Curved Islamic Star Patterns of Medieval Egypt and Syria

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Abstract

This paper discusses and illustrates seven of the most interesting and beautiful curved Islamic star patterns, found either on historic monuments in Egypt and Syria or illustrated in Bourgoin's 1879 book, entitled *Arabic Geometrical Pattern and Design* [1], or both. The patterns – some considered masterpieces from the Mamluk period – were found on bronze or wooden doors or carved wooden *minbars* and are comprised of circular arcs, alone or in combination with line segments. Many of these curved patterns also have analogous "twins" constructed only of line segments. We present plausible *Euclidean* constructions of three of the patterns, which may all be categorized as belonging to the *p*4*m*, *p*6*m* or *cmm* crystallographic groups.

Introduction

In geometric Islamic ornament, many planar surfaces tend to be covered by endlessly repeating patterns, most of which feature highly symmetric, intricate star polygon designs. The most commonly found geometric Islamic patterns are comprised solely of straight line segments. However, in the Middle East, and especially during the Mamluk period (13th – 16th centuries), some ornate star patterns were comprised of circular arcs, alone or in combination with line segments. Many of these curved patterns also have analogous "twins" comprised solely of line segments. In *Arabic Geometrical Pattern and Design* [1], Jules Bourgoin, a 19th century French architect, assembled a catalogue of over 200 Islamic patterns found on historic monuments in Cairo and Damascus, and 19 of these patterns include curved stars. This paper will examine a few of Bourgoin's most interesting curved star patterns, in addition to a few patterns that are not found in [1]. We will first look at three of the images not catalogued in [1] that were found by searching online photo archives, such as the *Thesaurus Islamicus Foundation Islamic Art Network Photo Archive* [2] and *Pattern in Islamic Art: The Wade Photo Archive* [3].

The first photograph (a cropped version of an image from [2]) is of the *minbar* (pulpit) of the *Khanqah wa-Masjid* (monastery-mosque of) *Barsbay* in Cairo, Egypt, dating to 1453 (Figure 1, on the following page). Considered a Mamluk masterpiece, the large carved wooden panel is inlaid with ivory. The entire pattern consists solely of circular arcs for the 12-stars (one whole, five halves and one quarter star) and their surrounding 5-stars and octagons. A skeletal version of this pattern (that is, if drawn without "over-under" interlacing) may be classified as *p*4*m* since it exhibits four-fold rotational symmetry with mirror lines in four directions (at 45 degrees to each other).

The second photograph, (a cropped version of catalog number *SYR*0507 [3]), shows a portion of a carved wooden door of the Great Mosque in Aleppo, Syria (either from the Ayyubid or Mamluk periods) (Figure 2). The pattern is also comprised solely of circular arcs, with two 12-stars in the center, towards the top and bottom of the rectangular panel, with each of these stars surrounded by eight adjacent 5-stars. At the corners of the panel are four quarter 8-stars, and at the midpoints of the two vertical edges are two half 8-stars. A skeletal version of this pattern may also be classified as *p*4*m.*

The third photograph, (a cropped version of an image from [2]), shows a bronze door – also considered to be a Mamluk masterpiece – of the Madrasa al-Sultan al-Nasir Hasan, dating to 1363 (Figure 3a). The pattern is comprised solely of circular arcs (except possibly for the two, small 7-gons at the center of the door). It consists of two large 16-stars in the center, towards the top and bottom of the rectangular panel, with each star surrounded by eight adjacent 5-stars. At the corners are four quarter 12-stars, and at the midpoints of the two vertical edges are two half 12-stars. A skeletal version of this pattern (shown in Figure 3b, a cropped version of Plate 131 from [1]) may also be classified as *p*4*m.*

Figure 1.

Figure 2.

Figure 3a.

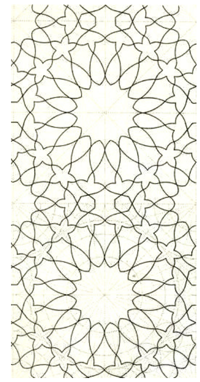


Figure 3b.

How did artisans centuries ago create these beautiful patterns? There are few written records to definitively answer this question although it most likely required a considerable amount of practical knowledge of geometry. Meetings were held between mathematicians and craftsmen starting in the 10th century, and "practical geometry" manuals instructing artisans in how to perform basic geometric constructions were written, so it seems possible that at least some artisans were capable of employing geometric construction techniques to generate patterns. In the next section, we'll consider and present plausible *Euclidean* (compass-and-straightedge) *constructions* (that preclude measurements of any kind) for four patterns representative of the nineteen sketches in [1] that are comprised of curved star patterns.

Bourgoin's Plate 1, 6-Star Designs

Bourgoin's Plate 1 is comprised of two analogous patterns; the one at the top of the plate is constructed only of line segments and consists of *regular* 6-pointed stars surrounded by *regular* hexagons joined vertex-to-vertex, as shown in the author's recreation (Figure 4a). By *regular* we mean that all sides and all interior angles are congruent, respectively. The bottom pattern of Plate 1, constructed entirely of curved 6-pointed stars and curved hexagons formed from arcs, is the "twin" of the top pattern (Figure 4b). An example of this curved 6-star pattern (Figure 4c) may be found in Aleppo as a carved stone lintel with over-under interlacing (Wade's catalog number *SYR*0329 [3]). The skeletal versions of the "twin" patterns in Plate 1 may both be classified as *p*6*m* patterns since the center of the 6-stars serves as a 6-fold rotocenter with multiple mirror axes possible through this point (in three directions, 60 degrees apart).

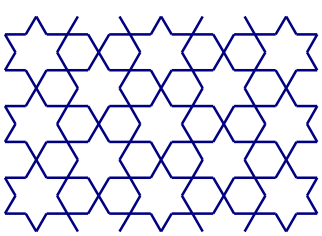


Figure 4a.

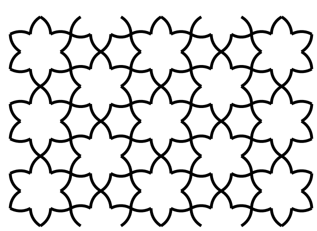


Figure 4b.

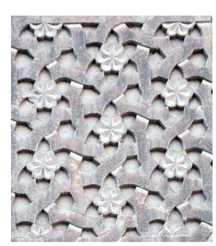


Figure 4c.

To construct the curved 6-star pattern, when using the line segment "twin" as a basis, is very easy. Construct a circle centered at the highlighted point with the radius defined by the given point (shown in Figure 4d). Create five more congruent circles in an identical manner (Figure 4e), highlight the arcs of interest, erase the now no-longer-needed circles and line segments (Figure 4f), which yields the curved 6-star (Figure 4g) with which one can now tesselate the plane to achieve the pattern of Figure 4b.

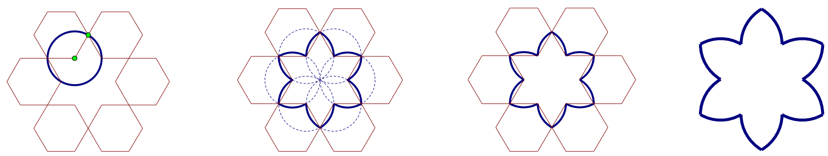


Figure 4d. **Figure 4e.** **Figure 4f.** **Figure 4g.**

Bourgoin's Plates 180 and 181, 10-Star Designs

Bourgoin's Plates 180 and 181 (see Figures 5a and 5b for the author's recreation of them) are also two analogous "twin" patterns. Both consist of *regular* 10-pointed stars surrounded by kite-shaped polygons; the former is constructed only of line segments and the latter is constructed solely of circular arcs. To create the {10/3} star polygons that appear in Plate 180, connect with line segments every third of ten equally-spaced points on a circle. The patterns of Plates 180 and 181 may both be classified as belonging to the *cmm* group since each has horizontal and vertical mirrors with two-fold rotocenters and some centers off the mirror lines. The author was unable to locate extant examples of either of these 10-star patterns in Egypt or Syria, however a modern-day creation of Plate 181 was found in Oman [4] and an example of the pattern found in Plate 180 may be found in Baghdad at the *Madrasah al-Mustansiriya* [5].

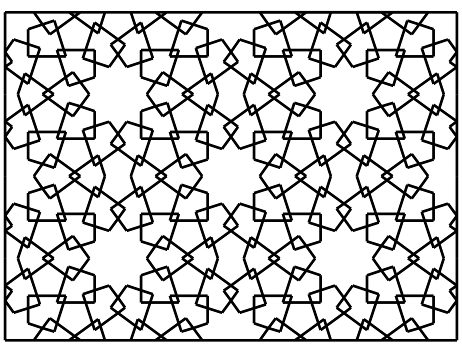


Figure 5a.

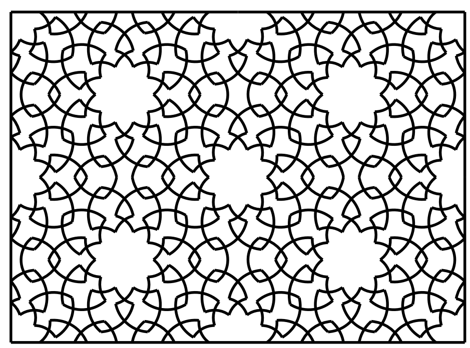


Figure 5b.

The construction of the curved star pattern of Plate 181 is slightly more of a challenge than it was for Plate 1. Starting with a rectangular *repeat unit* for the pattern of Plate 180, construct four line segments between existing points of the pattern, find the two points of intersection between each intersecting pair of segments and use these points as centers for two circles (Figure 5c on the following page). These line segments are no longer needed, so erase them. Next, construct a line segment between two existing points of a kite shape and find the point three-eighths of the way from one end of the segment (by finding multiple midpoints). This point may now be used as the center of one of the two large circles shown in Figure 5d; the other large circle may be constructed in the same way. Highlight the arcs of interest of these four circles to create the main curved *motif* for the 10 star (Figure 5e). Repeat this procedure to create the entire curved 10-star and then erase the circles (Figure 5f). With identical 10-stars centered on the corners of the *repeat unit*, one can create their *petals* in an identical manner as just described (Figure 5g). Then, to construct the remaining arcs needed to complete the *repeat unit*, construct two line segments (one vertical and one horizontal) between existing points and then use their point of intersection as the center of a circle, as shown in Figure 5h. Highlight the arc of interest and erase the circles, and then repeat this process five more times to yield Figure 5i. To construct the last arcs needed, construct a short line segment between existing points, and use its midpoint to construct a circle through an existing point (Figure 5j). Highlight the arc of interest, repeat the process one more time to get the last arc and then erase all the now no-longer-needed points, line segments and circles to yield the *repeat unit* (Figure 5k) which may be used to tesselate the plane and achieve Figure 5b.

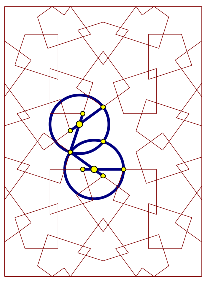


Figure 5c.

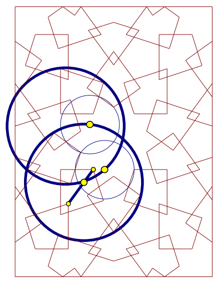


Figure 5d.

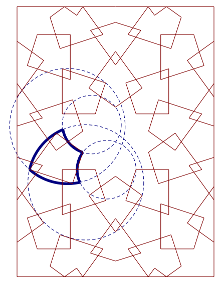


Figure 5e.

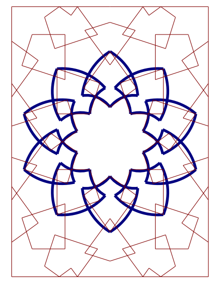


Figure 5f.

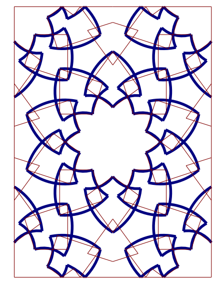


Figure 5g.

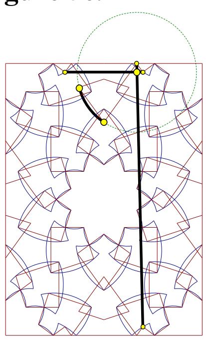


Figure 5h.

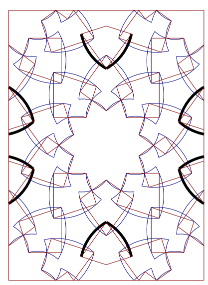


Figure 5i.

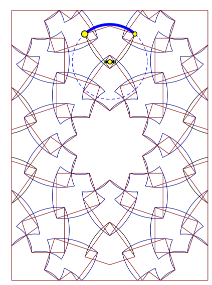


Figure 5j.

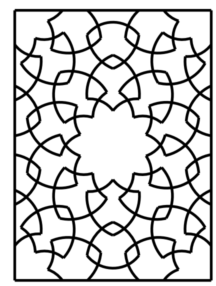


Figure 5k.

Bourgoin's Plates 120 and 121, 9- and 12-Star Designs

Bourgoin's Plates 120 and 121 are another set of analogous "twin" patterns that share the exact same mathematical structure. Both patterns consist of *regular* 12-pointed stars and "nearly" *regular* nine-stars. The polygons of the pattern in Plate 120 are made from straight line segments, thus forming both {12/5} star polygons and {9/3} star figures; while arcs are used to create the pattern of Plate 121. To create the {12/5} star polygons and the {9/3} star figures that appear in Plate 120's pattern, connect with line segments every fifth of twelve equally-spaced points on a circle and every third of nine equally-spaced points on a circle, respectively. A plausible *Euclidean* reconstruction of this pattern may be found in [6]. Both of the patterns from Plates 120 and 121 also appear to be from the Mamluk era and are found on bronze doors – the first at the *zawiya* (shrine) of al-Sultan Baybars II, dating to 1310 (Figure 6a), and the second at the Masjid al-Sultan al-Nasir Hasan, dating to 1363 (Figure 6c) – both are cropped images from [2]. The skeletal versions of both of the twin patterns of Plates 120 and 121 (reconstructed by the author and shown in Figures 6b and 6d) may be classified as *p*6*m*.

Constructing the curved star pattern of Bourgoin's Plate 121 is even more of a challenge than it was for Plate 181. For the first two constructions, the author used the analogous straight-line-segment version of the pattern as an underlying grid. For this pattern, we recreate the pattern "from scratch." To do so, start by constructing a *regular* hexagon and the midpoints of the line segments, as shown in Figure 6e. Also connect opposite midpoints and vertices with line segments and construct a circumscribing circle. Extend the segments emanating from the center until they intersect the circle, and then construct an angle bisector and its point of intersection with the circle (Figure 6f). This point may now be used to construct

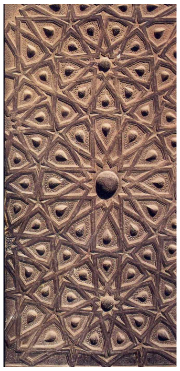


Figure 6a.

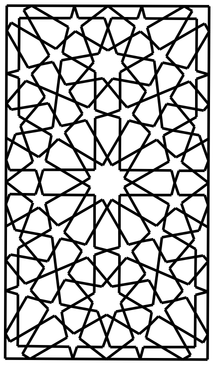


Figure 6b.

Figure 6c.

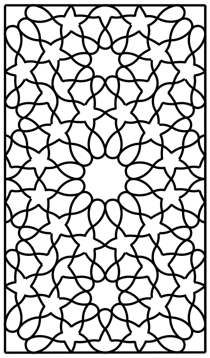


Figure 6d.

a circle using the points shown; then highlight the semicircular arc of interest. Erase the small circle and construct a diagonal line segment through two existing points (Figure 6g). The point of intersection between this segment and the angle bisector may be used to construct a circle through the point shown. Construct two more circles congruent to this one in the same manner and then highlight the arcs of interest to yield one complete point of the curved star (Figure 6h). Construct the remaining 11 star points in the same manner and a circle at the center of the star, as shown in (Figure 6i).

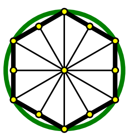


Figure 6e.

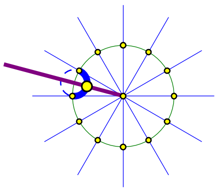


Figure 6f.

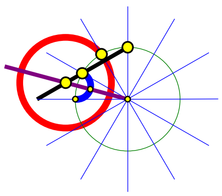


Figure 6g.

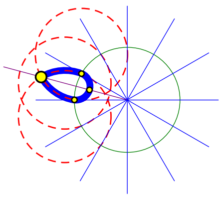


Figure 6h.

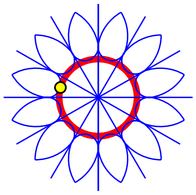


Figure 6i.

To determine a rectangular region containing the entire pattern found on the doors of Figure 6b, extend the vertical line through the center of the star and replicate the circle inscribed within the star four more times in both the upward and downward directions (Figure 6j). Where the outermost vertical circles intersect the vertical line segment, construct two perpendicular (and horizontal) line segments. Then, extend lines diagonally through existing points of the star until they intersect the perpendicular line segments. Connect these points of intersection to form two vertical line segments to form the requisite rectangle. Then extend line segments emanating from the center of the star until they intersect the rectangle's vertical sides. Finally, connect these points of intersection with the corners of the rectangle. To form the 5-stars surrounding the 12-star, start by constructing a circle through the existing points shown, and then highlight the arc of interest (Figure 6k). Repeat this procedure to yield all the arcs of Figure 6l. To create the "arrow" shapes, start by constructing four circles congruent to those used to determine the length of the rectangle (as discussed for Figure 6j) and highlight the arcs of interest (Figure 6m). Construct two more, slightly larger circles through existing points, and highlight the arcs of interest on these as well. Finally, to complete the 5-star, construct a larger circle, highlight the arcs of interest (Figure 6o) and then repeat the procedure to create 11 more 5-stars (Figure 6p). Since there are four quarter 12-stars centered on the rectangle's corners, we may construct the arcs there using the same process (Figure 6q). To create the curved top of the "arrow" shape, construct two "criss-crossing" line segments and use the point of intersection to construct a circle, and then three more congruent tangential circles (Figure 6r). Highlight the arcs of interest and then erase their circles. To create the 9-star, start by constructing another large circle centered on the rectangle that is tangential to an already-existing smaller circle (Figure 6s). Then construct a line segment and another circle centered on the point of intersection between this line and the previously-constructed large circle. Construct two additional circles congruent to this last one and then highlight the arcs of interest to form the outermost part of one petal of the curved

9-star (Figure 6t). Erase the now unneeded circles. To construct the inner portion of the 9-star, construct a circle using the highlighted points and trace the arc of interest (Figure 6u). Repeat these procedures to yield the rest if the 9-star petals (Figure 6v). To complete the “arrow” shape, construct two final circles and a segment and highlight the portions of interest, as shown in Figure 6w. Complete two additional arrows in the same manner (Figure 6x), repeat these procedures to fill in the remaining spaces and the pattern is complete (Figure 6d).

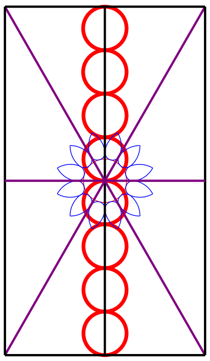
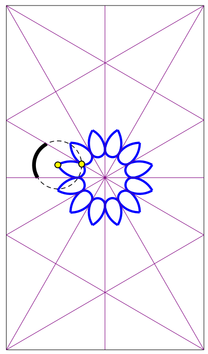


Figure 6j.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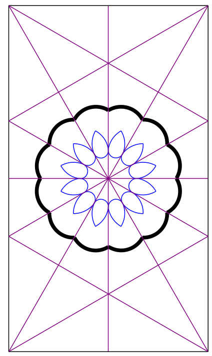
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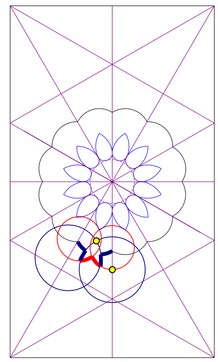
Figure 6m.

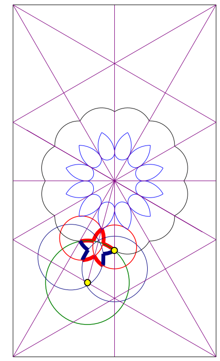
Figure 6n.

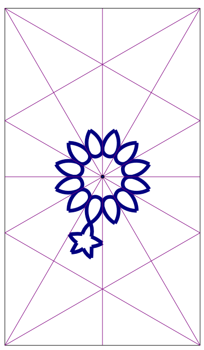
Figure 6o.

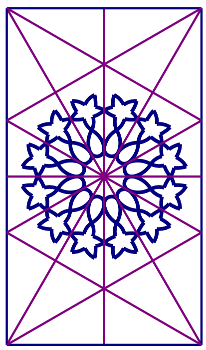
Figure 6p.

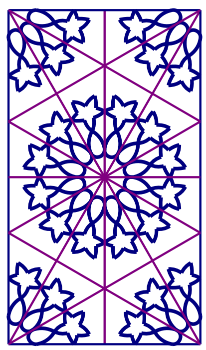
Figure 6q.

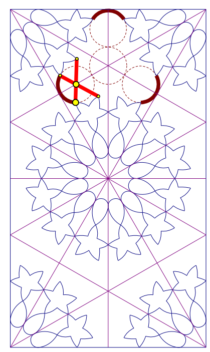
Figure 6r.

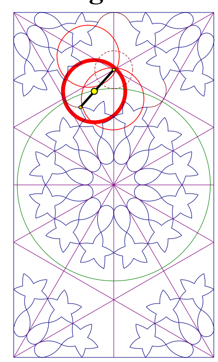
Figure 6s.

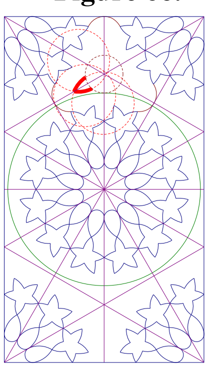
Figure 6t.

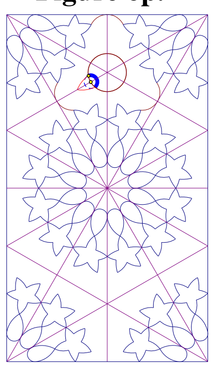
Figure 6u.

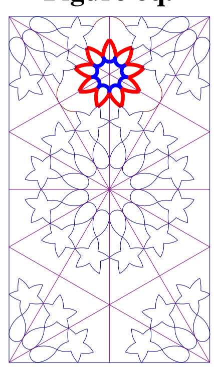
Figure 6v.

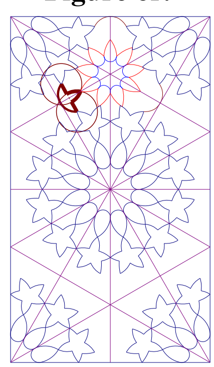
Figure 6w.

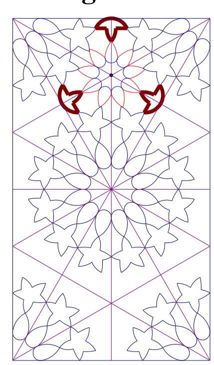
Figure 6x.

Bourgoin’s Plate 135, a 16-, 12- and 7-Star Design

The pattern of Bourgoin’s Plate 135 is an interesting one because it consists of half of a large {16/7} star polygon formed by line segments and a whole curved version of the same 16-star formed from arcs. This curved star is surrounded by large {12/5} and small {7/2} star polygons, both formed from straight line segments. To create {16/7}, {12/5} and {7/2} star polygons connect with line segments every seventh of sixteen equally-spaced points on a circle, every fifth of twelve equally-spaced points on a circle and every second of seven equally-spaced points on a circle, respectively. This pattern may be found on a wooden panel inlaid with ivory and zarnashan (a precious metal inlay in objects made of another metal such as copper [7]) on a *minbar* in the Masjid al-Amir Qijmas al-Ishaqi, dating to 1481, and is also considered a Mamluk masterpiece. See Figures 7a and 7b for a portion and a close up view of the curved 16-star of the minbar, respectively. The author's recreation of the pattern of Plate 135 is given in Figure 7c. The pattern may be classified as *p*4*m*.


Figure 7a.


Figure 7b.

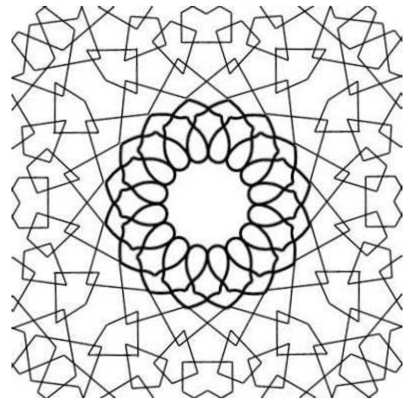
Figure 7c.

Due to constraints on the size of this paper, the author will only discuss the construction of the curved 16-star of Bourgoin's Plate 135. Start by constructing a square, its two diagonals, its center point and the two segments connecting opposite midpoints of the square's edges (Figure 7d). Construct an inscribed circle centered on the square's center and a line segment connecting this center with the point of intersection of the circle and one diagonal; and then construct this segment's midpoint. Next, construct a second circle centered on the square's center and through this midpoint, as shown in Figure 7e. Then construct an angle bisector and its point of intersection with the smaller circle. Construct a second angle bisector and its point of intersection with the smaller circle, which will become the center of an even smaller circle, shown in Figure 7f. Retain half of this circle and repeat this procedure to obtain the inner arcs of the star, as shown in Figure 7g. Also retain the three line segments and two points shown. Now, construct a vertical line segment connecting these two points, a point of intersection and a circle centered on this point of intersection, as shown in Figure 7h. In a similar manner, construct a second congruent circle, highlight the arcs of interest on these two intersecting circles to yield one of the star's points (Figure 7i).

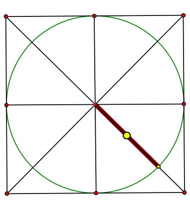
Figure 7d.

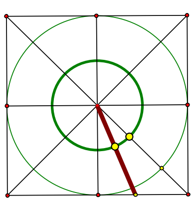
Figure 7e.

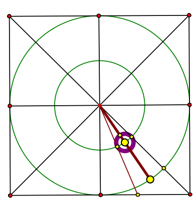
Figure 7f.

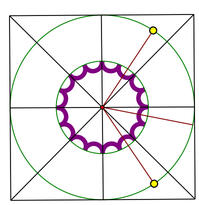
Figure 7g.

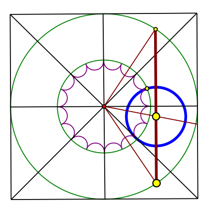
Figure 7h.

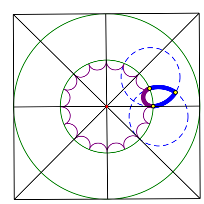
Figure 7i.

Repeat the process and erase the circles to complete all of the curved 16-star's points (Figure 7j). To create the petals and arrow shapes in the remaining space, construct three line segments emanating from the square's center and through existing points of the stars until they intersect the edges of the square; now connect two of these points with a line segment in order to form a third segment of a triangle and point of intersection, as shown in Figure 7k. Using the square's center and this intersection point, construct a circle concentric with the first (inscribed) circle constructed. Construct two small congruent circles centered on the circle just constructed at the points of intersection shown (Figure 7l). Construct a circle circumscribing the original square and one of its radii through an existing point until it intersects this circle (Figure 7m). Construct another circle centered on this intersection point and through existing points (Figure 7n). Highlight the arcs of interest on these three intersecting circles and hide the circles to yield the arcs forming the star's petals; repeat this all the way around the 16-star (Figures 7o and 7p). All that remains to be done is the construction of the "arrow" shapes. So, construct two more large circles centered on and through existing points (Figure 7q), highlight the arcs of interest (Figure 7r), and repeat this procedure to complete the curved 16-star shown in Figure 7s. The framework of the pattern in Bourgoin's Plate 135, consisting of straight line segments with the curved 16-star at the center (Figure 7t).

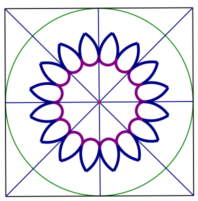

Figure 7j.

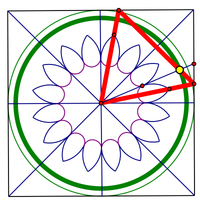

Figure 7k.

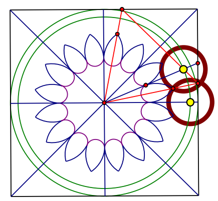

Figure 7l.

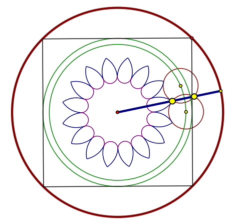

Figure 7m.

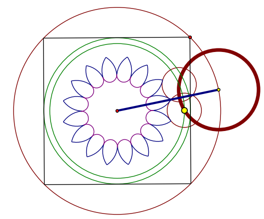

Figure 7n.

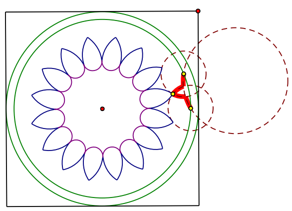

Figure 7o.

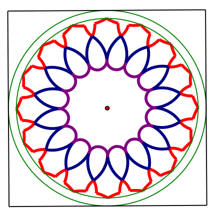

Figure 7p.

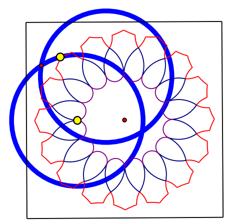

Figure 7q.

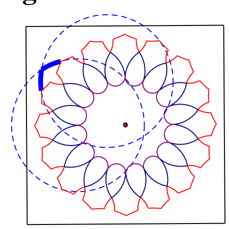

Figure 7r.

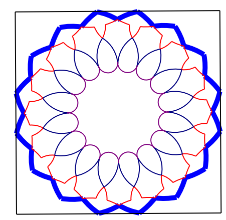

Figure 7s.

Discussion

This paper discussed seven interesting and beautiful curved Islamic star patterns, a few of which were were found on bronze doors and carved wooden *minbars* and are considered Mamluk masterpieces. Of the extant patterns found on historic monuments in Egypt and Syria, three were not documented in [1] and three were, two of which had analogous "twins." The designs discussed in this paper contained curved n-pointed stars with n = 5, 6, 9, 10, 12 and 16, but [1] also documents curved 8-star patterns as well. The one pattern of Plate 181 that the author has not yet been able to locate in Egypt and Syria, does appear as a mosaic in the Sultan Qaboos Grand Mosque (of Muscat, Oman) built in 2001. Of the four patterns reconstructed in this paper using *Euclidean* "point-joining" techniques, two (Bourgoin's Plates 1 and 181) relied on an underlying grid of line segments and two (Bourgoin's Plates 121 and 135) were reconstructed "from scratch." The patterns may all be categorized as belonging to either the *p*4*m*, *p*6*m* or *cmm* crystallographic groups.

References

[1] Bourgoin, J. (1973). *Arabic Geometrical Pattern and Design,* Dover Publications. Original 1879.

[2] Thesaurus Islamicus Foundation Islamic Art Network Photo Archive, available at http://www.islamic-art.org/PhotoArchive/.

[3] Pattern in Islamic Art: The Wade Photo Archive, available at http://www.patternsinislamicart.com

[4] http://renaissancetours.com.au/?attachment_id=1701

[5] https://www.google.com/search?q=al-mustansiriya

[6] Bodner, B. L. "From Sultaniyeh to *Tashkent Scrolls*: Euclidean Constructions of Two Nine- and Twelve-Pointed Interlocking Star Polygon Designs," in Nexus Network Journal, Vol.14 No.2 (autumn 2012).

[7] http://www.elazhar.com/mosque_eu/61.asp