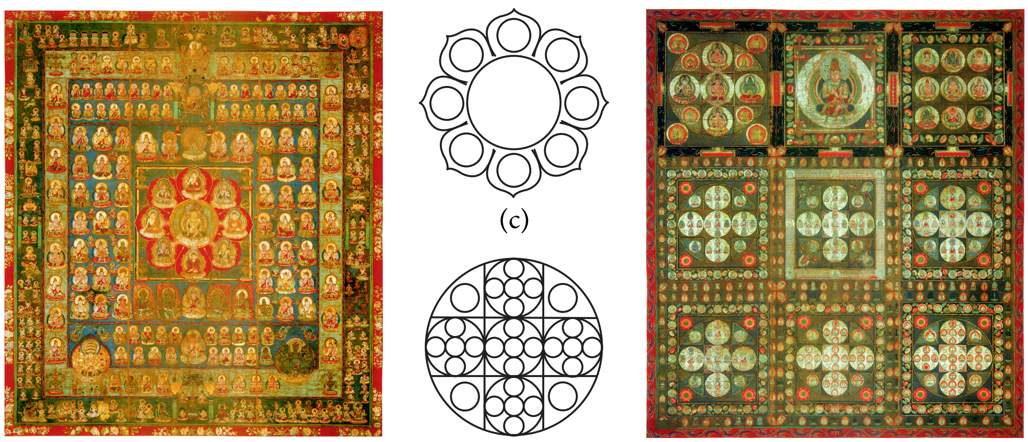
The Ryufuku-ji Temple (笠覆寺) in Nagoya, Japan, is a Buddhist temple, founded in the 8th century. The current layout of buildings was established in the 17th century and has been undergoing major renovations since 2019. The Congregation Hall, one of the new buildings constructed during renovation, was completed in 2022. The third author, the architectural designer of this hall, worked with the first author, a mathematician, and the second author, a programmer, to design a "modern ceiling painting" by referring to a mandala. The ceiling of the hall was designed with non-periodic symmetrical patterns and incorporated modern computational techniques. In this paper, we describe the geometric features of the design.

Figure 1: *Exterior and interior views of the Congregation Hall at Ryufuku-ji Temple in Nagoya, Japan. This building was completed in 2022. The authors designed the interior ceiling.*

Geometry of the Mandala

Ryufuku-ji Temple is a temple of Esoteric Buddhism. In Esoteric Buddhism, the universe is interpreted as the secret of Buddha. Its view is represented in a painting called a mandala. A mandala plays an important role in understanding Esoteric Buddhism and is worshiped in temples. There are various types of mandalas depending on the region and time. In Japan, *the Mandala of the Two Realms* were introduced from India and China in the 9th century and have been handed down to the present. This mandala consists of two paintings, called the Womb Realm (Figure 2(a)) and the Diamond Realm (Figure 2(b)), which face each other on the east and west sides of temples.

The mandala in the current era faithfully follows the composition of the original introduced in the 9th century. We may find specific geometric characteristics in the arrangement of hundreds of Buddhas. In the center of the Womb Realm (Figure 2(c)), Buddhas are arranged symmetrically and radially from the central Buddha. The Diamond Realm is composed of nine areas, and Buddhas are arranged to surround the central Buddha in a separate area (Figure 2(d)), and each Buddha is also surrounded by Buddhas in the same way, i.e., there is a recursive nature like the Sierpinski carpet. To incorporate the mandala's geometric essence, such as symmetry and recurrence, into the ceiling design, we adopted one of the most interesting results in tiling theory, the existence of tiles that only tile non-periodically [1, Chapter 10].



(a) (d) (b)

Figure 2: *The Mandala of the Two Realms: (a) the Womb Realm; (b) the Diamond Realm; (c) Schematic diagram of the central part of (a); (d) Schematic diagram of the central part of (b).*

The Ammann–Beenker Tiling

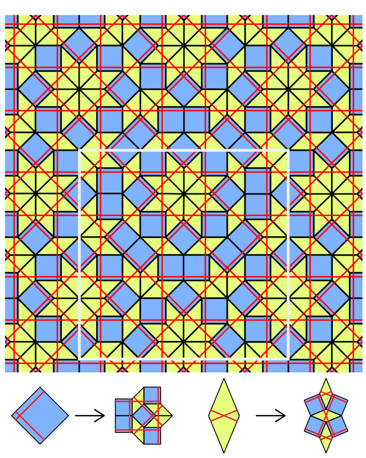


Figure 3: *The Ammann–Beenker tiling with the marked prototiles.*

Periodic tilings are often used from old age for the real world, such as decorating pavements or walls. The symmetry structure of periodic tilings can be classified by the wallpaper groups, which indicate that a periodic tiling cannot have n-fold symmetry except for the case with $n = 1, 2, 3, 4, 6$. However, extending the range to non-periodic tilings can have other rotational symmetries as the 5-fold symmetry of the Penrose tiling. In this ceiling design, we adopted *the Ammann–Beenker tiling* [2], which is a non-periodic tiling with 8-fold symmetry, the same as the Womb Realm Mandala.

The Ammann–Beenker tiling can be constructed by a recursive method like the Penrose tiling. As can be seen in Figure 3, the tiling consists of two prototiles (the square and the rhombus, called the Ammann aperiodic set A5 [1]). The tiling is constructed by iteratively replacing each prototile with the substitution rule in Figure 3, where we set a flower of the rhombic 8-petals as the initial state.

The Ammann–Beenker tiling also works for the actual construction. Because the hall is square, the design should be square-based. The Ammann–Beenker tiling meets this requirement. In fact, by marking the prototile with line segments, called Ammann bars [1], as in Figure 3, the four sets of bars with angles of $0, \pi/4, \pi/2$, and $3\pi/4$ are visible on the tiling. This means the tiles are aligned along the orthogonal grid.

The hall is 7.5m square. In this ceiling, aluminum panels are cut into tiles and fixed with joints, which are hung from the ceiling beams (Figure 4). As a constructional constraint, it was necessary to adjust the position of the joints to the beams. Moreover, the size of tiles had to be neither too small nor too large due to fabrication capability. Considering these requirements, we adopted the framed area in Figure 3 for our design. The reader will notice in Figure 3 that the center of the framed area does not coincide with the center of the 8-fold symmetry. The centric symmetry point concentrates the visitor's perspective, evoking a monotheistic meaning. This is not appropriate for Buddhism as a polytheistic religion, and we avoided it.

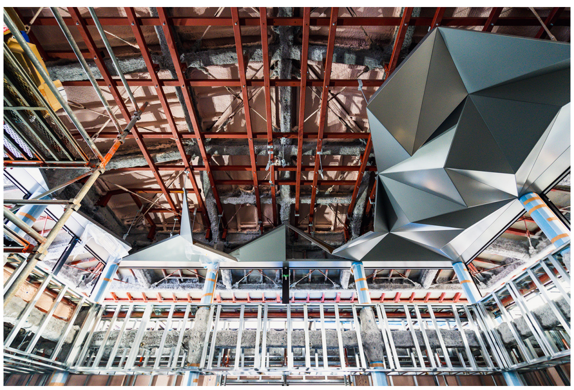


Figure 4: *Construction of the ceiling. The ceiling panels are suspended from the beams.*

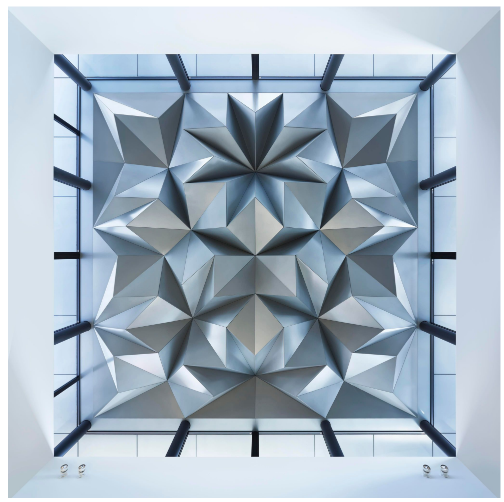


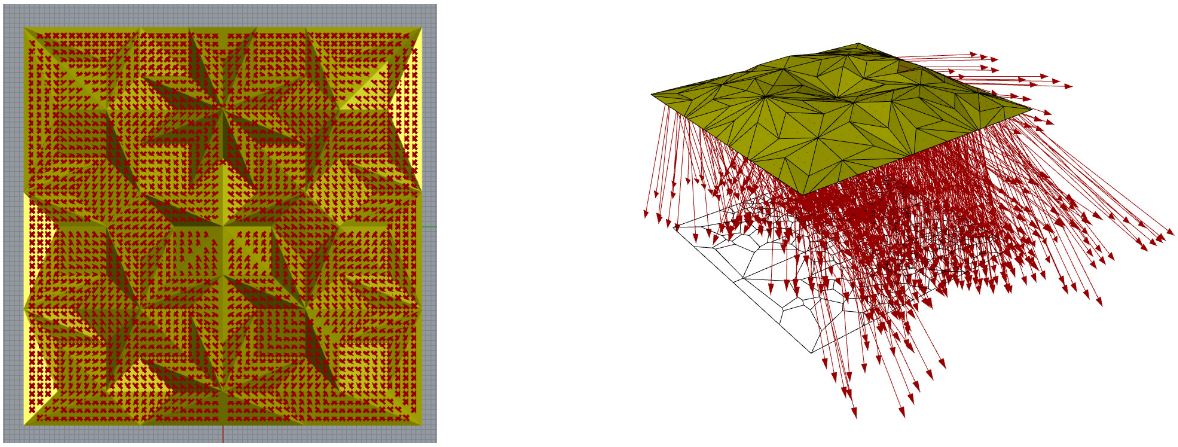
Figure 5: *The ceiling seen from the floor, photographed in 2022.*

Shape Optimization

The ceiling is not a flat tiling but an origami-like polyhedral surface, and this shape design has both aesthetics and functionality. The final shape is optimized for acoustics because the hall is used for religious discourses and music concerts. The presence of standing waves often causes acoustic problems. Standing waves are generated by the interference between two waves in opposite directions and are more likely to occur in square-shaped halls. In our design method, the shape was deformed in three dimensions to minimize standing waves.

This 3D tiling is constructed by moving the z-position of the vertices of the subdivided Ammann–Beenker tiling (i.e., a triangular mesh). For standing wave suppression, the reflection of the sound should be dispersed. To evaluate the scatter of the reflection, we carried out a sound simulation.

An acoustically worse situation that causes the standing waves occurs when the sound source is directly below the 8-fold symmetry point. Our simulation was carried out for this situation. We took the grid points on the ceiling as reference points (Figure 6(a)), and calculated the points where the primary reflections at the reference points would hit the floor as Figure 6(b). These points on the floor had to be scattered. We optimized the z-position of the vertices of the ceiling so that the points on the floor would be scattered. For



(a) **(b)**

Figure 6: *Sound simulation: (a) Reference points on the ceiling; (b) Primary reflections at the points (a).*

this optimization, we took the Voronoi diagram of the points on the floor and evaluated the areas of the Voronoi cells. To reduce the area dispersion, we used the standard deviation of the areas as the objective function and minimized it using a genetic algorithm. In order to ensure the aesthetics and ease of installation, constraints such as the folding method (mountain fold or valley fold) and boundary conditions were added for this optimization. Precise simulation tests showed that optimizing the ceiling shape suppressed standing waves better than in the case of a flat ceiling.

Summary and Conclusions

In this architectural project, we have connected modern science with ancient Buddhism to create a new design using current technology. The Ammann–Beenker tiling has rich geometric features with a square-based grid structure, which is aesthetically and practically suited for architecture. Moreover, the shape optimization method we have proposed would be helpful in applications other than acoustics. We expect our design approach works well in other cases.

Acknowledgements

This work was partially supported by JSPS KAKENHI (Grant Number 20K03540).

References

[1] B. Grünbaum, G. C. Shephard, "Tilings and patterns." W. H. Freeman and Company, New York, 1987.

[2] E. Harriss. "Images of the Ammann-Beenker Tiling." *Bridges Conference Proceedings*, Donostia 2007, pp. 377–378. Available online at http://archive.bridgesmathart.org/2007/bridges2007-377.html.