# Exploring Rotational Symmetries and Triangles through Dance and Body Movement: Maths in Motion


Lena Nasiakou[1], Saara Lehto[2], Svetlana Goranova[3], Kerry Osborne[4] and Kristóf Fenyvesi[5]

[1] Olde Vechte Foundation, Ommen, The Netherlands; lenasiakou@gmail.com
[2] Dept. of Mathematics and Statistics, University of Helsinki, Finland; saara.lehto@helsinki.fi
[3] Fun Mathematics LTD, Sofia, Bulgaria; sveta.newforest@gmail.com
[4] Experience Workshop STEAM Movement, Finland; kezzaozza@gmail.com
[5] Finnish Institute of Educational Research, University of Jyväskylä; Experience Workshop STEAM Movement, Finland; info@experienceworkshop.org



## Abstract

In this workshop, we share the main results of the "Maths in Motion" (MiM) Erasmus+ educational project (2017-2019), concerning how dance and body movement can be used as tools to teach mathematics. The MiM-approach is based on embodied cognition and opens up new horizons for students, teachers, and even parents by offering simultaneous experiences with the structural, spatial, rhythmic and symbolic dimensions of mathematics through body movement. We will introduce two teaching modules developed during the project. These modules follow in the footsteps of the writing, workshops, and performances of Schaffer and Stern in the field of mathematics and dance over the past few decades. The modules guide and encourage the participants to create performance art pieces that articulate mathematical concepts.


## Workshop Outline

This workshop consists of a 15-minute introduction to the "Maths in Motion" (MiM) Erasmus+ project, two half hour movement activities combining body movement, embodied learning and mathematical understanding, and a 15-minute discussion and reflection at the end of the workshop. In the movement activities, workshop participants are invited to experience two learning modules of MiM: one dealing with rotational symmetries through a creative group assignment, another for experiencing the properties of triangles through embodied methods.

## Activity 1: Dancing Snowflakes

"Dancing Snowflakes" is a participatory and experience-oriented activity about rotational symmetries, geometry and creative movement. In addition to learning about symmetries, the activity reinforces cooperation, problem solving skills, coordination of movement, and creativity. In the activity, participants get to experience geometry and symmetry as parts of a larger symmetric figure and to create their own movement-based performance piece: a short dance that explores rotational symmetries. The activity is 30 minutes long.

Our activity as a whole closely resembles the activity *Threesies* in Chapter 5 of the book *Math Dance with Dr. Schaffer and Mr. Stern* [19] together with the workshop presented by Schaffer and Stern at Bridges 2010 [20].

**1 Warm-up:** The learning goals of the warm up are exploring basic mirroring symmetry, working together with a partner, learning to observe and copy others' movements, communicating through movement, getting comfortable with touch, awakening creativity for movement, and exploring different possibilities of body positions with a partner.

**1 Warm-up Activity 1. Mirroring:** Participants are asked to form pairs and try out mirroring. With slow melodic music on the background, one person leads and the other follows as a mirror image of the leader. The leader and the follower are asked to switch roles, and the pairs try to keep their movements continuous despite the switches. If possible, the pairs try to find a state of continuous movement and attention where neither leads nor follows, but both are tuned into their joint movement (Fig. 1). The participants are recommended to not talk during this exercise, but let themselves to fully concentrate on the movements of their body.

**Figure 1 a, b:** *Maths in Motion mirroring exercise led by Experience Workshop at IC Codogno School, Italy in 2018.*

**1 Warm-up Activity 2. Rotational symmetry:** Keeping the same pairing, one is now designated as the leader, the other is the follower. When the leader lifts the left part of their body, the follower lifts the left part of their body, as well. The concept of rotational symmetry is introduced here. Then roles are switched.

**1 Warm-up Activity 3. Bodily connections:** Participants work in triplets to find different bodily connections between three people. Different kinds of connections are encouraged using first arms and legs and then whole bodies. This activity is partly inspired by the *How Many Ways to Shake Hands?* activity by Schaffer and Stern, which they presented at Bridges 2010 [20]. After the exercise, each group is asked to show their favorite connection. Different artistic aspects of the connections are discussed together. Notice is drawn also to the possible mathematical and symmetric properties of the connections created.

**2 Main Part:** Learning goals of the Main Part are exploring and learning about rotational symmetries, working in a larger group, and solving problems with bodily connections. The participants can experience how to move from one position into another as a group, learning about making a choreography, learning to memorize moves, moving in synchronization with others, learning to observe geometric figures in our own bodies, etc.

**2 Main Part Activity 1. Rotational symmetries:** In groups of six, participants are asked to start by forming a circle facing towards each other and holding hands. With slow ambient music on the background, the groups are asked to find different positions that have rotational symmetry for the group. Groups are then asked to find ways to move from one such position into another. If groups are creatively stuck, different prompts are offered (Fig. 2).

**Figure 2:** *Maths in Motion Dancing Snowflake activity led by Experience Workshop at IC Codogno School, Italy in 2018. Photo: Giada Totaro.*

**2 Main Part Activity 2. Everyone is a choreographer:** Once creativity is flowing, the groups are asked to form a short dance of their symmetry ideas. Groups get to pick their favorite positions and moves and decide how the different symmetries flow from one into another, how the dance starts and ends, and how their different movements are performed. Groups are asked to memorize and practice their choreographies.

**3 Closing:** Learning goals of the closing are getting artistic satisfaction through performing and watching the produced symmetry dances, bringing the mathematical ideas inside the activities to the forefront, further exploring the phenomenon, etc.

**3 Closing Activity 1. Acknowledgement:** All groups are asked to perform their symmetry dances for the others to see. If the groups agree, the performances can be filmed from above, so that everyone gets a chance to see how the symmetries work and what the artistic effect of the dance is.

**3 Closing Activity 2. Reflection:** In groups (here participants can be divided into groups with different interests) discuss what each participant experienced and felt during the activity and what mathematical concepts and skills were learned during the activity. Groups can discuss how they would teach this activity and how they would change or develop the activity. What would be the different possibilities for further study from a mathematical or an artistic viewpoint?

**3 Closing Activity 3. Sharing:** Groups share their thoughts and ideas with everyone.

## Activity 2: Get Angled!

"Get Angled" is a participatory and experience-oriented activity about angles, their measurement and problem solving. The activity's emphasis is on finding out under which conditions of three angles' measurements can a triangle be formed. In the activity, participants get to experience creative movements in groups of two and three and to come up with solutions for tasks using their innate body knowledge. The workshop is 30 minutes long.

**1 Warm up:** The learning goal of the warm up is exploring different types of triangles using parts of the body, from the largest to the smallest.

**1 Warm up Activity 1. Different levels:** Participants are asked to form different postures with their bodies on each of the 3 levels, bottom, middle and top. The postures can include crab kick, gorilla hop, etc. (Fig. 3)

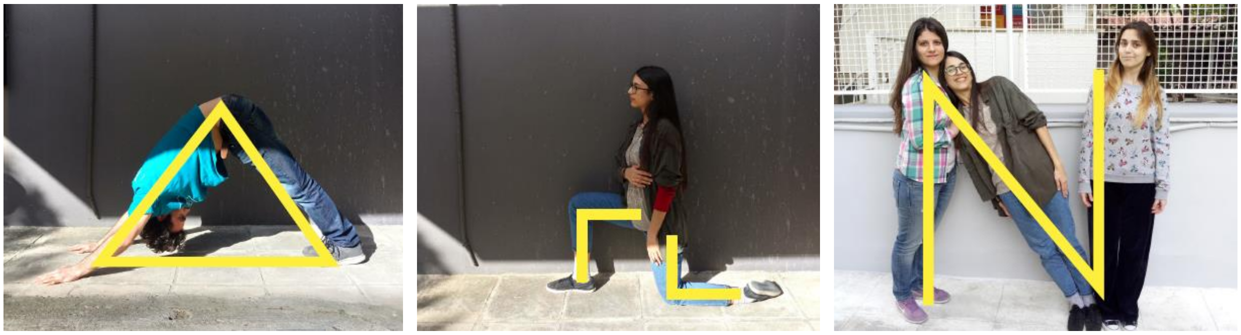

**Figure 3 a, b, c:** *Maths in Motion Different Levels exercise by SciCo, Greece. Low level (a), middle level (b) and high level (c).*

**1 Warm up Activity 2. Shape triangles:** Participants shape triangles with different parts of the body. They are suggested to look for the smallest and for the biggest triangle. How many triangles can they form with different parts of the body?

**1 Warm up Activity 3. Secret triangles:** All participants stand in the room free from chairs. Each participant chooses two others, without letting them know. While moving, each participant pays attention that they are always at equal distances from their chosen participants.

**2 Main Part:** Learning goals of the main body are three: first, to represent different types of angles with the span of the arms. Second, to understand in a creative, embodied way, that three random angles do not necessarily form a triangle and third, to discover the property of a triangle in terms of the sum of its angles.

**2 Main Part Activity 1. Span of the arms:** A rope is placed across the centre of the workshop space in a straight line, separating it into two zones. On one side of the line is the Acute Zone, on the other side is the Obtuse Zone. Participants can freely and independently experiment with forming acute angles with outstretched arms while moving about within the Acute Zone. They are just as free to experience making obtuse angles with the span of their arms within the Obtuse Zone. However, when moving across the delineated zone's boundary, participants must freeze on the line in a right angle until another participant's supplementary right angle comes to join them. The two supplementary right angles now total 180 degrees and both participants are released from the line, allowed to flow freely and independently into into the other zone. If the workshop space has room corners that are architecturally right angles, then participants can go to these corners to use them as 90 degree reference points. Participants place themselves into the right angled corners, stretch out their arms to get their bearings as to what is 90 degrees, and then step away adjusting their arms accordingly to be sure their arm angles are greater than a right angle in the Obtuse Zone and less than a right angle in the Acute Zone.

**2 Main Part Activity 2. Triangles attempt:** Once participants have practiced changing the span of their arms for different types of angles, they are asked to freeze in space and keep the angle they have. Without moving their arms, they form triplets with two other participants and try to form a triangle using their predetermined angles formed by the position of their arms. Participants realize that it is not always possible to form a triangle with three random angle measurements.

**2 Main Part Activity 3. 180 degree zone:** Participants in groups of 3 have to figure out a solution to the following problem. The rope or a line on the floor represents an angle of 180 degrees. In groups of three make three angles with your arms that fit exactly inside this 180 degree angle. Can you make a triangle with these angles? What does this tell you about triangles? Possible solution: In groups of three, one participant steps on the rope splitting the room and holds his/her hands wide open (parallel to the rope). The second team member steps next to the first and together compose an angle stretching arms to form two angles with an overall sum of 180 degrees. The third member goes between the other two, making them to reduce their angles in order to keep the overall sum of 180 degrees. Without moving the positions of their hands, participants now move to form a triangle with sides formed by participants' hands. They observe that the three angles ideally form a triangle.

**3 Closing:** Learning goals of the closing are to give the different groups chance to present their ideas and to learn other solutions to the given task.

**3 Closing Activity 1. Acknowledgement:** All groups are asked to perform their solution as a sequence of different body sculptures. In performing the sculptures participants start from the attached rope on the floor and end as an embodied triangle.

**3 Closing Activity 2. Reflection:** In groups (here participants can be divided into groups with different interests) discuss what each participant experienced and felt during the activity and what mathematical concepts and skills were learned during the activity. Groups can discuss how they would teach this activity and how they would change or develop the activity: what would be the different possibilities for further study from a mathematical or an artistic viewpoint?

**3 Closing Activity 3. Sharing.** Groups share their thoughts and ideas with everyone.

## Methodology

The methodology behind the MiM-project has been co-created by an international group of teachers, dancers, science communication experts, researchers, mathematicians between 2017-2019 (project website: https://oldevechte.com/international-projects/partnership/). 56 teachers and more than 200 students have tested the methodology during the development period. The positive results of the evaluations can be accessed in *Maths in Motion: the toolkit*, a book that has been published in the framework of this project [14].

*Maths in Motion: the toolkit* contains six learning modules, two of which are presented in this paper. All modules were created during MiM training events in Ommen, the Netherlands by diverse groups of experts from the fields of dance, mathematics and education. Each module was created with a specific target group in mind and each was tested accordingly in kindergarten, elementary school, middle school and high school.

All the MiM-modules follow a certain structure with a Warm-up, a Main part and a Closing. The Warm up has several purposes: to lure the participants into learning in an engaging way, to physically warm up the body, to enhance creative movement and to place the participants in a playful mood, which can be effective in improving attitudes towards mathematics [2].

In the Main Part, the learners explore and gain a deeper understanding of mathematical concepts through the movement of their bodies. In this stage the experience provokes the insights and reactions needed to move toward learning in a more conscious way. This can be achieved through different embodied activity components, which offer the frame for an experiential process of translating mathematical vocabulary into embodied vocabulary and vice versa. Some embodied activity components are:

- Using one or more of the senses to observe and interact with the world.
- Being aware of certain parts of the body and body positions.
- Being in a particular location.
- Moving from one point to another.
- Studying objects from various physical perspectives.
- Moving certain parts of the body.

- Experiencing different qualities of a movement.

- Moving with respect to, and in interaction with, other participants.

- Interacting with concepts through objects and elements of the world.

The MiM-methodology is based on interaction [11] and discovery [13]. Both of these imply seeing mathematics not as a product, but as a process, and therefore supporting participants in concentrating on their own engagement and progress with the respect of differences in individual learning styles. In respect to this, every MiM Closing includes acknowledgement, reflection and sharing.

Acknowledgement is achieved by the learners presenting their creations and the results of their work through movement. This part brings the whole group together and offers an opportunity to inspire and be inspired.

In order to pay attention to differences coming from various learning styles, reflection on learning is guided with questions according to Kolb's Experiential Learning Theory-model [9]. The reflection is a crucial part of the learning process, because while creating and observing connections between physical experience and abstract thinking, the "understanding is more resilient and can stretch to fit in new situations" [19].

Sharing of thoughts and ideas with the whole group supports the teacher's understanding of participants' learning and empowers the learners to get new insights through other people's reflections.

The goal of MiM is primarily to enhance mathematics education. However, we acknowledge that a much higher goal is to be strived for when combining two disciplines. Arts have often been used as tools for mathematics education, but an equal balance between the two should be aimed for, and dance and mathematics should both get to benefit from the multidisciplinary context as was pointed out by Moerman in Bridges 2018 [12]. For example the paper by Schaffer, Thie and Williams in the same conference shows that mathematics can also be used as a tool to benefit dance [21]. Although, the core work in MiM has been done with the view of benefitting mathematics education, we feel that several aspects of the MiM-modules offer interesting tools for the use of dance art as well. For example in *Activity 1: Snowflake Dance* learning goals (symmetries, geometry, creative movement, cooperation, problem solving skills, coordination of movement, and creativity) are all useful tools for dancers and choreographers, and, in fact, the module could be viewed as a module for teaching dancers choreographic skills.

Finally, we would like to note that the name of our project, "Maths in Motion" might get confused with a similar term "Math in Motion" which is widely used for projects combining mathematics with a variety of fields including music, visual arts or origami. However, we felt it was important to keep the name of our Erasmus+ project, funded by the European Commission also in the name of this workshop paper.

## Why Maths in Motion?

The project Maths in Motion brought together several experts from the fields of mathematics, dance and education, who all shared the feeling that dance and mathematics have something in common and that dance would therefore be an excellent aid in mathematics education. Our feeling is supported by research.

Medicine and contemporary philosophy have pointed out that the human body and brain are inseparably linked [10, 16]. Cognitive neuropsychology provides evidence of the brain-body relationship's positive effects on learning; physical activity may positively affect cognitive skills, academic performance of physically active children has shown several improvements, and evidence suggests that regular physical activity is linked to structural and functional changes in the brain that can play a primary role in achieving better intellectual results [1]. Despite these findings, mathematics and science education are still strongly based on mind-body dualism. Learning mathematics is most often regarded as a solely intellectual activity. Students in mathematics classes are still typically sitting at their desks, expected to use "their brain only". They are being told to exclude all the external stimuli from their attention and instructed to focus "only on the task in front of them". More focus on embodied mathematics education is thus needed.

Indeed, studies of embodied mathematics education yield positive results. To name a few, we know that integration of dance and movement in mathematics instruction for students diagnosed with learning disabilities, emotional, behavioral disorders, attention-deficits, and hyperactive disorders, can play a key role in increasing their mathematical performance [2], that body movement and dance can enhance learning and the understanding of mathematical concepts [15], and that incorporating embodied methods using the whole body can lead to better learning results than incorporating methods using merely thinking or even drawing [18]. Overall, structural, spatial, rhythmic and symbolic dimensions of mathematics can be learned through body movement and presence [7, 8, 17, 23]. The MiM-project built upon the core of this research. The project has also greatly benefited from the work of several dance, mathematics, learning and embodiment experts, like Schaffer et al. [19, 20], Renesse et al. [17], Gerofsky [7], Bingham [3], and many others.

## Conclusions

MiM project's team members are already using the approach and *Maths in Motion: the toolkit* [14] in formal and informal learning situations and in teacher education. The feedback we get during our experiments are suggesting that our approach can effectively contribute to STEAM-based [5] mathematics educational practices. The MiM-approach can primarily address both the development of Gardner's multiple intelligences [6] and Burnard's multiple creativities [4] hand-in-hand, through embodiment and movement-based learning strategies. Our next goals include to integrate our approach into multidisciplinary and multi-sensory learning programs and we would like to explore the further potentials of sports-based mathematics learning and employing technology (e.g. tracker software, GeoGebra, Micro:bit, fitness wristbands, electronic textiles, soft robotics, pedometers) for motivation and data-collection in movement-based mathematics learning programs in general. MiM has also provided inspiration for projects incorporating dance and mathematics in artistic works. MiM team members are working on dance, choreography and productions based on the connection between dance and mathematics. We find this an intriguing direction for development as well.

## Acknowledgements

This paper is one of the results of the 3-years-long "Maths in Motion" Erasmus+ project, supported by the European Commission in the framework of Strategic Partnership for Innovation in School Education. The following organizations are participating in the realization of the project: Olde Vechte Foundation, The Netherlands; Experience Workshop, Finland; Fun Mathematics, Bulgaria; SciCo, Greece; IC Codogno, Italy; Scoala Gimnaziala No 20 Galati, Romania; Balletskolen Holstebro, Denmark.

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