Exploring Szpakowski's Linear Ideas

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Abstract

The Polish architectural engineer Waclaw Szpakowski (1883-1973) spent over fifty years of his spare time creating intricate line drawings—each one a single line that starts on the left side of the page and winds its way across its surface, bending only at 90° angles and finally ending up on the right side. In this paper, we discuss his linear ideas and present our efforts to map them (and also our own Szpakowski-esque designs) onto cylinders, tori, Möbius strips, and the faces of cubes.

Introduction

In 1968, Waclaw Szpakowski (1883-1973) wrote a "self-commentary" (reproduced in [3]) to accompany a remarkable series of line drawings that he had worked on for over fifty years (1900-1951). Within, he described his self-imposed rules: "Every drawing features a single line which begins as a straight line ... By deflecting the line's course from its original direction at a given point, we create a break ... The line can only break in either of two directions—right or left, and these are relative, defined in reference to the progression of the act of drawing and the direction of the line's course preceding the break ... By repeating this procedure we obtain a continuous unicursal line composed of straight line segments between each pair of breaking points" [3, p. 14]. Figures 1 and 2 display our reproductions of two of Szpakowski's less intricate pieces, *A9* (circa 1924) and *D1* (1925). In each case, Szpakowski created the final version in ink on tracing paper, having done extensive preliminary work in note books and on graph paper. For our reproductions, we superimposed Szpakowski's designs on grids of shaded squares in order to highlight their mathematical structure.

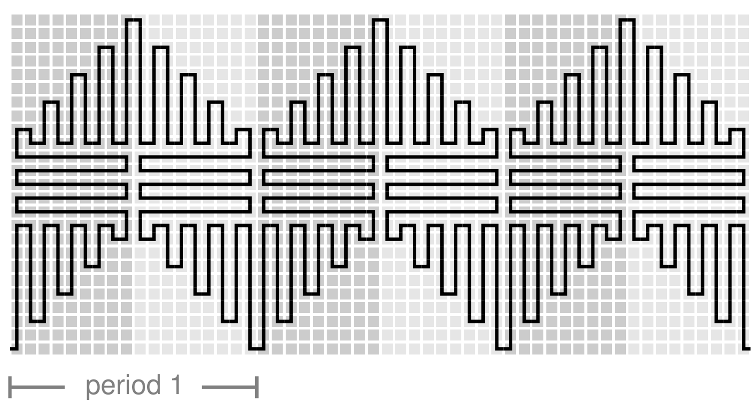

Figure 1: *A reproduction of Szpakowski's A9 (circa 1924).*

Szpakowski's commentary also contains instructions for viewing his *linear ideas* (his preferred term for his drawings): "The linear ideas possess a certain 'inner' content which can be discovered by making the eye follow the course of the broken line, from one break to the next, proceeding from the line's starting point, just like one would put together written words of individual letters ... A single glance would not be enough. This

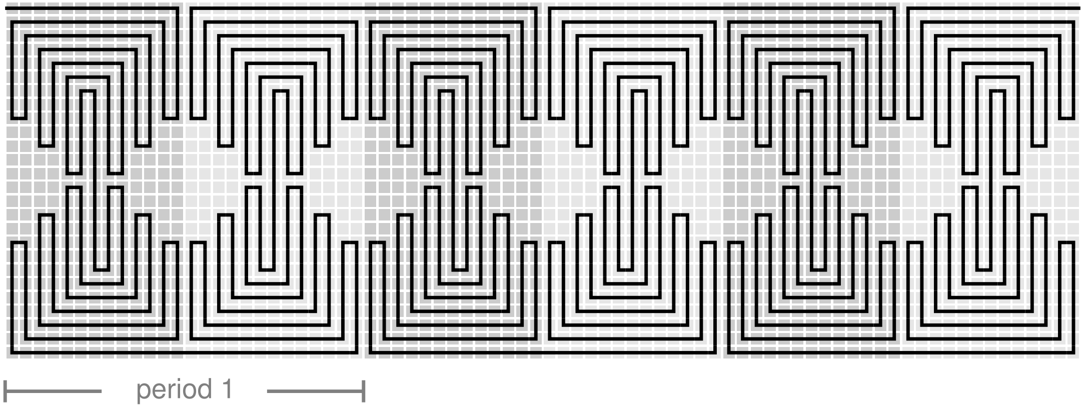

Figure 2: *A reproduction of Szpakowski's D1 (1925).*

would mean ignoring the author's purely mental and thus invisible thinking process which is fundamental to creative activity" [3, p. 16].

We believe that any mathematician who comes across one of Szpakowski's drawings will not need to read his instructions. Even without them, the mathematician will certainly notice that Szpakowski's linear ideas are simultaneously simple curves, Hamiltonian paths on grid graphs (in which the vertices are the shaded squares visited by the path), and unicursal mazes (i.e., labyrinths). They are also, typically, both symmetric and periodic. Accordingly, the mathematician will be compelled to read Szpakowski's pieces from left to right (like Western texts and musical scores). And in the process, they will marvel at how, in each case, the line interacts with itself. They will take delight in how Szpakowski suggests shapes and constructs patterns through the careful positioning of bends (what he calls breaks) and empty space.

We have written this paper for two reasons. First, we want to introduce the Bridges community to Szpakowski's wonderful artwork. We came across it on May 22, 2018 via a tweet by Geoff Manaugh, which led us to one of his blog posts [4], which in turn brought us to a *Paris Review* article by Sarah Cohen [2] (a review of a 2017 exhibition of Szpakowski's work held at the Miguel Abreu Gallery [1]), and ultimately, a breathtakingly beautiful catalogue of a 2015 exhibition held in his hometown of Wroclaw, Poland [3].

Second, we want to share the results of our efforts to map Szpakowski's linear ideas (and also our own Szpakowski-esque designs) onto cylinders, tori, Möbius strips, and the faces of cubes. According to Anna Szpakowski-Kujawska (one of Szpakowski's daughters), "When he [Szpakowski] found something that interested him, he would get completely absorbed in it, to the point of oblivion—he was transported into a world of his own, in the center of the universe" [3, p. 294]. The senior author (Bosch) can certainly identify with this!

Cylinders, Tori, and Möbius Strips

If we place multiple copies of a Szpakowski design on a long strip of paper and then tape the left edge to the right edge, we will end up with a cylinder. If, before taping, we roll the strip into a tube, we will obtain a torus. And if instead we give the paper an odd number of half twists before joining its edges, we will produce a Möbius strip. In this way, we can map Szpakowski's designs onto cylinders, tori, and Möbius strips.

Figure 3 displays renderings (executed with the Rhinoceros 3D software package) of Szpakowski's *D1* design mapped onto a cylinder and a torus. In each case, we used two full copies of the design, or equivalently, six periods.

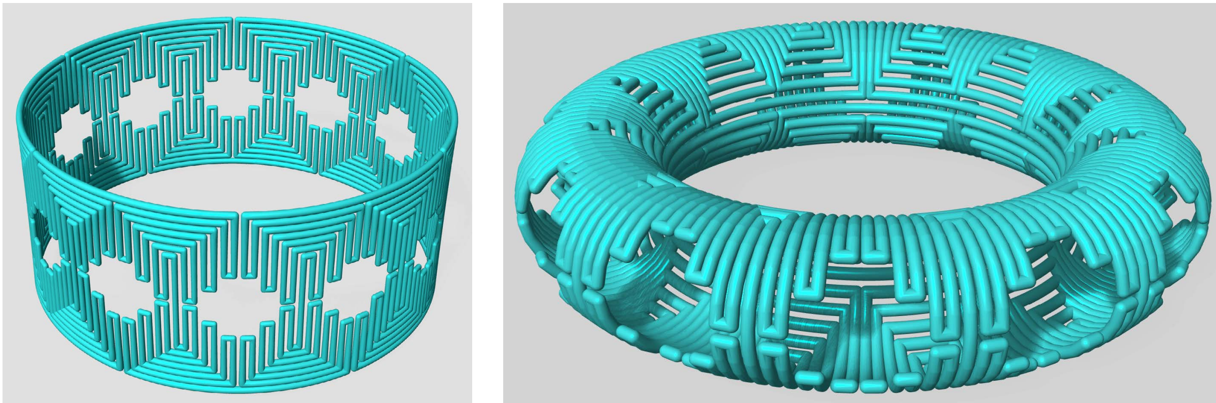

Figure 3: *Szpakowski's D1 mapped onto a cylinder and a torus.*

Figure 4 displays a rendering of Szpakowski's design *D1* mapped onto a three-half-twisting Möbius strip. For this piece we used two and one half copies of the design (seven and one half periods).

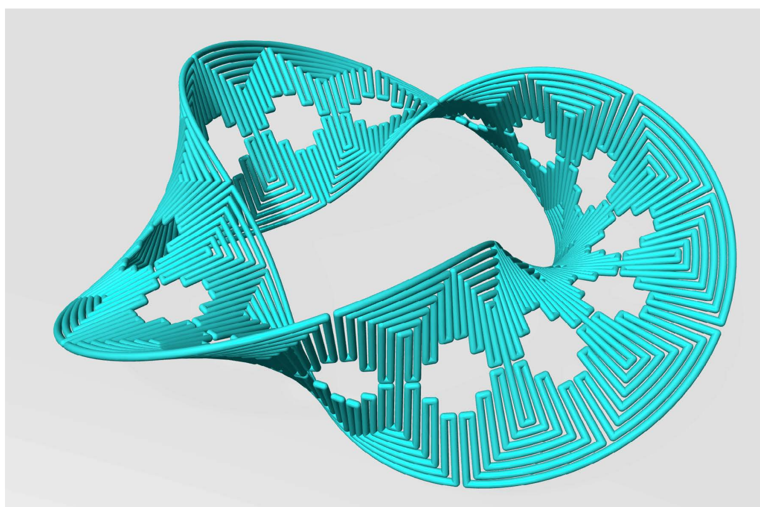

Figure 4: *Szpakowski's design D1 mapped onto a three-half twisting Möbius strip.*

For the 3D printed piece (made by Shapeways) shown on the left side of Figure 5, we mapped 12.5 periods of Szpakowski's design *D1* onto a self-intersecting Möbius strip that makes five one-quarter twists on its first revolution around its axis and then another five one-quarter twists on its second revolution. It is approximately ten inches in diameter. After it has completed its ten one-quarter twists on its two-revolution journey, the strip's leading and trailing edges (the right and left edges, when the strip is flat) meet each other at a 180° angle. The holes in *D1* enable the design's line to pass through its empty spaces in a way that is reminiscent of Rinus Roelof's Moebius torus and Moebius ring pieces [5]. The 3D printed piece shown on the right side of Figure 5 is similar but based on a modification of Szpakowski's design *A9*. It is about eight inches in diameter.

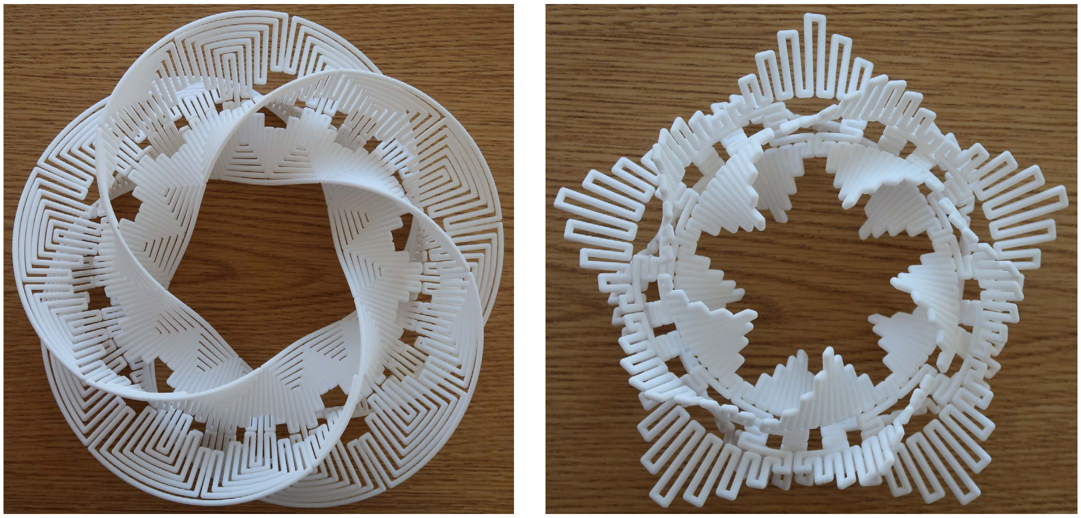

Figure 5: *3D printed versions of Szpakowski's design D1 (left) and a modification of A9 (right) mapped onto self-intersecting Möbius strips in which the strip makes five one-quarter twists on its first revolution around its axis and another five one-quarter twists on its second revolution.*

Two-path Szpakowski-esque Designs

Our design displayed in Figure 6 is different from those of Szpakowski: it is *not* a Hamiltonian path on a rectangular grid graph, as it has not one but two paths that connect the rectangle's left and right edges.

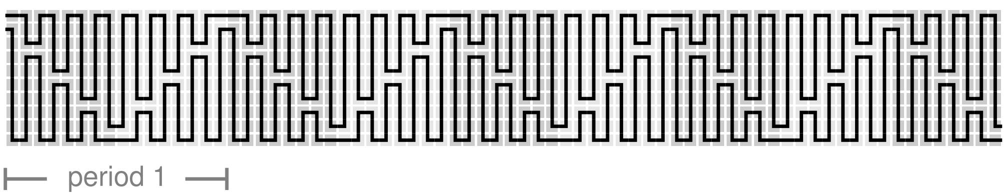

Figure 6: *A design with two paths that connect the left and right edges.*

Yet if we take this two-path design, twist it an odd number of times, and then join the left edge to the right edge (forming a Möbius strip), the two paths will become a single cycle, a Hamiltonian cycle on the Möbius strip. Figure 7 contains two photos of a 3D printed version of this object (approximately six inches in diameter). The photo on the left shows the object in its resting position. The photo on the right demonstrates the flexibility of Shapeways' "white versatile plastic" material, which the company formerly marketed as "white, strong, and flexible." This photo of the stretched out object also reveals that the object is topologically equivalent to a trefoil knot.

Figure 8 displays two additional two-path Szpakowski-esque designs, and Figure 9 shows photos of them in 3D printed form. In each case, the 3D printed object is a single non-self-intersecting loop of plastic, and in each case, we formed the object by mapping 12.5 periods of the design onto a self-intersecting Möbius strip that makes two revolutions around its central axis and one one-quarter twist per revolution. The empty spaces in the designs allow the plastic loops to avoid themselves. They also allow the plastic loops to rest on themselves when placed on a table top, which gives the 3D printed objects greater structural integrity than one might expect them to have.

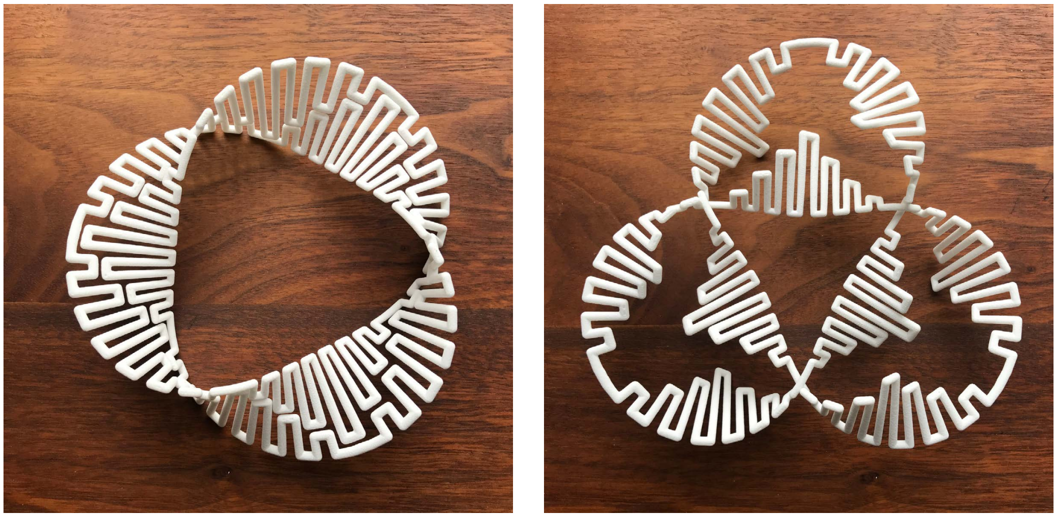

Figure 7: *Two views of a two-path Szpakowski-esque design on a three-half-twisting Möbius strip.*

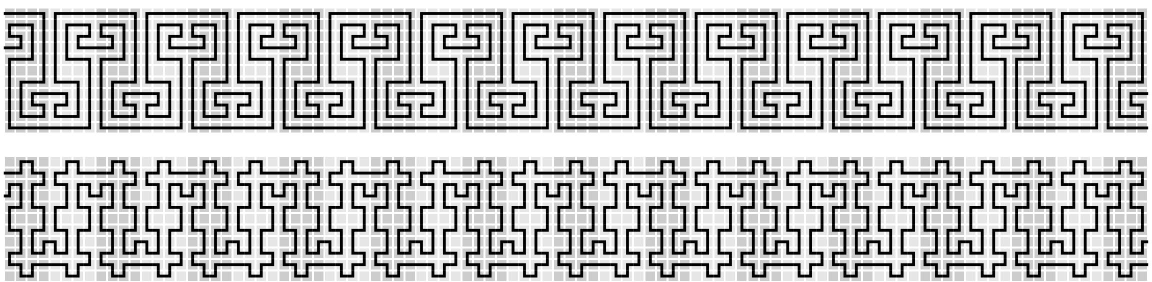

Figure 8: *Two two-path Szpakowski-esque designs.*

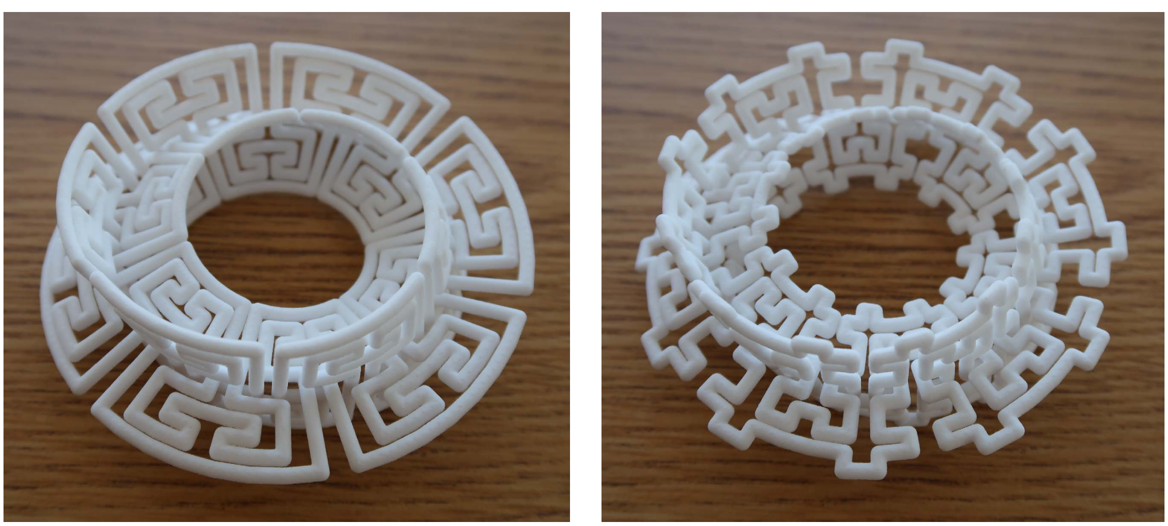

Figure 9: *Two 3.6"-diameter 3D printed two-path Szpakowski-esque designs.*

Toroidal Szpakowski-esque Designs

Our design displayed on the left side of Figure 10 is also different from those of Szpakowski. On a rectangular grid graph, it is not a Hamiltonian path. Nor is it a two-path design. But if its top edge is considered to identify with its bottom edge (and vice versa), it can be seen to form a Hamiltonian path that connects the left and right edges. And if the right edge is then joined to the left edge, the Hamiltonian path becomes a Hamiltonian cycle on a torus. By using multiple copies, we obtain a toroidal Szpakowski-esque design, which also forms a Hamiltonian cycle on a torus. For the 3D printed object shown on the right side of Figure 10 (with a 3D printed ball inside the torus), we used 12 copies of the design.

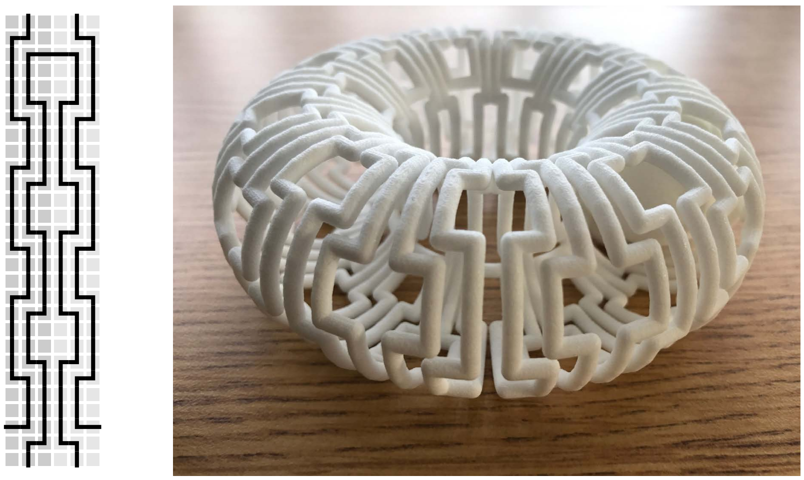

Figure 10: *A toroidal Szpakowski-esque design.*

Touroborean paths

Figure 11 shows two views of a 3D printed *touroborean path*, a Hamiltonian path on a torus that is eating itself, the "offspring" of a torus and an ouroboros (a snake eating itself). The design is Szpakowski's *D1*.

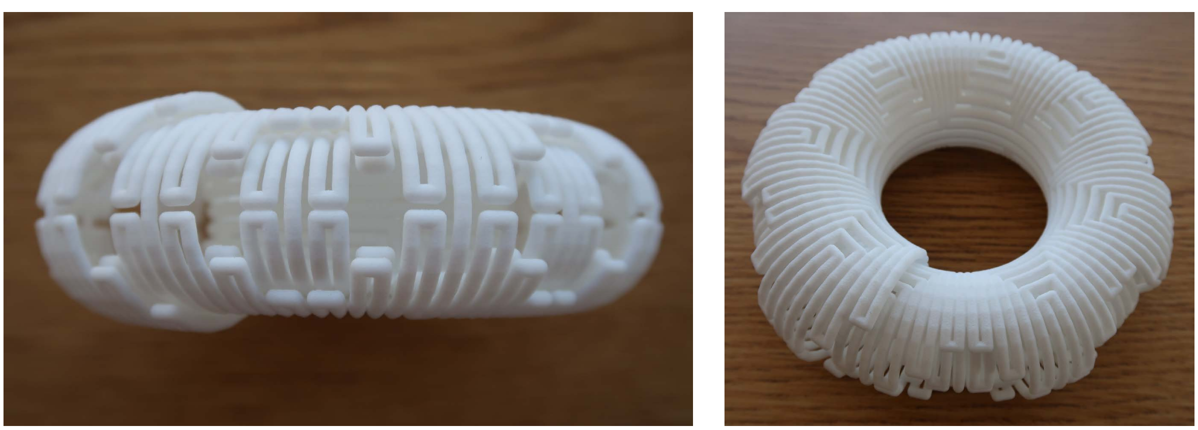

Figure 11: *Two views of a touroborean path.*

Cubes

It is also possible to map Szpakowski-esque designs onto the faces of a cube. The left side of Figure 12 shows a slightly modified portion of *A9* mapped onto a net for a $9 \times 9 \times 9$ cube, and the right side shows a rendering of the resulting object. In the net, the dark gray squares correspond to the edges of the cube. If we follow the line, starting at the bottom edge of the front face (the center of the net), we see that the line goes from the cube's front face to its left face and then to its top, back, right, and bottom (and then returning to its starting point). When we modified *A9* and placed it on the net, we followed two self-imposed constraints. We wanted the resulting object to have 120° rotational symmetry about one of the cube's long diagonals, and we wanted all of its faces to be the same, modulo symmetry.

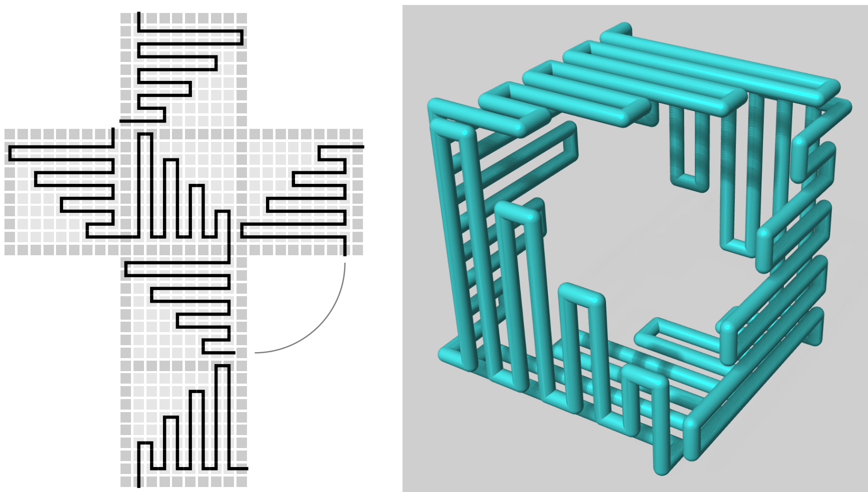

Figure 12: *A Szpakowski-esque design mapped onto the faces of a* $9 \times 9 \times 9$ *cube.*

Figure 13 displays two additional designs that satisfy these self-imposed symmetry constraints (and visit the faces of the $9 \times 9 \times 9$ cube in the same order as in the Figure 12 example).

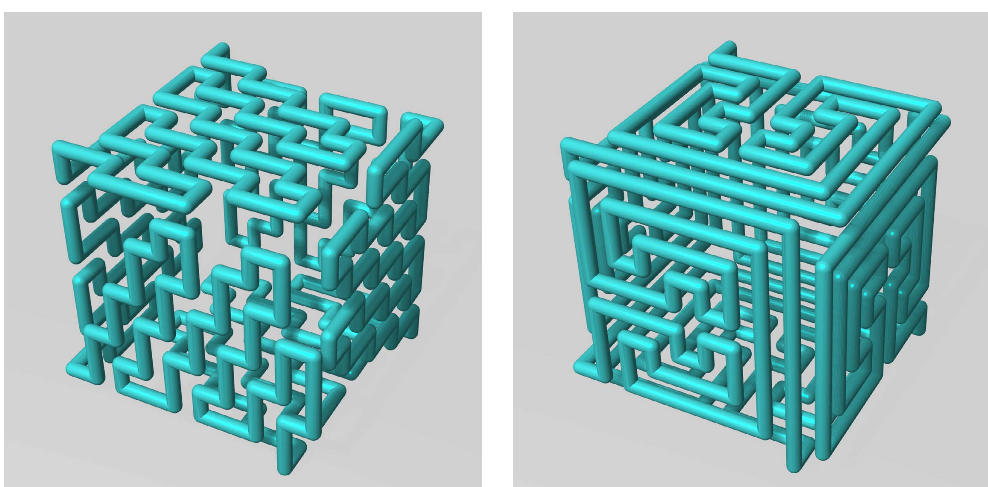

Figure 13: *Szpakowski-esque designs mapped onto the faces of a* $9 \times 9 \times 9$ *cube.*

And finally, Figure 14 displays a Szpakowski-esque design mapped onto the front-facing faces of a collection of cubes. The bottom left square of the net (shown on the left) corresponds to the bottom left face of the cubes on which the object rests. As was the case with our cubes, this piece is a single loop and has 120° rotational symmetry.

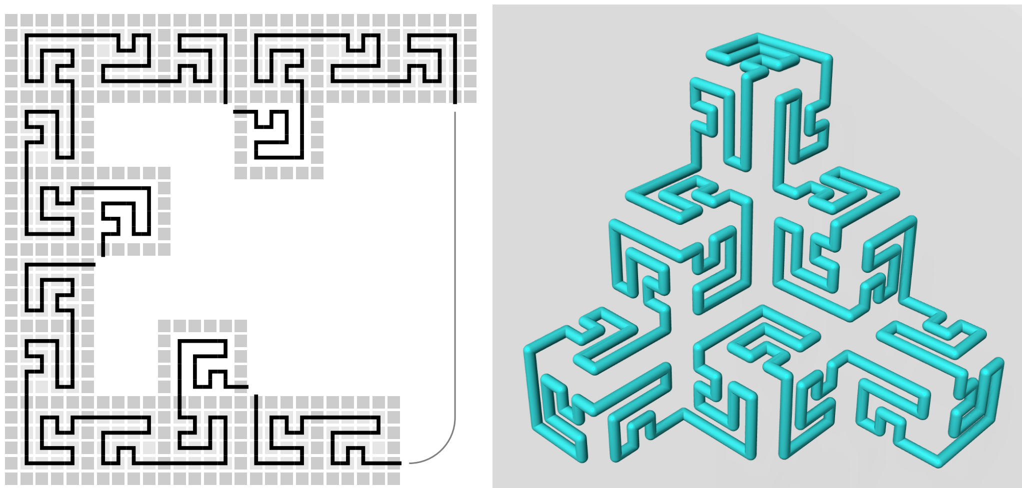

Figure 14: *A Szpakowski-esque design mapped onto faces of a collection of cubes.*

Acknowledgements

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