Make Music Visible, Play Mathematics

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Abstract

The workshop is based on interdisciplinary and interactive activities that links mathematics and music and presents mathematical and musical ideas through hands-on and multisensorial approaches. Its goals are to encourage maths teachers to inspire students by making maths audible as well as visible, and music teachers to use a geometrical method to foster a new approach towards music theory. Participants learn how the creative discovery process can be used to form an atmosphere of experimentation and play that can positively affect people attitudes about mathematics, music and learning.

Introduction

Music and geometry has always had more contact points than we can ever imagine, but nevertheless music theorists has focused attention on counting and ratios. As the great 17th-century German mathematician Gottfried Leibniz wrote: *"Music is the sensation of counting without being aware you were counting."* But there is more to this connection than counting. As the French baroque composer Rameau declared in 1722: *"I must confess that only with the aid of mathematics did my ideas become clear."*

So, is there really a link? Or is it crazy to try to connect the creative art of music with the steely logic of mathematics? Certainly the grammar of music – rhythm and pitch – has mathematical foundations. While the combinations of notes we have been drawn to over the centuries can all be explained through numbers, music is more than just notes and beats. It is in putting the notes together to create that we believe the true connection between mathematics and music reveals itself. Harmony draws from basic numerical relationships has visible connections with geometric shapes [1].

Mathematics is about structure and pattern. The geometric approach to music that we put forward allows educators to involve student in the kind of process that emphasizes the parallel application of complementary viewpoints and encourages an interactive learning involvement through hands-on and multisensorial approach. As a result, a direct connection can be made between the processes of developing practical construction solutions for geometric problems in mathematics and the natural way of construct chords in music [2]. Moreover, the geometry hidden behind the chords can also be source of amusement and play outside educational establishments.

The workshop aims to encourage maths teachers to inspire students by making maths audible as well as visible, and music teachers to use this geometrical method to foster a new approach towards music theory where to show how music could be effectively displayed. Carrying out activities such as those we propose in this workshop, we received positive feedback by people involved. We have observed that they become deeply connected with maths and music when they take an active part in building visually engaging objects. Their interest in mathematical and musical ideas increases as questions naturally arise during the construction. Furthermore, when the main phases of the "design" process are reproduced in the form of a problem-solving activity, the structure can provide first-hand experience in developing harmonic composition skills concerning practical, geometric and aesthetic aspects.

Participants could observe how the problem-solving frameworks for two different approaches – mathematics and music – are actually very closely related to one another. They engage in a multisensorial path to music theory and those learning explorations are easily transferable to interdisciplinary classroom contexts, scalable for different ages and abilities. The overall expectation is that participants apply some of the techniques used to their own practice, for theoretical understanding or simple pleasure. Our method would like to serve as a new model for mathematics and music teaching. Workshop activities give insight to two possible outcomes: participants will discover, understand and put to use connections between creative music investigations and mathematic concepts; on the other hand, participants will reflect on how creative discovery can positively influence the learning process study of music and mathematics.

The workshop is open to anyone interested in mathematics or music theory and its relationship with mathematics. No specific knowledge of mathematics and music are required.

Things We Need to Know Before We Get Started

The chromatic scale is a musical scale with 12 notes, each a semitone above or below its adjacent notes. During the workshop, we use the chromatic scale C, C#, D, D#, E, F, F#, G, G#, A, A#, B. If we put these 12 notes evenly distributed around a circle, we obtain what is called "chromatic circle" (Figure 1).

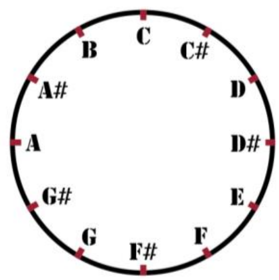

Figure 1: *Chromatic circle.*

The angle formed by the rays connecting the center of the circle with every two adjacent notes is always constant and 30° wide. So, a semitone corresponds to this angle of 30 degrees. This means that any space between two notes, called "interval", can be represented by an angle of $k \cdot 30$ degrees, where k is an integer such that $|k| \leq 12$ and shows how many semitones there are between the two notes. If k is positive, we move clockwise around the circle; if k is negative, counterclockwise.

In this way, we can also calculate the measure of any segment connecting any two notes around the circle by using the Law of Sines and the fact that, in a circle, the angle at the center is double of the angle at the circumference when angles have the same circumference as base. For example, the measure of the segment connecting C-E is equal to the product between the diameter of the circle and the sine of 120° = 4·30°. This will be useful somewhere along the way to confirm if the triangles we construct in section 2 are scalene, isosceles or equilateral.

Workshop Structure

We have divided the workshop into five sections. Each section follows a three-part pedagogical model: a minds-on activity, a hands-on activity, and a consolidation. There are also optional extensions that suggest questions for people to delve deeper into the subjects involved. But given the time constraints at the Bridges Conference, we present a comprehensive framework of the activities without these optional extensions.

Each section is structured to encourage the process of discovery. Indeed, during the workshop, musical chord is not taken as already preexisting or classified through harmonic and algebraic rules as happens in teaching methods used in the most of conservatories, universities or music schools. Participants build up the musical chord, by all means, starting from its basic ingredients (pitch, note, interval) across an effective listening and geometrical experience, where by musical chord we mean three or more consonant notes played together.

Section 1 – Do they play nice?

Workshop participants have a "BoomWhacker" (BW) associated with a specific note and they can listen to its sound by beating their own BW on the palm of their hand or any other body part they fancy. Then, they can start walking around the room and, whenever two people meet together, they must greet by playing their BWs against each other. Some "meeting" will be more interesting and pleasant than others.

This introductory activity is a playful way both to engage participants to know each other better and to start playing with notes and intervals. The aim is to discover what intervals are pleasant to hear by answering the question: *"Do they play nice?"* At the end of the activity, the intervals perceived as pleasant are essentially (and culturally) those of minor third/major sixth (+3, -3), major third/minor sixth (+4, -4), perfect fourth/perfect fifth (+5, -5) and augmented fourth/diminished fifth (+6, -6).

Section 2 – Build up the triangles

Participants have at their disposal straws, scissors, tape and a sheet with a chromatic circle. They can use the straws to obtain segments of a size corresponding to the size of the pleasant intervals discovered in Section 1, by placing one end of the straw at their note and cutting the other end in correspondence of the note that gives the desired interval.

After that, workshop participants should answer questions like "How many triangles can we build up using those pleasant segments?" or "What kind of triangles do we get?" or "Are all perfectly inscribed in the chromatic circle?".

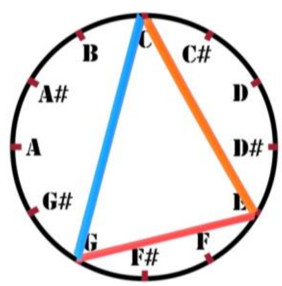

Figure 2: *One of the four possible inscribed triangle (C major).*

Only 4 of the triangles thus obtained are perfectly inscribed in the chromatic circle (Figure 2), and each of them represents a musical chord: the first Scalene Triangle (4-3-5) represents a Major Chord, the second Scalene Triangle (3-4-5) a Minor Chord, the Isosceles Triangle (3-3-6) a Diminished Chord and the Equilateral Triangle (4-4-4) an Augmented Chord. For example, the C major in Figure 2 is a 4-3-5 Major Chord.

Section 3 – What happens if…

It's time to explore. Workshop participants have to figure out what happen if they rotate the four triangles/chords obtained within the circle choosing a different starting note every time. They have a direct experience of how rotation changes the shape position, but doesn't affect the chord real "essence" [3]. At this point, they should be able to recognize the four triangle shapes only by listening to the sound they produce playing the three notes together. They actually find out that those triangles/chords sound like Major, Minor, Diminished and Augmented.

Then, they try to discover what happen if we now symmetrise triangles/chords relative to the diameter passing through the note that named the musical chord or to an axis passing through one side of the triangle. In this case, each chord changes in an intrinsic manner so that it becomes another chord [4]. Workshop participants have to explore how triangles/chords change by working in group with the aid of a worksheet and the materials created in the previous sections. During this activity, they should identify what similarities and differences occur both from a geometrical and musical point of view.

Section 4 – It's time to play

Participants choose a song from a list proposed and try to understand what kind of geometry lies beyond by analyzing all the triangles/chords present in that song harmonic structure. After having discovered song harmonic sequence, they play and sing the song together with the others.

Section 5 – It's time to compose

By using different triangles/chords, participants give space for creativity and try to create all together an harmonic composition. Consequently, each participant becomes a "geometrical/musical" composer for the occasion. For the remaining duration of the workshop, just like a chef, they cook a tasty dish for ears and eyes by using suitable ingredients (notes), proper tools (BW or other instruments) and some geometric recipes.

Conclusions

We are convinced that hands-on workshops of this kind can change the mindset and confidence of people towards mathematics and music. The collaborative aspect of this type of construction and discovering activities encourages participants to engage in authentic conversations about maths and music connections. Furthermore, this interdisciplinary and interactive approach involves creativity and promotes a positive attitude to open-ended problem solving that could serve as a paradigm for other interdisciplinary courses and settings.

The different stages of work enable workshop participants to conceptualize, model, or analyze topics relevant to basic mathematics and music theory and relevant to these fields integrated pedagogical approach as well. The workshop is perfectly adaptable to inquiry-based, playful learning and to experience-oriented pedagogical approaches in this respect. Consequently, if we use these activities for educational purposes, an endless number of complex and multidisciplinary topics can suddenly be studied in an active, entertaining and engaging way, without forgetting the aesthetic and artistic aspects.

The workshop is part of an innovative educational interdisciplinary path from primary to upper secondary school in which discovery, creation, playing and interdisciplinarity play a central role. At the same time, it would like to be a new proposal for teaching basic music theory and harmonic composition through geometry within music schools. We are convinced that both mathematics and music can gain mutual benefit from this approach. Furthermore, the workshop lay the foundations for further developments and educational insights such as the study of the most important musical scales in geometric-musical terms, the analysis of musical chords with more than three notes (polygons with more than three sides) and the study and creation of dodecaphonic music and geometry.

References

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