Drill Jigs for Wooden Ball-and-Stick Models

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Abstract

Custom 3D-printed drill jigs are introduced, allowing the placement of holes at precise locations on wooden balls. These simple devices, together with widely available tools and materials, are all that is needed to create custom ball-and-stick models. Design principles for drill jigs are discussed, as are tips for working with them and examples of works amenable to these construction techniques.

Introduction

From molecular model sets to Tinker Toys, Synestructics, and Zometool, node-and-stick modeling kits have long facilitated the fabrication of 3D edge models. Zometool in particular has blunted the impetus to fashion such models from scratch, for it is versatile, precise, and straightforward to work with. But the Zome system has inherent limitations. In the words of Paul Hildebrandt, "[Zome] components are an excellent medium for 3-dimensional sketches, but not ideal for permanent installations. The material is subject to UV degradation and the primary colors are corny (although a great shorthand for 2-, 3-, and 5-fold symmetries)" [5].

There are many other reasons to consider a custom ball-and-stick model. If a structure has pairs of intersecting edges, wooden dowels can be notched like Lincoln Logs to cross discretely, while a Zome model would require extra ball joints or 3D printed "cross-bobs." Edges in a custom model can be cut to any desired length, and construction is not limited to the edge directions of the Zome system. Holes in ball joints can be placed only where needed. Edges of different colors, diameters, and materials can be employed to give emphasis to certain aspects of a structure (see Figure 1).

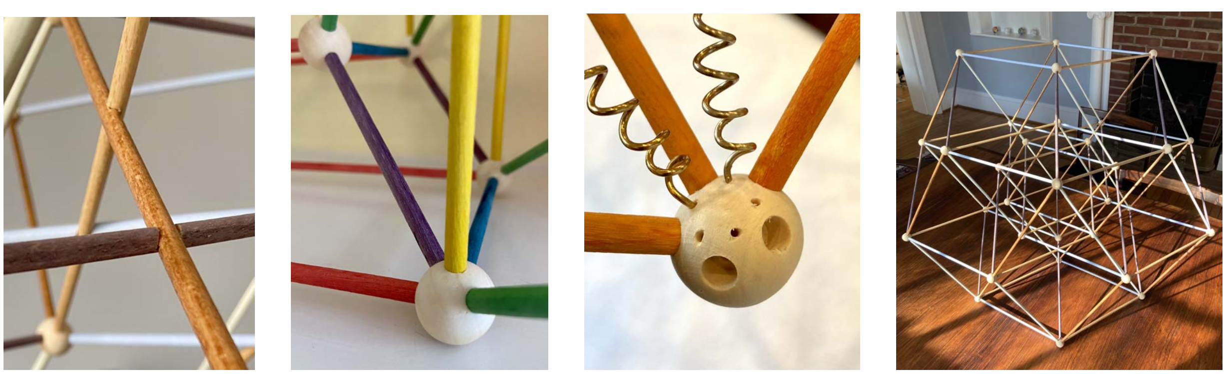

Figure 1: *Left: Notched edges facilitate crossings. Center-Left: Ball joints and color-coded edges in a hypercube model. Center-Right: Ball with edges of two diameters. Right: Ball-and-stick model of the 5-cube with a rhombic dodecahedron convex hull, made with a single drill jig.*

George Hart and others have used 3D-printed ball joints with wooden dowels to build custom models (e.g., see Figure 11 in [3]). But our focus here will be on drilling wooden craft balls. With the requisite drill jig in hand, a wood ball can be fashioned in a fraction of the time needed to 3D print it, and wood components are both long lasting and aesthetically appealing. A single jig will often suffice for drilling all the different vertex configurations in a model.

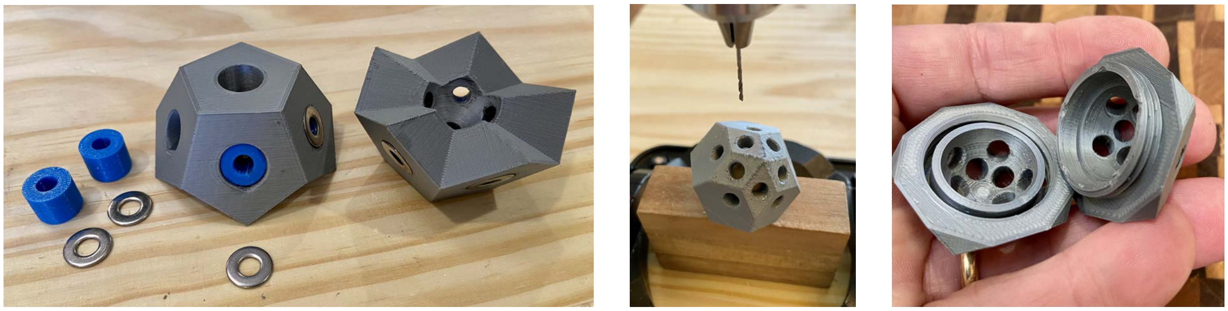

Figure 2: *Left: Jig for a handheld drill utilizing two washers separated by a spacer for precise alignment. Center: Jig pinned at its base in a drill press. Right: Jig with a threaded interface.*

The Drill Jig

The drill jig is a 3D-printed structure that serves as the interface between the drill bit and a wooden ball. Some examples are shown in Figure 2. A craft ball (widely available in lots of 50 or 100) is placed snugly between the two halves of the jig, which are then clamped or screwed together. While these balls are not perfectly spherical, all that is required is that they be held firmly. A small rubber o-ring can be used as a spacer to firmly seat an irregular ball. Circular openings in the jig are used to drill holes at precise locations on the surface of the ball. If using a handheld drill, metal washers may be permanently affixed to the jig with two-part epoxy to keep the drill bit from wandering. For greater precision with a handheld drill, two such washers with a cylindrical spacer between them can be used to keep the drill bit radially aligned.

Sizing I have used 3/4 inch diameter balls exclusively, which seem to fit nicely into a jig cavity of diameter 19.7mm. The minimal angle between pairs of adjacent dowels, together with the diameter of a dowel, limit how many holes can be placed on a ball. It is critically important to procure and carefully measure all your parts, including the drill bit and any washers, before designing a drill jig, as it serves as the bridge between all these components. 3/16 inch diameter dowels may require a 5mm or 13/64 inch diameter drill bit, depending on the lot. A digital caliper is useful.

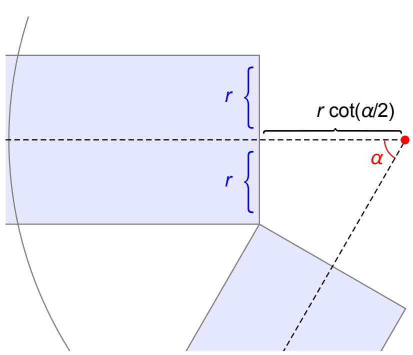

Figure 3: *Hole setback as a function of dowel radius r and minimum angle α between dowels.*

Hole depth Adjacent holes, if drilled too deep, will puncture into each other. This will make it difficult to insert all the dowels to a uniform depth. On the other hand, holes should be as deep as possible to ensure accuracy and rigidity in the completed model. It follows that any given configuration of hole axes has a unique optimal hole setback, and its formula is simple: If a dowel has radius r, and if α is the minimal angle between pairs of adjacent dowels, then the bottom of each hole must be set back a distance of $r \cot(\alpha/2)$ from the center of the ball. See Figure 3.

The jig body A drill jig should have the property that all hole entry points on its outer surface are equidistant from the center of the spherical ball cavity. Call this common distance R. The drill bit should then penetrate $R - r \cot(\alpha/2)$ into the jig. Note that this will ensure uniform edge lengths (between ball centers) when using dowels of uniform length, irrespective of how far out of round an individual wooden ball may be: A dowel of length d will give a distance $d + 2r \cot(\alpha/2)$ between ball centers. It will also simplify the drilling process, as all holes are drilled to a common depth from the surface of the jig.

Tip for handheld drilling: A piece of duct tape on the shaft of the drill bit may be used to mark the penetration depth. A "flag" of tape protruding from the "pole" of the bit (see Figure 5, right) will sweep away sawdust from the washer when the correct depth has been attained. Whether using a handheld drill or a press, it is a good idea to place a short dowel pin into the first hole before drilling additional holes to ensure the ball does not move.

With one additional assumption, the outside polyhedral shape of the drill jig will be uniquely determined by the configuration of its hole axes: We will require that for each hole axis, the jig has a pair of opposite parallel faces containing the ends of this axis, with these faces orthogonal to the axis. This will ensure that when the jig is placed face-down on a table, its top face will be parallel to the table surface, so a vertically oriented drill bit can be used to bore radially into the ball. With holes in the jig on both ends of each axis, a dowel pin can be placed in the jig's bottom hole to secure it to the table, while a drill press is used at the opposite top hole of the jig. See Figure 2, center. Note that a jig held together with a threaded interface (Figure 2, right) works beautifully and is handy for keeping all pairs of parallel face planes accessible.

So how does one determine the polyhedral outer shape of such a jig? Place the origin at the center of the jig's spherical cavity. Begin with a finite collection of vectors of length R all lying in the same half-space when rooted at the origin, so that each vector is aligned with a desired hole axis. These are the *generating vectors* for the jig. For each generating vector $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, there is a unique plane orthogonal to $\mathbf{v}$ passing through the point (v_1, v_2, v_3), and a second parallel plane containing the point $(-v_1, -v_2, -v_3)$.

As long as there are at least three generating vectors in general position (none lying in the plane determined by the other two) these pairs of parallel planes will enclose a convex polyhedron. This polyhedron has exactly one face per drill hole, and so provides an outer jig body shape with maximal face plane area orthogonal to each hole axis, and hence maximal stability for drilling. Finding the vertices and face structure for this polyhedron is a special case of the *vertex enumeration problem*. It can be efficiently computed with the Avis-Fukuda pivoting algorithm [1]. I've included a supplementary *Mathematica* notebook with my own code, which can be downloaded from the Bridges Archive and used for this computation.

For example, suppose we wish to construct a jig whose hole axes are the 3-fold and 4-fold symmetry axes of the regular octahedron, as shown in Figure 5. The resulting jig's outer surface is a truncated octahedron, but it will not be regular, as the square faces must be as close to center as the hexagonal faces. Indeed, it bears striking resemblance to 2300 year old 14-sided dice found in China [4].

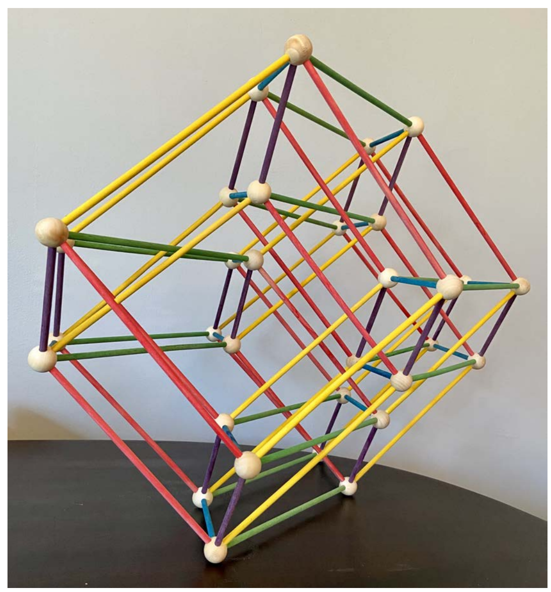

Figure 4: *5-cube edge model.*

Zonohedra and Hypercubes

Ball-and-stick models of zonohedra are particularly amenable these construction techniques, as they are based on predetermined edge directions. The n generating vectors for a zonohedron, after being scaled to uniform length, are precisely the generating vectors described above for producing its drill jig, that is, a jig that can produce the ball joints required to build its edge model. In fact, such a jig may be used to construct an edge

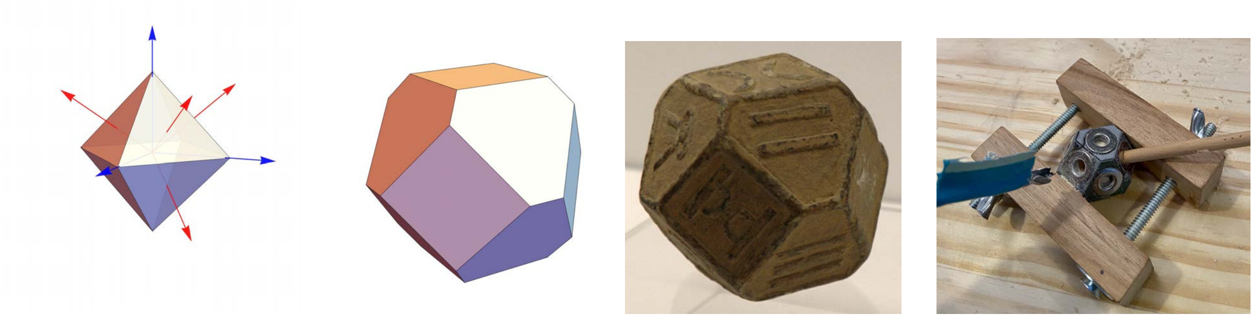

Figure 5: *Left: Octahedron with generators. Center-Left: Drill jig body for these generators. Center-Right: 14-sided die from China's Warring State Period [4]. Right: Drill jig in action.*

model for the entire n-hypercube enclosed by this zonohedron. (See [2] for the requisite background). Every hole axis in the jig has two ends, and precisely one end of each must be drilled to obtain a vertex of this hypercube. There are 2^n ways to do this, and these are precisely the required vertices for the hypercube. Indeed, when drilled in this way, each ball need only be translated, without rotation, to its location in the model: Two balls differing only in which end of a particular axis was drilled can accept a dowel along that axis.

In practice, if we permit rotation of the ball joints, there are fewer than 2^n vertex hole configurations. For example, the five-dimensional hypercube shown in Figure 4 uses five of the seven axes of the jig shown in Figure 5, and there are seven distinct vertex configurations needed, up to rotation, for this model. By contrast, the five-dimensional hypercube shown in Figure 1, based on five of the six 5-fold symmetry axes of the icosahedron, has only four distinct vertex configurations.

Summary and Conclusions

A custom drill jig takes some care to create, but once in hand provides an efficient means to produce custom ball-and-stick models. After the parts have been measured and a collection of axes has been specified, one determines the minimal angle between pairs of adjacent axes to determine hole depth, and uses the Avis-Fukuda algorithm (or the supplementary *Mathematica* notebook) to determine the outer shape of the jig body. It is straightforward from here to use a CAD program to design the jig.

References

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