Beautiful Geometry: a STEAM Experience in a Professional Development Course for Mathematics Teachers

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Abstract

Mathematics and art have a connection that goes far beyond proximity. In fact, they are two symbiotic areas shaping a beautiful, logical, systematic, and breathtaking world. One of the works that exposes their intrinsic connection, in a clear, rigorous and simple way, is the book *Beautiful Geometry* by Eli Maor and Eugen Jost. To a greater or lesser degree, we all feel the power that art exerts on our minds, both at a sensory and at a cognitive level. In fact, art is one of the privileged areas for the emergence of creativity. Combining beauty with rigor and reason, and based on the book mentioned above, we designed a professional development course for mathematics teachers, following a STEAM approach. The description of this course, illustrated with part of the graphic/artistic material produced by the participants, is the main object of this paper.

Introduction

Mathematics is seen, by a considerable number of students, as a set of incomprehensible abstract microworlds regulated by a certain set of rules and described by formal languages that are difficult to unravel. Not surprisingly, high levels of stress, boredom, and lack of motivation invade math classes, making a teacher's mission nearly impossible. Unlike mathematics, art is seen as a world that transports gratifying emotions. Given the intrinsic relationship between art and mathematics, bringing art, among other areas, into the math classroom should be an excellent strategy for getting students to have a more positive attitude toward mathematics. There are strong pieces of evidence that this is the case [1][3].

Based on this assumption, the idea of designing a professional development course for mathematics teachers arose, based on the book *Beautiful Geometry* [9], written by the mathematics historian Eli Maor and illustrated by the artist Jurgen Jost. *Beautiful Geometry* contains more than sixty colour plates illustrating a wide range of geometric or numerical patterns and theorems, accompanied by clear explanations, proofs, and brief accounts of the history and people behind each.

In this paper, we present the results of a professional development course for in-service mathematics teachers where the participants explored several topics of the book by Maor and Jost [9] and created several artworks with mathematical content. The professional development course addressed in this paper is part of a set of professional development courses for mathematics teachers promoted over the last several years. Following the ideas presented by Borko et al. [2], these courses are intended to be "opportunities grounded in a conception of learning to teach as a lifelong endeavour", that should be both pleasurable and rewarding. At the same time, we try to move towards Eisner's conception of a practice rooted in the arts [5]: teachers take on the role of an artist, they are given time to explore, to create and to surprise themselves. A STEAM approach was adopted. The integrated combination of math, art and technology allows the creation of hotspots to guide participants in creating critical and creative learning scenarios [4].

Description of the Course

The professional development course in focus, titled "Geometria Criativa" (Creative Geometry), took place at the University of Aveiro, in 2021, from April 24th to May 26th. Like all professional development courses for Portuguese teachers, it was recognized by the national scientific and pedagogical council for teacher's professional development (Conselho Científico-Pedagógico da Formação Contínua), being registered with the number CCPFC/ACC – 108327/20. This course lasted for 25 hours and had 22 participating teachers who taught mathematics from grades 5 to 12. Given the range of grades taught and the specificity of each level of teaching, different activities were proposed for teachers of different levels (grade 5–6 and 7–12). The proposed activities (based on chapters of the base book) were designed considering the mathematical contents and their appropriateness and didactic transposition to the classroom, for different levels.

In the first session (3 hours face-to-face), the course trainers explored a set of mathematical topics with artistic applications: Voronoi diagrams, Lissajous figures and Steiner's chains. In all cases the topics were explored from a mathematical point of view, with the aid of GeoGebra applets, and complemented with artistic applications (mostly from renown artists but also created specifically for the course). Whenever possible, the trainers identified school curricula topics that could be explored by the teachers in their classroom, stemming from the artworks presented. In the second session (3 hours face-to-face), the participants had the opportunity to develop an applet using GeoGebra, about a specific topic of their teaching practice (participants were divided into groups according to their teaching levels). Next, the participants (individually or in groups) selected one or more topics from the book *Beautiful Geometry* [9]. They had to study it and prepare a presentation, mostly following the content of the book, but also exploring the topic further and, whenever possible, linking it with their teaching practice (4 hours asynchronous session). The third session of the course consisted of all the presentations prepared by the participants (5 hours face-to-face).

In the second part of the course, the participants were asked to develop an individual project consisting of one or more artworks stemming from a mathematical concept previously explored in the course or closely related. Two sessions were devoted to providing help on creating the underlying drawings using GeoGebra (2 by 90 minutes online/synchronous). Given the dynamic features of GeoGebra, the objects created there, according to certain mathematical rules, allow manipulations in real time, creating configurations that are not expected, either from the mathematical point of view or from the aesthetic point of view. The challenge of combining the aesthetic feeling with the mathematical explanation of the base configuration chosen and created in GeoGebra, solidifies and expands the participants' mathematical knowledge. The following session was a practical session of artistic nature (4 hours face-to-face). The participants chose different techniques for their artworks: watercolors, acrylics, soft pastels, string art, and mixed media, among others. Most of the participants had some pre-existing artistic skills, which they used. However, some participants had no previous skills and were assisted by the trainers and other participants in acquiring some basic skills (they used mostly soft pastels). In the last session (3 hours face-to-face), the participants presented their artworks to the class, giving a brief explanation of its mathematical content.

This course was an opportunity for learning or developing artistic skills while at the same time witnessing how other more skilled participants carried out the artistic process. Though mathematics teachers do not, in general, possess high artistic skills, this course served as an opportunity to go through an artistic process. As a result, participants revealed they felt motivated to interact with the arts teachers and engage in future interdisciplinary projects in their schools.

Results

In this section, we show photos and scans of a selection of the artworks produced by the participants. In addition, the author of each work is identified. All pictures were taken by the authors of this paper with the permission of the participants. The projects are organized according to the mathematical content covered. In the end of the section, we provide some results of a final questionnaire answered by the participants.

Line Designs

Julius Plucker (1801–1868) realized that a curve need not be regarded as a set of points and it can be described by a set of tangent lines [9]. Thus, curves can be depicted through sets of straight lines and this fact has inspired what is now known as string art. One such curve is the parabola which can be visualized by segments drawn from equidistant points on two intersecting straight lines. Figures 1, 2 and 3 show line designs made by three of the participants, using parabolas and epicycloids.

Figure 1: *String artworks by Carla Rentes: cotton threads on canvas, 20×20 cm (intersection of three parabolas on the left, parabola on the right).*

Figure 2: *String artworks by Carla Machado and Carla Rentes: Flower made of parabola arches, cotton threads on canvas, 30×30 cm (left), Cardioid, cotton thread on a wooden circle, 30 cm diameter (right).*

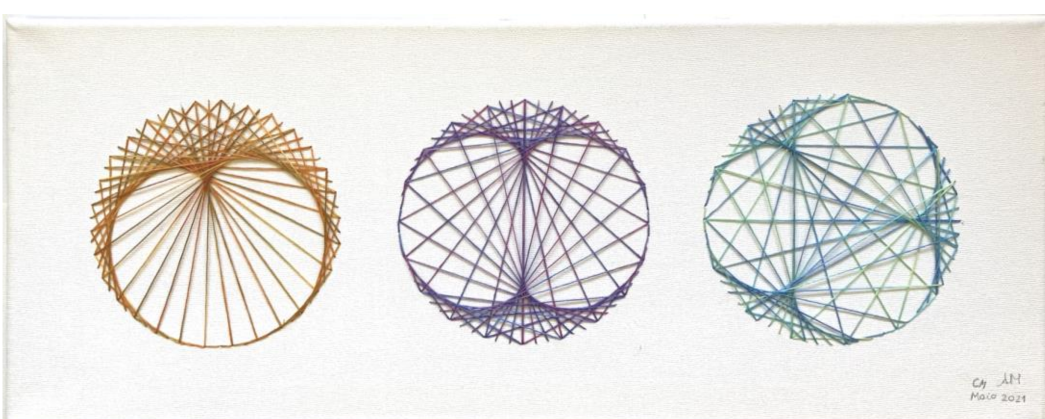

Figure 3: *String artwork by Ana Mourato depicting epicycloids: cotton threads on canvas, 50×20 cm (left to right: cardioid (k=1), nephroid (k=2) and trefoil like epicycloid (k=3)).*

Voronoi Diagrams

Given a set of *n* generating points (sites), a Voronoi diagram is a partitioning of the plane into convex polygons such that each polygon contains exactly one of the sites and every point in a given polygon is closer to its site than to any other [10]. Voronoi diagrams may be visually very attractive and are a useful application to explore the concept of bisector in schools. Several participants explored Voronoi diagrams and Figures 4 and 5 show some of the resulting artworks.

Figure 4: *Artworks by Ana Deolinda Silva (left) and Yennifer Martins (right) based on Voronoi diagrams: tucked fabric on Styrofoam, 30×30 cm (left), glued textured paper and ink, 42×30 cm (right).*

Figure 5: *Artworks by Isabel Moutinho based on Voronoi Diagrams: watercolor on paper, 42×25 cm (left), 42×28 cm (right).*

The two artworks of Figure 5 are based on the Voronoi diagrams depicted in Figure 6.

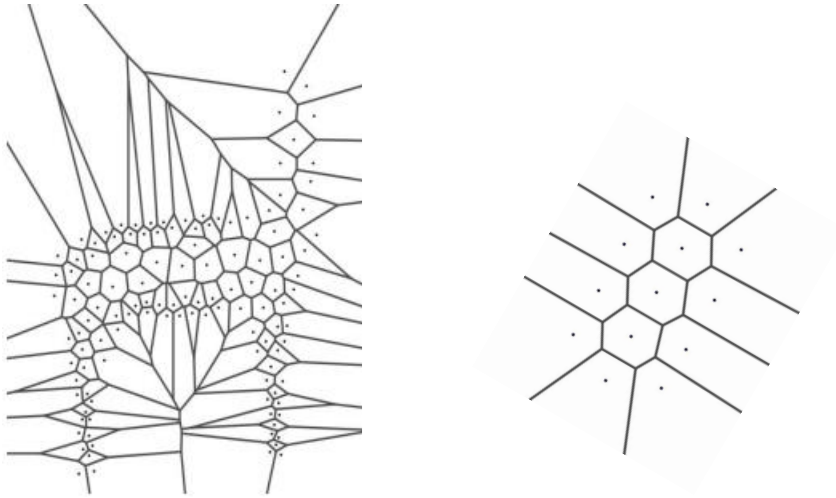

Figure 6: *Underlying Voronoi diagrams used in the artworks of Figure 5.*

Circle inversion

Figure 7 illustrates two artworks by one of the participants who explored circle inversion freely, combined with rotations and reflections.

Figure 7: *Artworks by Susana Rainho: soft pastels on paper, 26×25 cm (left), acrylic on paper, 26×25 cm (right).*

Steiner's Porism

Jacob Steiner proved in 1826 that given two nonconcentric circles, one lying entirely inside the other, it is always possible to construct a chain of secondary circles, each touching the circle preceding it in the sequence as well as the two original circles, in such a way that the chain closes upon itself, the last circle coinciding with the first [9]. Steiner's chains are visually very attractive and served as the basis for the artworks of three high school mathematics teachers participating in the course (see Figures 8 and 9).

Figure 10 shows the construction of the *porism* used in Figure 9 through a print from GeoGebra using circle inversion, and a detail where the embroidered contours are visible.

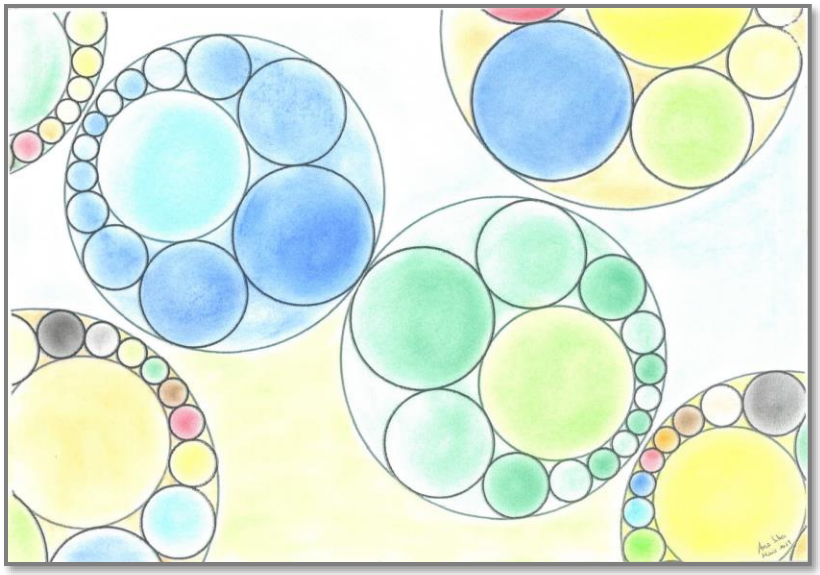

Figure 8: *Artwork by Ana Cristina Silva: soft pastels on paper, 42×30 cm.*

Figure 9: *Artworks by Carla Rentes (left) and Teresa Mena (right): acrylic on canvas, 50×50 cm (left), acrylic on canvas with contours embroidered in cotton threads, 50×50 cm (right).*

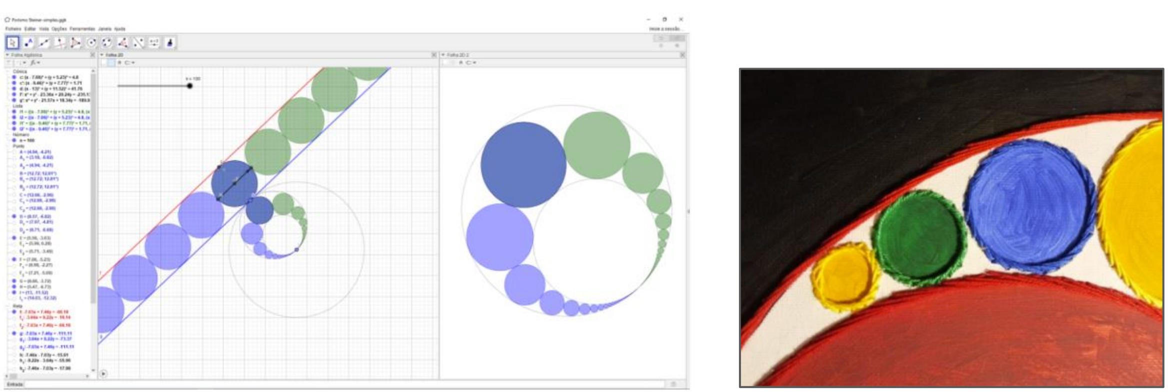

Figure 10: *Details of the artworks of Figure 9: GeoGebra printout (left) and embroidering detail (right).*

Lunes of Hippocrates

The lune of Hippocrates (named after Hippocrates of Chios) is a lune bounded by arcs of two circles, the smaller of which has as its diameter a chord spanning a right angle on the larger circle. Equivalently, it is a non-convex plane region bounded by one 180-degree circular arc and one 90-degree circular arc [11]. One of the participants of the course created a painting based on Hippocrates' lunes shown in Figure 11.

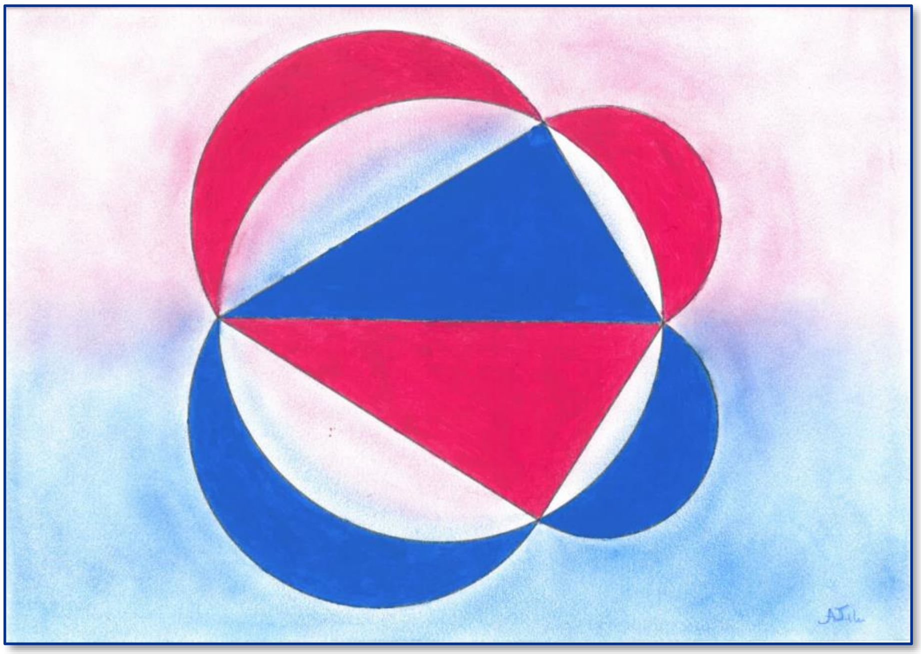

Figure 11: *Artwork by Ângela Teles: soft pastels on paper, 42×30 cm.*

Final Questionnaire

In the end of the course, teachers filled in a questionnaire. They were asked to rate, through a five-point Likert scale (1-very negative; 5-very positive), their general evaluation of the course, and to give, also through a five-point Likert scale (1-not at all; 5-very much), their agreement with a set of statements regarding several aspects. Next, we provide the heatmap of the responses to some of the statements.

	1	2	3	4	5
What is your global evaluation of the course?	0	0	0	4	17
The course contributed to increase my mathematical knowledge.	0	0	0	4	17
The course contributed to increase my skills with GeoGebra.	0	0	2	8	11
The course contributed to improve my teaching practice	0	0	0	8	13
I find it interesting to use an artistic approach in the teaching of mathematics.	0	0	0	6	15
The use of an artistic approach during the course stimulated the emergence of collaborative teaching experiences, involving teachers and students from different areas.	0	0	0	6	15

Summary and Conclusions

In the professional development course described in this paper, a STEAM approach to teaching and learning was adopted, integrating mathematics, arts, and technology. The main objectives of the course were: to contribute to the scientific/pedagogical updating of teachers in the area of mathematics; to develop skills in the use of dynamic geometry software, namely GeoGebra; to stimulate the development of the ability to establish relationships and interconnections between mathematics and other areas of knowledge, in particular the visual arts; to use an artistic approach to the teaching of mathematics. Each participant created a piece of art with mathematical content and using GeoGebra. The participants showed great commitment in the accomplishment of the proposed tasks and projects. Through the analysis of the final questionnaire and by observing their performance during the course, we may conclude that the overall evaluation of the course was very positive and that the objectives of the course were achieved.

The conclusions withdrawn from this study are in line with those reached in [6] [7] and [8]. Overall, the activities developed have proved to be successful examples of interdisciplinary methodologies that bring into the teaching of mathematics usual procedures in the teaching of the arts, associated with the use of technology. We believe that these teachers have been strengthened in their capacity to develop multidisciplinary tasks and projects with their students. The STEAM approach used in this professional development course, involving the interconnection between mathematics, technology and the arts, promotes a positive attitude towards mathematics and thus fosters motivation to learn it.

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