Knot Designs Based on Rhombille Tiling Notations

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Abstract

A textile practitioner works in collaboration with and a mathematician to examine textile knot practice and create novel knot designs using mathematical tiling methods, in particular the rhombille tiling. This paper reveals ways in which tiling notations were used to visualize novel patterns and structures.

Introduction

The previous exploration of tiling notations as design tools for a textile knot practice has shown a successful use of Wang tiles or topological unit squares with colored edges to generate several novel two-tone structures and patterns of connected reef knots (Figure 1) [4]. It has led to the question whether the underlying tiling can be adapted from simple square grid into the rhombille tiling to design new knottable patterns and structures. A textile practitioner (Nimkulrat) and a mathematician (Nurmi) are working together to explore this question.

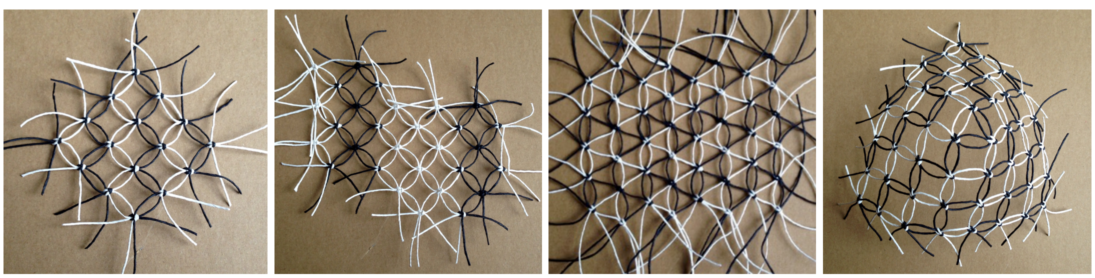
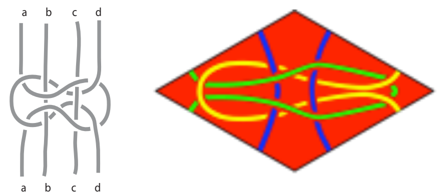


Figure 1: *Two-tone structures and patterns of conneected reef knots [4].*

Rhombille Tiling Notations and Textile Knot Designs

The rhombille tiling is a Euclidean tessellation of rhombi, whose four sides have the same length and with two 60◦ and two 120◦ angles (topological type 3.6.3.6), that resembles the axonometric projection of cubes and can be formed by subdividing hexagons into three rhombi per hexagon [1]. Each angular point of the rhombille tiling has either six rhombi meeting at their acute corners, or three rhombi meeting at their obtuse corners [2]. It is one of 56 isohedral tilings by quadrilaterals identified by Grunbaum and Shephard as P_4-42 [3]. The characteristics of isohedral tilings are that they 1) consist of only one shape of tile and 2) have translation symmetry, which demands that the tiling repeats itself periodically over the whole tiling. The use of rhombille tiling in textiles is mostly seen in quilting known as the "tumbling blocks" pattern whose three-dimensional effect is achieved by the use of light, medium, and dark fabrics.

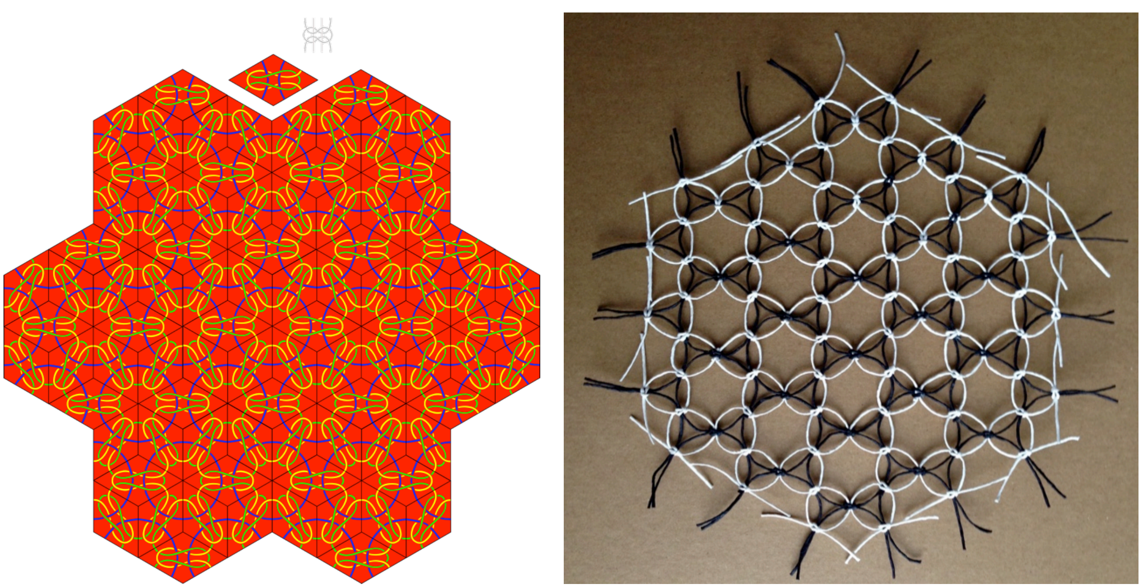
To test whether rhombille tiling notations are applicable as design tools for knot textile practice, we employed the previously used method of identifying the reef knot pattern with four strands as a unit cell—in this case a rhombus unit (Figure 2). Each unit or quadrangular tile was marked with a reef knot: blue for the middle, passive strands and yellow and green for each of the active strands. These colors code what strand enters and exits the tile, and in which position. This was to ensure the continuity in designs of knot structures.



(a) (b)

Figure 2: *(a) Diagram of the reef knot with four strands and (b) a rhombus unit cell of the reef knot whose passive strands are in one color and active strands are in two different colors.*

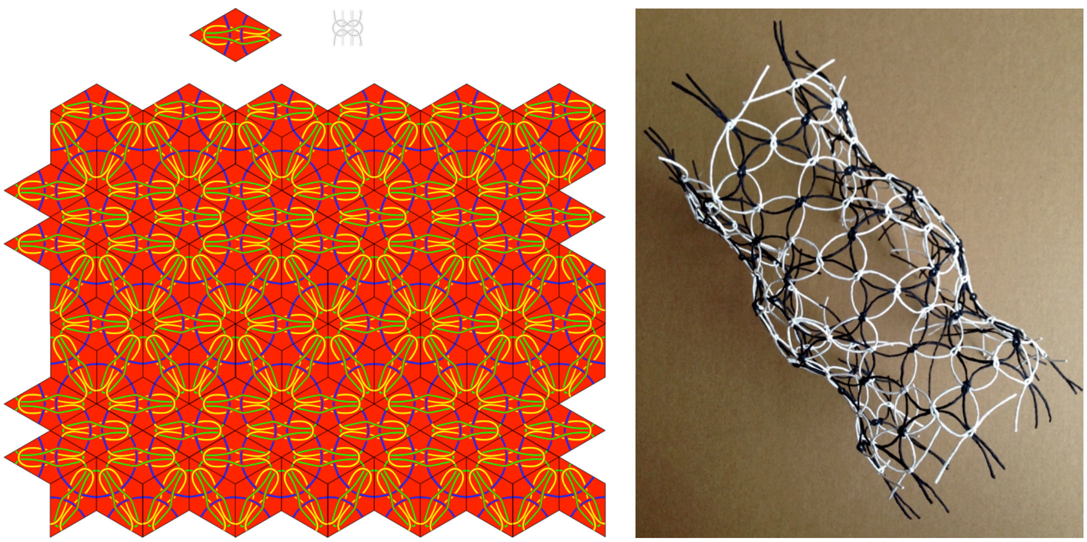
We believed that a variety of novel knot patterns and structures might be created using identical rhombi of the only variation shown in Figure 2b, which is a characteristic of isohedral tilings. First, we explored the possibilities by applying the rhombille tiling rule that each vertex has either six rhombi (all 60◦) meeting at their acute corners, or three rhombi (all 120◦) meeting at their obtuse corners to create a rhombille notation (Figure 3a). We attempted to place the tiles in a way that all marks (i.e. colored strands) matched, passing over the edge of the tiling from each tile to its adjacent tiles. However, as Figure 3a shows, whilst the middle blue-colored strands could pass over the edge of the whole tiling and create circle shapes, the yellow- and green-colored strands could not do so. Only one of three tiles at every obtuse corner could have all colored strands match their touching tiles. The reason for this inconsistency was the odd number of edges around the vertex at an obtuse corner. The mismatching yellow and green colors at the obtuse corners contributed to only half of the acute corners having all colored strands of six touching tiles match. Regardless of the color matching, all strands appear continuous. Nimkulrat used black and white paper string, black for blue (passive) strands and white for yellow and green (active) strands to knot Figure 3a. Figure 3b confirmed the knottability of this notation. On observing the knotted piece, we can see that the black strings do not always remain passive, but occasionally are active in tying knots in order to form the circle shape to continue the knotting process. If the material used for the blue strands were in a ring shape instead of string, the blue strands would remain passive throughout the knotting process.



(a) (b)

Figure 3: *(a) Rhombille tiling notation knottable into (b).*

To explore more possibilities using this single shape of tile, we combined the rhombille tiling with a different type of isohedral tilings (P_4-55), of which all tiles appear in the same orientation, to create the tiling notation shown in Figure 4a. Although all knots' strands looked continuous, this combined tiling rule ignored the congruity of knot strands' colors, meaning that the sides of two tiles could be adjacent even when strands' colors did not match. As a result, the blue strands did not always act as the passive strands in this notation. Nimkulrat utilized Figure 4a as a tool to knot the piece in Figure 4b, which curled and became three-dimensional as it was being made. Whilst black circles of six knots were generated in the area in which the rhombille tiling rule was applied, those of five knots appeared in the area of the P_4-55 tiling.



(a) (b)

Figure 4: (a) Rhombille tiling notation combined with a different type of isohedral tilings knottable into (b), a three-dimensional knotted piece

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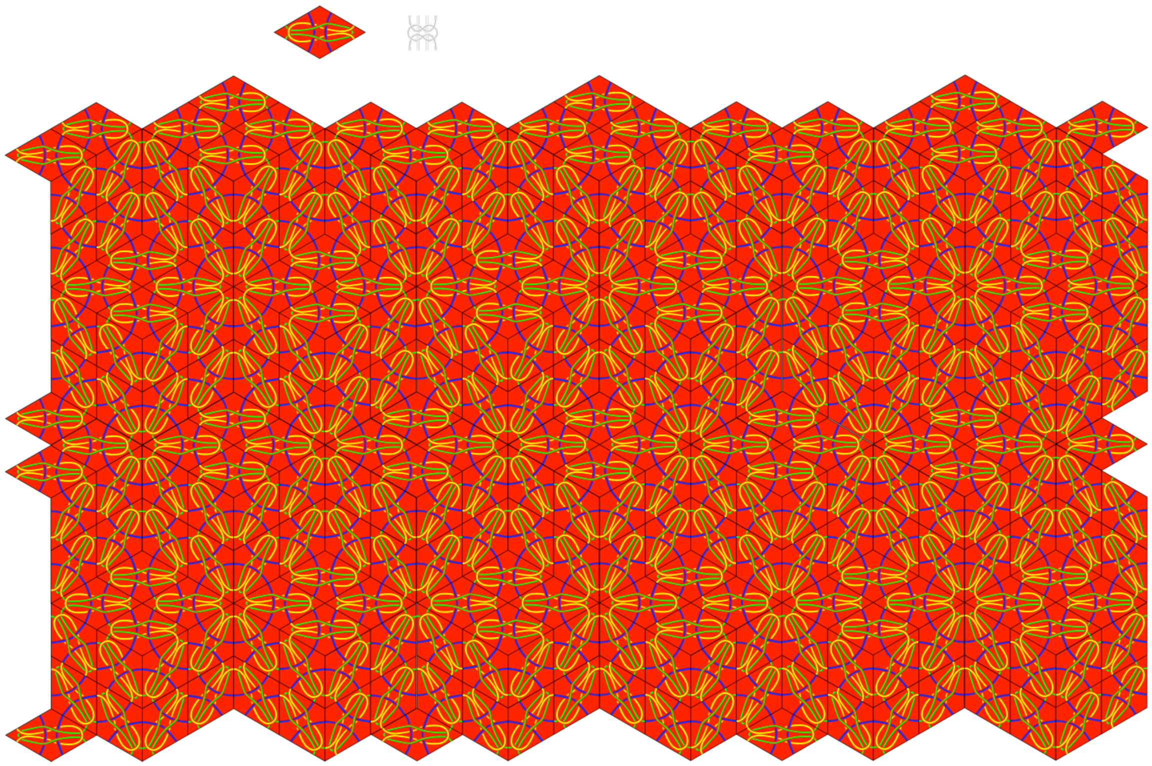
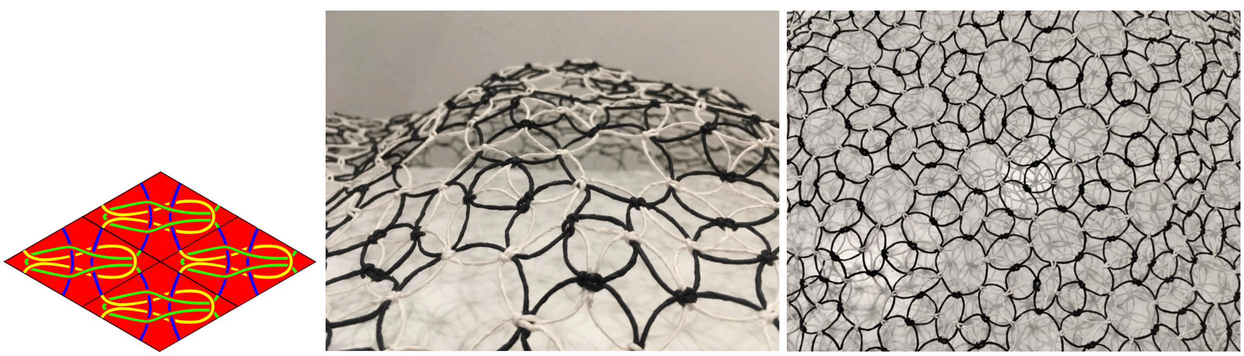
The next notation integrated the P_4-55 into the rhombile tiling, by using identical larger rhombi (Figure 5b), each enclosing four rhombus unit cells in Figure 2b placed in the same orientation, to create a rhombille tiling notation (Figure 5a). Using of this notation to knot black and white paper string was more challenging than the previous notations; it required a numbering system at the start of the knotting process. Again, the knotted piece naturally became three-dimensional (Figure 5c & 5d).

Conclusions

This paper has shown how rhombille tiling notations can be used as tools in a textile knot practice. We have used this method to design three novel two-tone structures, two of which are three-dimensional. These three-dimensional structures occur due to the application of a different type of isohedral tilings into rhombille tiling notations. There is much potential for further work. For example, nine or 12 unit cells of the single type of rhombus tiles can be put together to form a larger rhombus that can then be tiled according to the rhombille tiling rule. The underlying tiling could easily be adapted from the periodic rhombille to the aperiodic Penrose P3 tiling.



(a)



(b) (c) (d)

Figure 5: *(a) Rhombille tiling notation of unit cells of four rhombi (b) knottable into (c)(d) a three-dimensional knotted piece.*

References

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