Impossible Folding Font

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Abstract

We present a series of seemingly impossible objects made by cutting and folding a square piece of paper, in the style of hypercards. Each object is in the shape of a letter or numeral, thereby making an "impossible folding font".

Introduction

Humans are fascinated by *impossible objects* — physical objects that obviously exist yet appear to be impossible. For extensive examples, see Sugihara's award-winning collection [12]. The *International Puzzle Party*, an annual forum for serious puzzle collectors, has a session devoted to impossible objects, which can only be attended by those who have already created something "impossible".

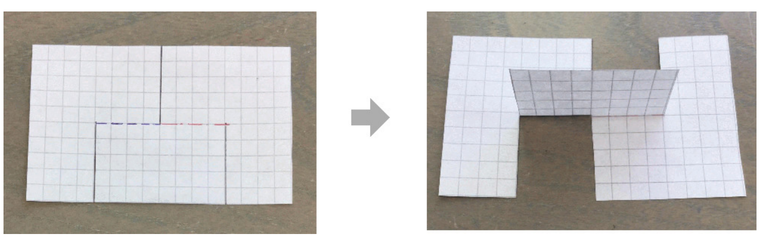

Figure 1: *How to make a hypercard.*

Figure 2: *Magical hypercards.*

One elegant form of the impossible object is the *hypercard* [1, 5, 4, 9, 10, 11], as popularized by Martin Gardner in 1978 [6]. Hypercards are typically made from one complete playing card or index card, and yet seem to be impossible. Figure 1 shows an example: (1) cut along the black solid lines, (2) mountain fold along the red dash-dot line, and (3) valley fold along the blue dashed line. When we fold both crease lines by 90°, we obtain the hypercard, which seems to be impossible at first glance. Using this type of trick, many different impossible cards, such as the one in Figure 2, have been designed and sold in magic shops [4]. Gardner describes hypercards as a kind of minimalist sculpture [6].

Why do these objects seem to be impossible? One reason is that the sheet has been twisted, and twisting is necessary to form the 3D shape, while the resulting folded state does not seem to be twisted. A key feature

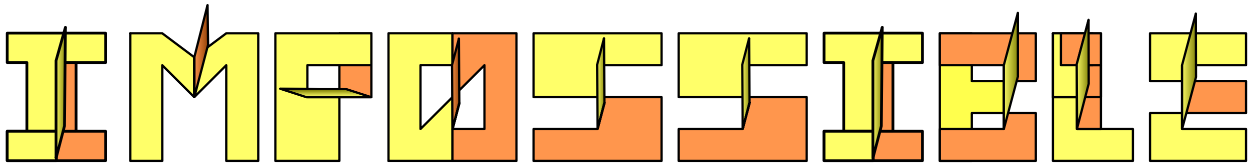

Figure 3: *"IMPOSSIBLE" written in our impossible folding font.*

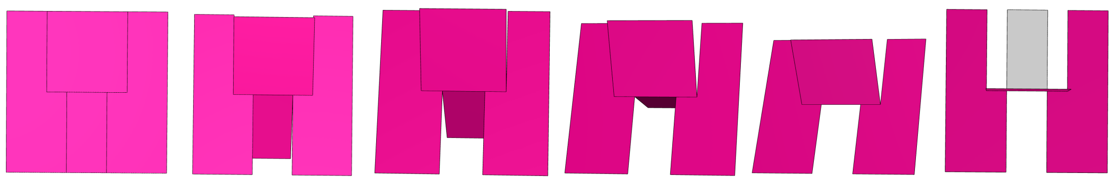

Figure 4: *Folding of "W", generated by Origami Simulator. The folding percentages are 0%, 20%, 40%, 60%, 80%, and 100%. Magenta and gray represent the two sides of the paper.*

of the folded state is a vertical flap extending from a horizontal plane, with the property that the vertical flap overlaps other material when rotated in either direction onto the horizontal plane, thus seeming to be impossible from a single sheet of material. We call such a folded state a *one-flap impossible folding*.

In this paper, we design several new one-flap impossible foldings that together form an *impossible folding font*. From the viewpoint of origami design, the symbols of the impossible origami font require several nontrivial origami/paper craft techniques. From the viewpoint of computational origami [3], this framework can be seen as a natural extension from 2-dimensional origami and a natural intermediate model to general 3-dimensional origami. In our designs, we use a square sheet of paper with crease lines on horizontal, vertical, and 45° diagonal lines. Each folding angle is orthogonal, that is, ±90° or ±180°.

Figure 3 shows some text written in our font. Can you figure out how each letter can be made from a square of paper? For example, "W" can be folded as shown in Figure 4, which is one of the simplest patterns.

Notation and System

In each hypercard in our impossible folding font, we use a square sheet of paper and draw our cuts and creases using a 16×16 grid. For each crease line, as in classic origami diagrams, we assign *mountain* or *valley* to describe the direction of folding. However, in our context of 3D folding, to fix the folded state, we also have to assign a folding angle to each crease line.

To represent such a crease pattern with mountain–valley assignment and folding angles, as well as the pattern of cuts, we follow the representation used by *Origami Simulator* developed by Amanda Ghassaei [7, 8]. This simulator not only adopts the *FOLD* file format [2], but also allows input to be in a particular form of SVG[1]. We use the latter format to draw our examples, which we briefly explain.

In each cut/crease pattern, a red line indicates mountain fold, and a blue line indicates valley fold. A black line indicates the boundary of the paper, and a green line indicates a cut or slit. In the SVG format, the folding angle of each crease is set by its opacity. In particular, 1.0 (fully opaque) indicates 180° (fully folded), while 0.5 (half opaque) indicates 90° (which makes red and blue appear pink and light blue respectively). Some of the foldings can be decomposed into a sequence of folding steps that avoid self-intersection, but we do not know whether this is universally the case.

[1]SVG stands for Scalable Vector Graphics, which is a data format for vector graphic data.

In the real design process, we first design each symbol by cutting and folding a physical sheet of paper. Then we trace the pattern using a vector drawing program (Inkscape) and save it in SVG format. Origami Simulator then allows us to visualize the 3D folded letters, as well as animate a continuous folding process. These renderings are great in an interactive setting, but for easier-to-see still images, we drew the 3D folded states (again in Inkscape) using oblique projection (with out-of-plane lines represented by 15° rotation at 85% scale and gradient shadows). Yellow and orange represent the two sides of the paper.

Impossible Folding Fonts

Here we show our impossible folding typeface in two fonts: the 3D folded state and the cut/crease pattern (as described above). Each folded state is a puzzle: how can it be cut/folded from a square of paper? Each cut/crease patterns is also a (straightforward) puzzle: what letter/numeral does it fold into? To encourage the reader to play with these puzzles, we present each font separately. Figures 5, 6, and 7 show the folded font (drawn at a uniform scale), while Figures 8, 9, and 10 show the cut/crease patterns (drawn at a different uniform scale). Figure 11 shows real-world foldings of the uppercase letters.

Acknowledgements

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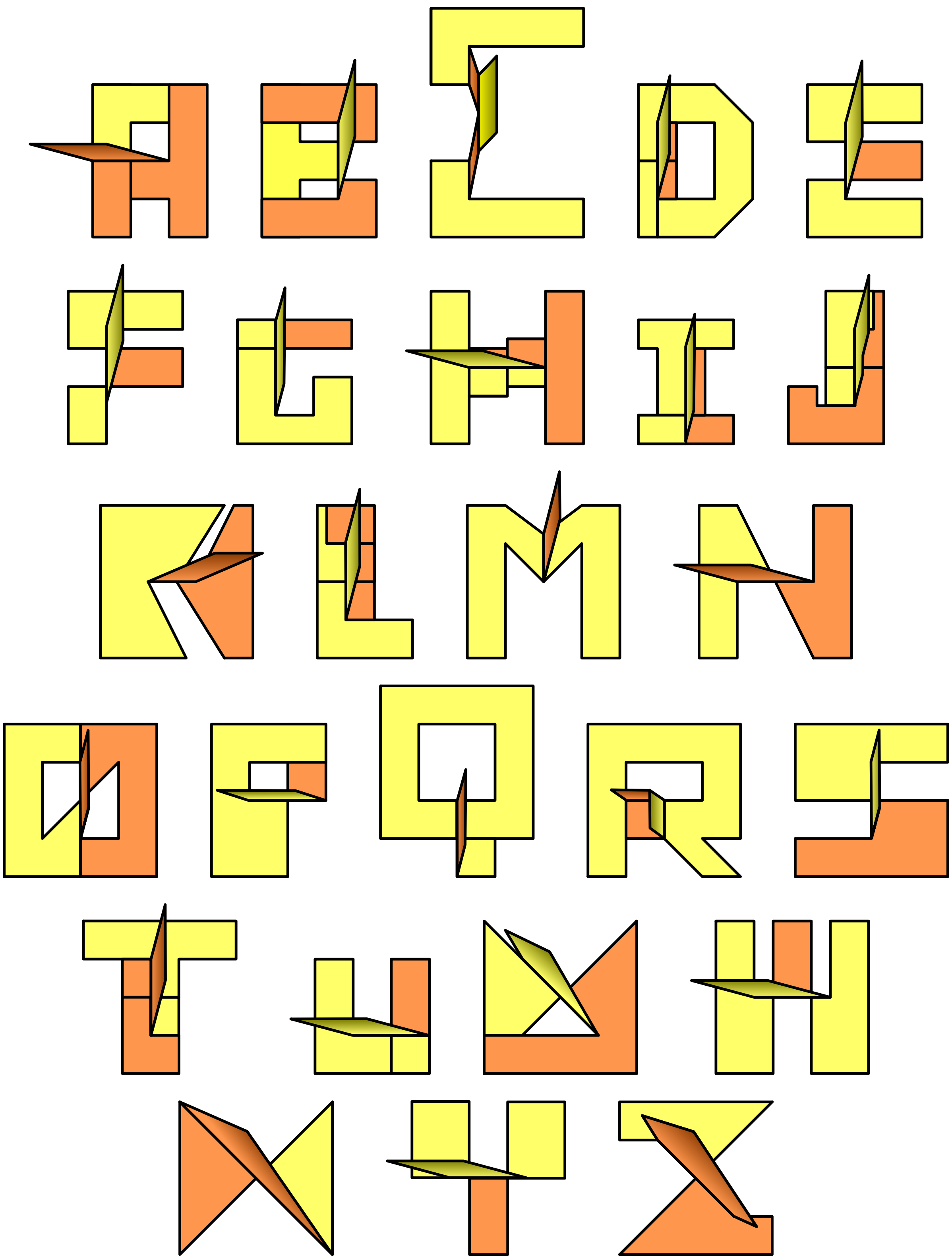

Figure 5: *Uppercase impossible folding font, folded into 3D.*

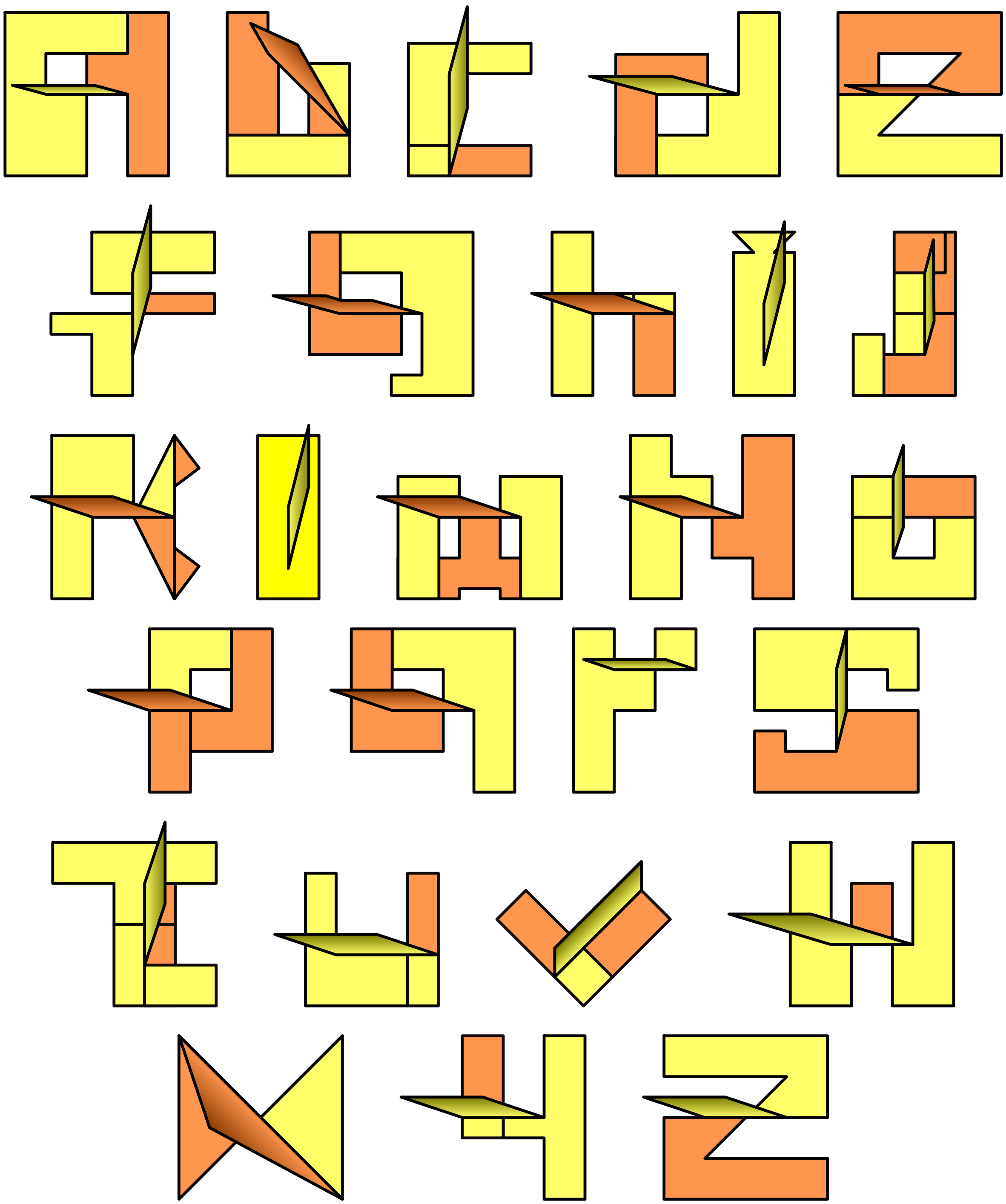

Figure 6: *Lowercase impossible folding font, folded into 3D.*

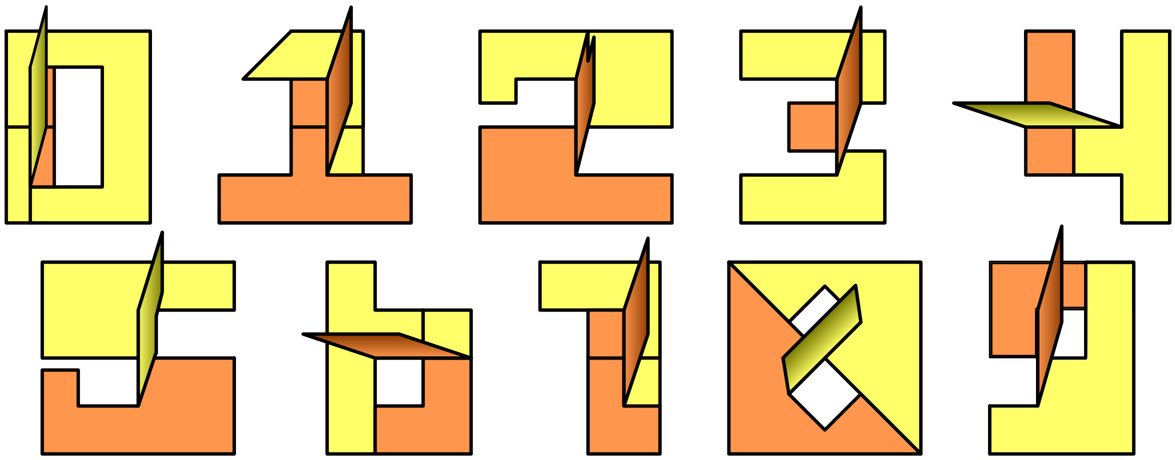

Figure 7: *Numeral impossible folding font, folded into 3D.*

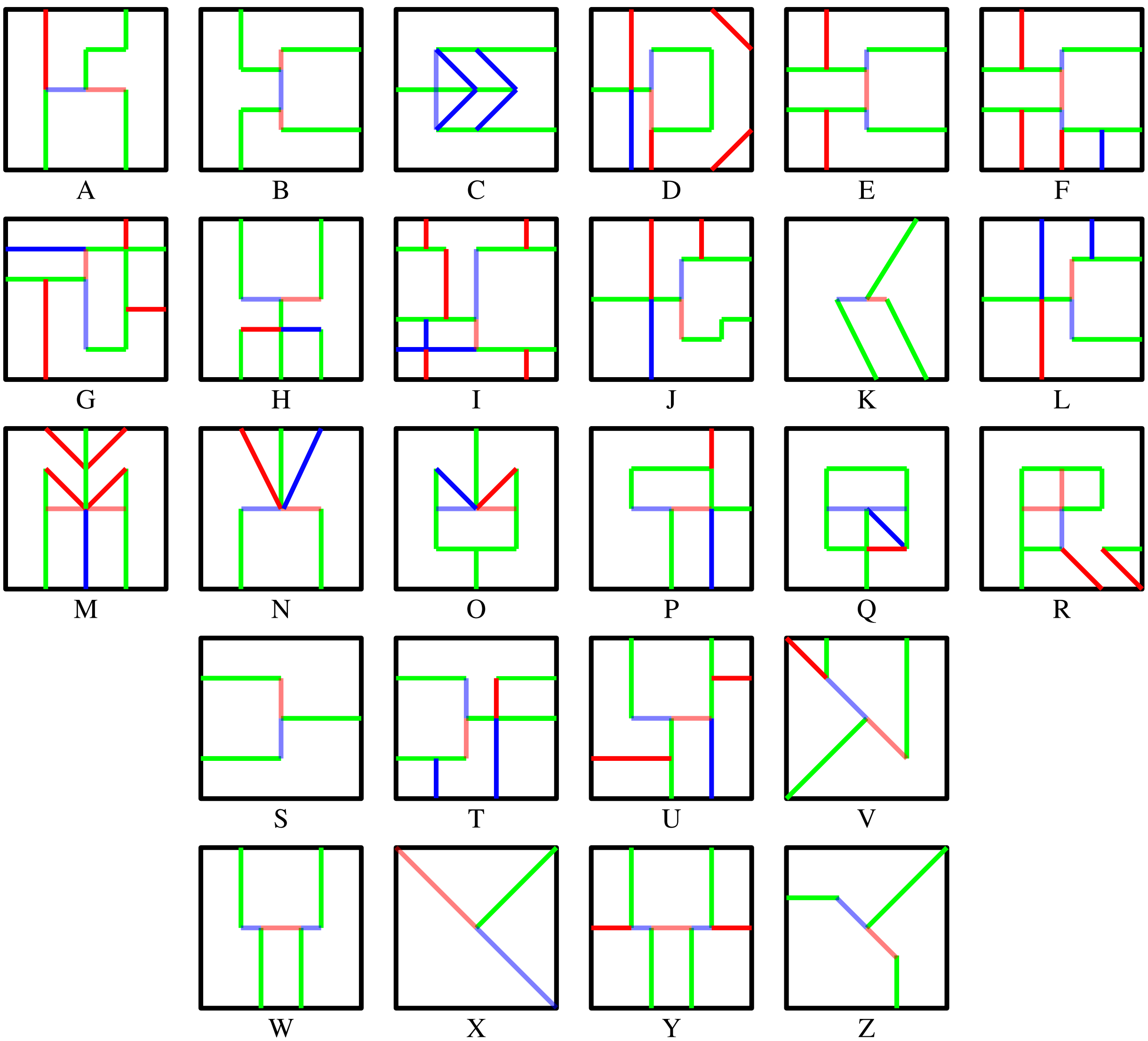

Figure 8: *Uppercase impossible folding font, cut/crease patterns.*

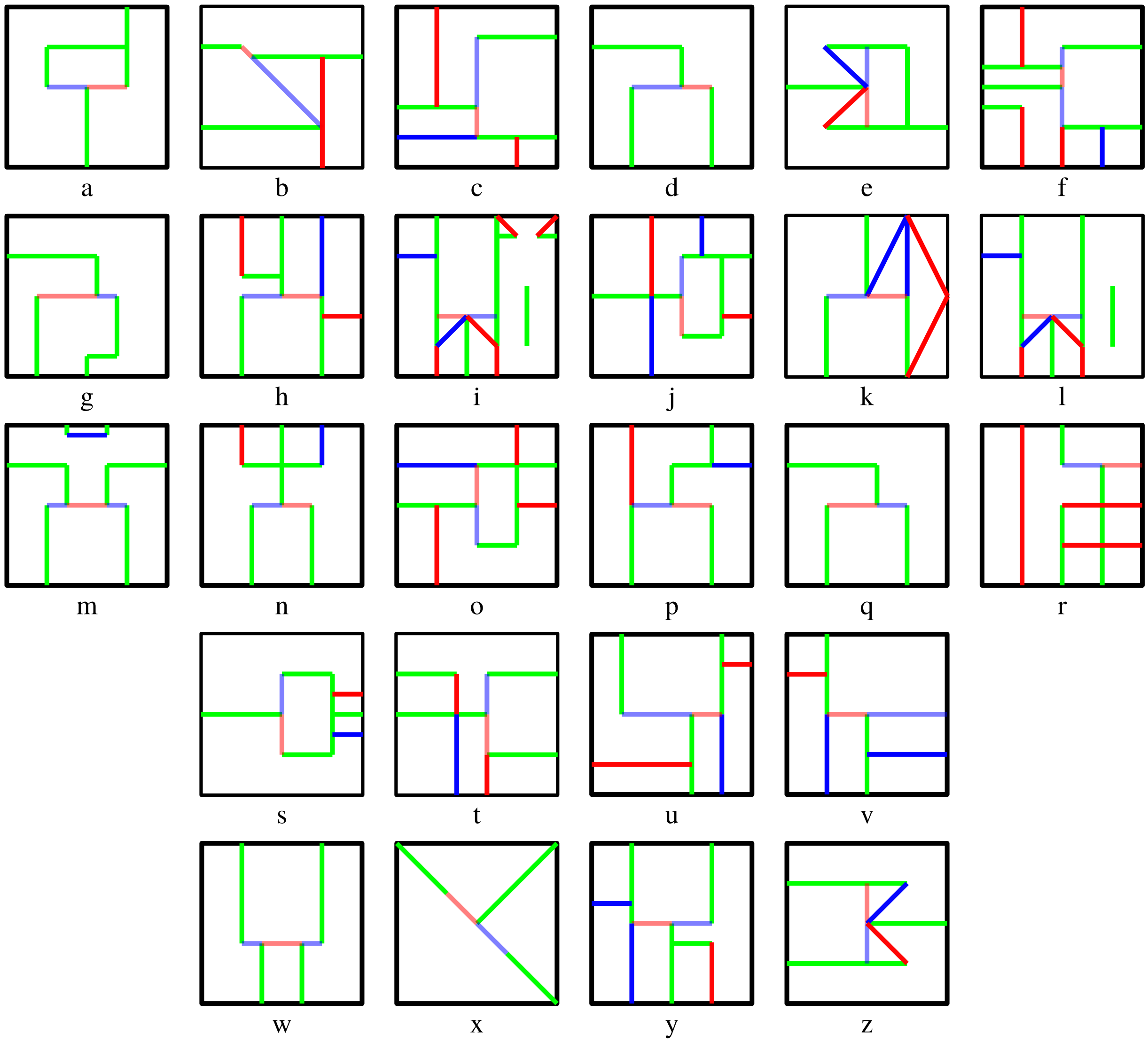

Figure 9: *Lowercase impossible folding font, cut/crease patterns.*

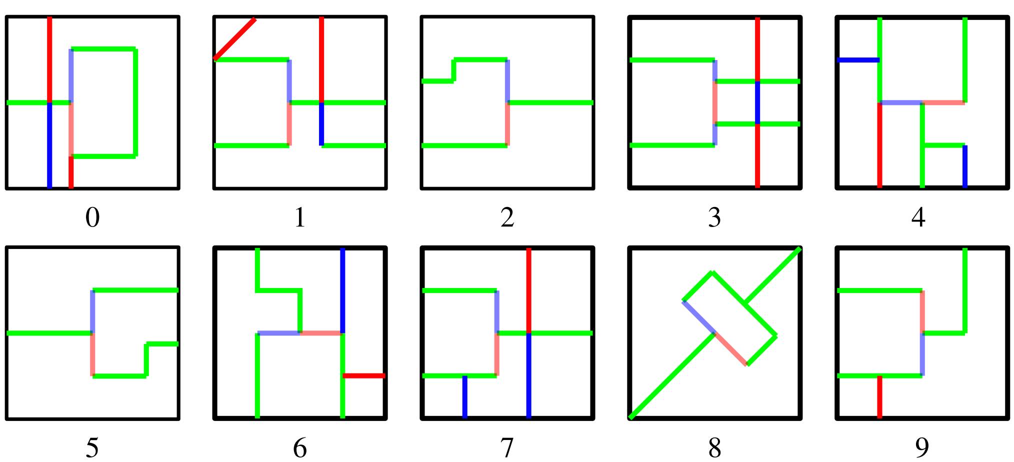

Figure 10: *Numeral impossible folding font, cut/crease patterns.*

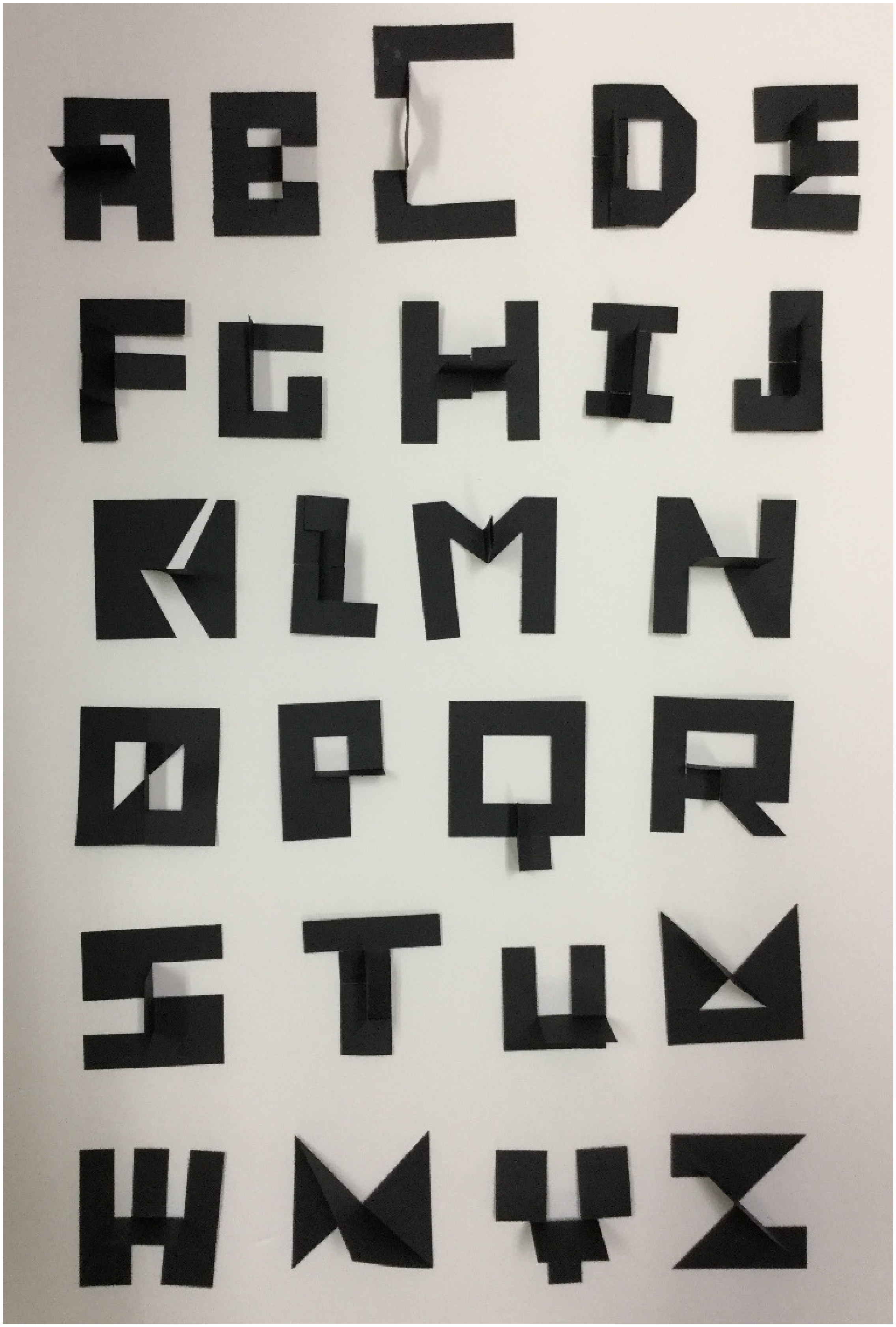

Figure 11: *We laser-cut sheets of black paper and folded them into the uppercase impossible font.*