

# Baskets for the Mathematics Classroom

S. Louise Gould  
Department of Mathematical Sciences  
Central Connecticut State University  
1615 Stanley Street  
New Britain, CT, 06050, USA  
E-mail: goulds@ccsu.edu

## Abstract

The special  $45^\circ$ - $45^\circ$ - $90^\circ$  and  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles appear many places in the school curriculum. The baskets described in this paper are good illustrations of the importance of these special triangles, which occur naturally in weaving because of the symmetries enforced by the weave. The baskets also incorporate examples of the relationships between certain solid angles and the angles in the sides that meet to form it. Weavers from cultures throughout the world have used the two and three directional plaiting techniques described. This paper provides detailed descriptions of the structure of these baskets and suggests experiments that students and teachers might do with them in the classroom.

## Introduction

Some students in secondary school mathematics classes notice that they spend a lot of time working with the special  $45^\circ$ - $45^\circ$ - $90^\circ$  and  $30^\circ$ - $60^\circ$ - $90^\circ$  right triangles. It seems to them that the teacher uses the special triangles because they produce convenient, uncomplicated numeric results rather than because of their intrinsic value. These triangles are, in fact, very important in the real world because they are the ones that occur in connection with the periodic symmetries of the plane. The craft of weaving provides a dramatic practical example of this phenomenon, and basket weaving, in particular, leads to some 3-dimensional consequences of the occurrence of these triangles in particular configurations. Baskets where these angles arise very naturally have been used by many cultures throughout history (see Rossbach [5], Glashausser and Westfall [2], Laky [3] and Pendergrast [4]).

Rossbach observes:

Baskets have been called the most destructible of all textile constructions. Unlike ceramics, few ancient baskets have managed to survive. Yet through the persistence of tradition, through the extreme conservatism which has made basketry virtually changeless, baskets have withstood all the destructive forces to which they are so subject: moisture, heat, fire, mold, insects, wear. Although the life span of an individual basket is very short, another basket takes its place. Baskets have been replaced, over and over and over, unmodified, unimproved, unchanged. So, through the centuries, baskets have shown a remarkable endurance. See Rossbach [5].

Thus there is a rich cultural history of basket making. Basket makers use a wide variety of techniques including twining, rib structures, plain weave and twill, and three-rod or four-rod wale. They also use diverse materials including bamboo, grasses, reed, cane, bark, wood splints, cardboard, plastic and metal. Basket makers learn from ancient traditions and employ whatever material is at hand.

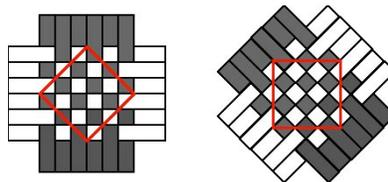
**Square and Hexagonal Baskets.** This paper focuses on two particular kinds of baskets. The weave structure of the base determines the shape of each of these baskets. The structure established on the base continues up the sides of the basket. One of these containers is a basket with a square base that is circular at the top. The other basket is hexagonal at the base and circular at the top (see Figure 1). Westfall’s articles in *Shuttle Spindle & Dyepot* [7 and 8] inspired experiments with using these baskets in the mathematics classroom. The baskets in Figure 1 are made of readily available materials. A number of manufacturers make very thick, easy-to-stitch interfacing that has heat activated fusible material on both faces typically used to make hats, cloth bowls and boxes. Students can adhere fabric to both sides of this interfacing, and then use a rotary cutter to cut strips of uniform width. It is best to secure both edges of the strips with a zigzag stitch to keep the edges from fraying during the weaving. The handle of the square to round basket in Figure 1 is clothesline rope covered with fabric. It is fastened to the basket with hand stitches that attach it to the sides in Archimedean spirals.



**Figure 1:** A square to round basket (left), and a hexagonal to round basket (right).

### Basket Construction

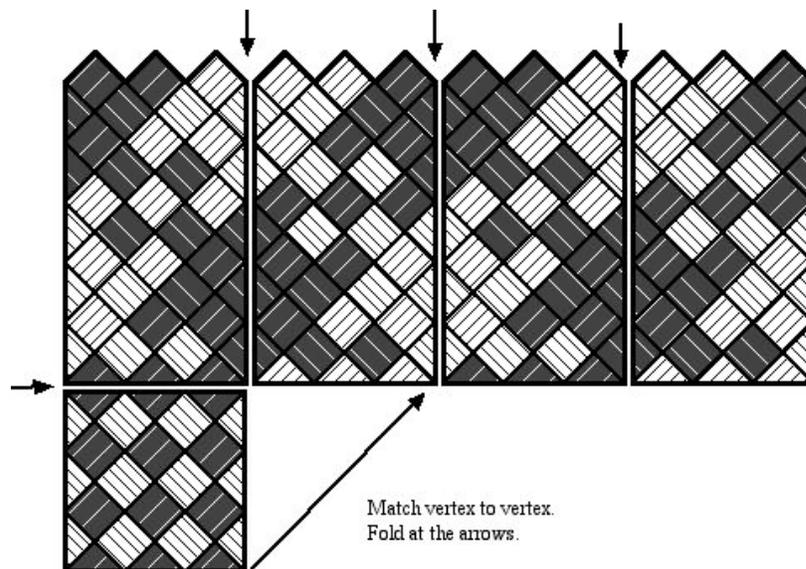
**Square to Round Basket Construction.** The basket with a square base has a plain weave with two layers throughout. Usually all of the ribbon-like strips are of uniform width and thickness. The strips can be different colors in either one or both directions. This is called “color and weave” technique. In Figure 2 all the strips in one direction are black and the strips in the other direction are white. A square base with the same even number of strips in two perpendicular directions is then folded from midpoint of one side to the midpoint of the next consecutive side (see the fold lines in Figure 2). These folds form another square that appears to be woven on the diagonal. Figure 2 shows the square base as woven and then rotated with the weaving appearing to be along the diagonal.



**Figure 2:** The square basket base as woven (left) and rotated (right).

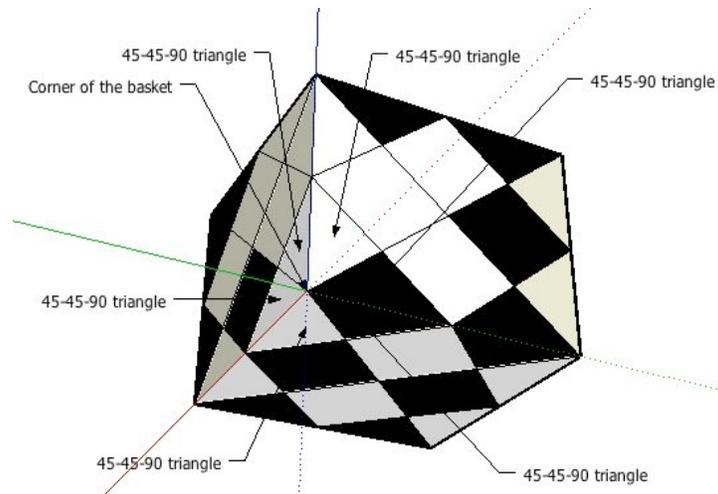
The original set-up of the strips determines if this is a “right-handed” or “left-handed” basket. Unless the weaver uses different coloring patterns in one or both directions this handedness does not influence the outcome of the basket until the construction of the saw-toothed edge at the top. Depending upon where the weaving stops it may be necessary to work clockwise or counter clockwise to finish the top conveniently.

The sides of the basket are perpendicular to the bottom with the strips from the bottom continuing diagonally up the sides in a natural fashion and to an arbitrary height without requiring the addition of any new strips or the termination of any strips. Some strips are parallel to each other on the bottom but become perpendicular to each other as the bottom folds to start the sides. Figure 3 shows the sides and base of a basket with white strips in one direction on the base and black strips in the perpendicular direction. The lines on the strips indicate the lengthwise direction of the strips. The reader can make a working model of the basket by cutting out the diagram below and taping it together. The basket pictured is 5 rounds tall (not including the saw-tooth edge at the top). The basket weaver can decide how tall the basket will be to the nearest  $\frac{1}{2}$  of the diagonal of a square by choosing to weave a different number of rounds.



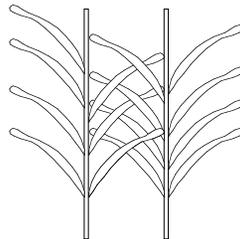
**Figure 3:** A “cut and tape” basket that shows all sides and the base. The lines on the strips indicate the direction the strips travel.

The natural tendency of the weaving material to remain straight causes the top of the basket to gradually become round, but the basket begins as a cube. Why is that? A close inspection of the angles at one corner vertex tells the tale. Figure 4 shows three planes that form the corner of the basket. Touching this corner are six  $45^\circ$ - $45^\circ$ - $90^\circ$  triangles that pair-wise form three right triangles. The angles around the vertex add up to  $270^\circ$  so that a  $90^\circ$  angle is removed from  $360^\circ$  giving us the required corner. The parallel strips on the bottom turn to become perpendicular to each other as they weave up the sides.



**Figure 4:** *The angles at the base of a square basket.*

**Examples of Square Baskets from the Literature.** There are a number of interesting examples of baskets constructed using similar strategies. Members of the Navajo School of Indian Basketry showed an interesting variation of this basket using date palms in their 1903 book on Indian Basket Weaving [6]. Placing four fronds in the ground as the corners of a square, they would use plain weave to weave the adjacent leaves with each other. After several rounds they would trim the leaves off the top of the stalk and turn the direction of the leaves to weave the bottom of the basket. They would trim the tops of the stalks to form feet for the basket. They would then upend the basket on its new feet, trim the leaves at the other end to enable them to weave the other ends of the leaves to form a graceful point at each side of the top of the basket (see Figure 5).



**Figure 5:** *Fronds woven between stalks to start a square to round basket.*

An exhibit in the Barbados Museum shows a basket (more than 4 feet tall). The basket consists of an inverted outer basket to keep flies out and an upright inner one to filter vegetable matter out of the mashed sugar cane. Current Barbadian weavers have used this museum basket to design a small model of a basket inside of a basket (see Figure 6 ) for the tourist trade. It consists of an inverted basket 8 ½ inches tall with a similar smaller upright basket inside such that the square base of the smaller basket is tangent to the circular “top” of the larger basket. The smaller basket has twine attached to opposite ends of a diameter of its circular top. The twine passes through opposite vertices of the square “bottom” of the outside basket so that it may hang from a hook. For decorative purposes, the basket weavers use colored palm strips and do their weaving using twill designs rather than plain weave. These nested baskets are in the collection of the author.



**Figure 6:** Two views of the basket inside of a basket. The inner basket is held in place by twine on diametrically opposed positions of the circular top that passes through opposite corners of the square “base” of the outside basket.

**Basket with a Hexagonal Base.** Instead of using two sets of strips as in the square base basket, the hexagonal basket uses three separate groups of strips woven at 60° angles to each other. Each strip passes over two and under one of the strips from a second direction and under one and over two of the strips of the third direction. Thus this basket is three layers throughout. It requires significant concentration to keep track of which strip is on top.

Pendergrast [4] calls this weave three-directional plaiting, plaited closely and reports that it is called *anyam gila* the mad weave in Malaysia. He relays a story told by a Mrs. L. E. Bland in 1906 that female prisoners in the Singapore prison were made to learn this weave and they called it the mad weave because it was difficult to learn and was “quite calculated to drive a beginner mad.” Pendergrast shows an example of a tupa (winnowing trace) worked in this weave from Boti village, West Timor, Indonesia. He also shows a specimen from his collection that is an urn cover. Shaped like a hexagonal tower it has woven protruding decorations at each level and a three-directional plaited bird at the top complete with tail and comb.

While the photographs in Figure 7 a-c appear to be of a different basket than the one in Figure 8 c, they are not. The strips on this basket are three colors on one side and a three different colors on the other side so that what you see in Figure 8 c is the reverse side of Figure 8 a-b after the sides are turned up to form the hexagonal bottom and the six lateral sides.



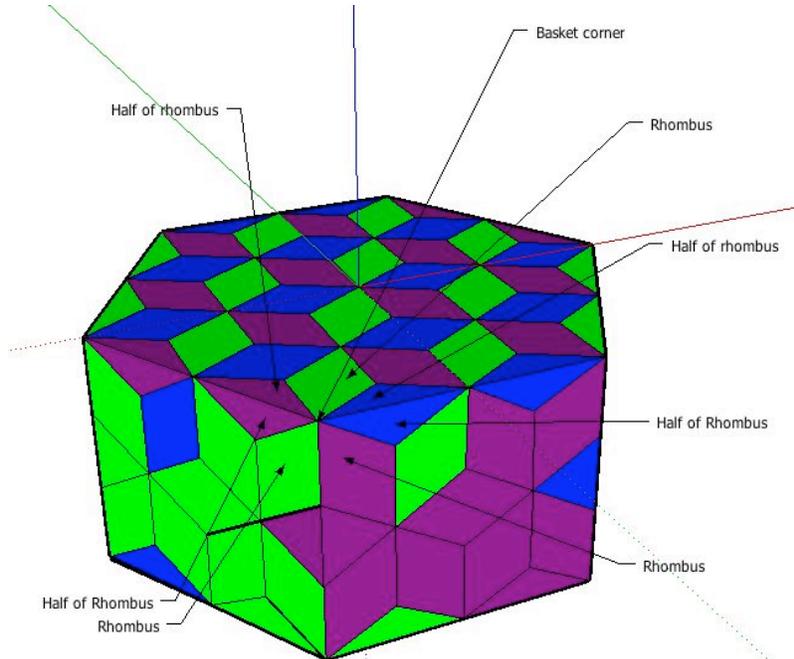
**Figure 7 a-c:** The first three steps in three directional weaving.



**Figure 8 a-c:** *The weaving continues towards the completion of the bottom of a hexagonal basket*

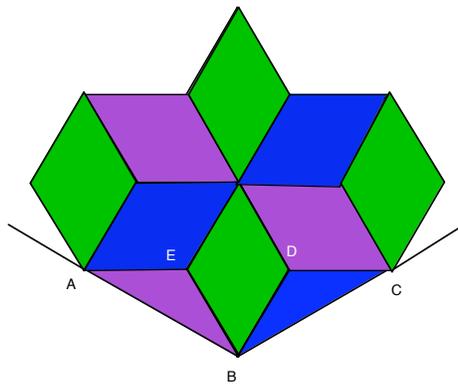
This weave requires the same multiple of four strips for each of the three directions. Since an even number of strips is perpendicular to the base at each side of the hexagonal base and half of the strips from each neighboring side must become the strips to weave at  $30^\circ$  for the lateral sides, there must be a minimum of four strips in each direction. If there are more than four strips on a side,  $2n$  for example, then each adjacent side of the hexagonal base must contribute half that number or  $n$  strips to weave at  $30^\circ$  angles. Thus there are  $2n+n+n=4n$  strips in each direction for some  $n$ . When the weave is done correctly you should see a pattern that looks like the quilt pattern *baby blocks* made up of three rhombi at each vertex forming a tessellation of hexagons on each side of the bottom of the basket.

How is it possible to have a hexagonal shape for the bottom of the basket and rectangular sides to the basket that are perpendicular to the plane of the bottom and at a  $120^\circ$  angle to each other (see Figure 9)? The lateral faces have corner angles consisting of a  $60^\circ$  angle from a rhombus and a  $30^\circ$  angle from an adjacent  $30^\circ$ - $120^\circ$ - $30^\circ$  triangle. This accounts for the right angles at the corners of the lateral faces.



**Figure 9:** *One corner of the base of a hexagonal basket.*

Figure 10 shows just the top view of that corner if the basket were upside down. Finding that  $m\angle ABE=30^\circ$ ,  $m\angle EBD=60^\circ$ ,  $m\angle DBC=30^\circ$ , and  $m\angle ABC=120^\circ$  explains the construction.



**Figure 10:** *One of the angles of the base of the hexagonal basket.*

When we drew the model for the square basket it was possible to cut out Figure 3 and fold it into a model. A square was removed in the lower right hand corner to make this possible. A possible exercise for a class is to draw a diagram for a model of the hexagonal basket that could be folded and taped to make a model of the hexagonal basket. Students can determine the measure of the angle that must be removed to make the lateral sides meet each other at a dihedral angle of  $120^\circ$  and permit them to be perpendicular to the plane of the base of the basket.

**Historical Models of the Hexagonal Basket.** In addition to the *anyam gila* previously described, examples of three directional weave are frequently found in baskets that have the strips spaced far apart. The weaving in these baskets has a different over and under pattern than the three directional plaiting closely plaited. The Shelburne Museum in Burlington Vermont has a Shaker basket in three directional weave (see Figure 11).



**Figure 11:** *Two views of a Shaker basket with open worked three directional weave.*

Many Asian people still use such baskets for carrying poultry to market, see [4]. Native American tribes historically used them for washing and draining laundry, see [2].

Gerdes [1] describes a similar hexagonal weave used by people in Madagascar, Cameroon and Zaire to make fish traps and transport baskets. Typically these weaves begin along the rim of the top of the basket. Long strips are wrapped around a circular rim at  $60^\circ$  angles to the rim to form two strips to weave at  $120^\circ$  angles to each other. The strips in the third direction are parallel to the plane of the rim. This hexagonal weave is open and quite rigid because of the direction of the weaves at each intersection. See [1] pages 112-114.

## Conclusion

Baskets deriving their shape directly from their geometry can provide interesting explorations for the classroom. Students can try their hand at designing and making the baskets. Students can even use such cost-free materials as strips cut and folded from recycled brown paper bags. They can explore how the width and number of strands affect the volume of a basket. They can compute the surface area of a basket to estimate how much material is necessary to make the basket. Some might even begin to see that basket weaving has a mathematics of its own.

Another topic for students to explore is the coloring of the strips. Gerdes [1] has detailed a number of textile and basketry weaving schemes using two directional coloring patterns. Coloring patterns in three directions are much more complex. The development of a Java applet that permits color sampling for virtual or cut and tape hexagonal baskets would be a useful tool for exploration.

## References

- [1] P. Gerdes, *Geometry from Africa: Mathematical and Educational Explorations*. Washington DC: The Mathematical Association of America, 1999.
- [2] S. Glashausser and C. Westrall, *Plaiting Step-by-Step*, New York, Watson-Guption Publications, pp. 13-33. 1976.
- [3] G. Laky, Plaiting: Non-Loom Weaving. *Shuttle Spindle & Dyepot*. Issue XII No. 2 West Hartford CT: Handweavers Guild of America, Inc., pp. 33-35. 1981.
- [4] M. Pendergrast, *Māori fibre techniques: Katahi hei tama tā tama*. New Zealand: Reed Publishing, pp. 66-71. 2005.
- [5] E. Rossback, *The Nature of Basketry*. West Chester, PA: Schiffer Publishing Co, pp. 10-11,134-140. 1986.
- [6] The Navajo School of Indian Basketry, *Indian Basket Weaving*. New York, NY: Dover Publications, Inc., 1971. This is an unabridged republication of the work originally published by Whedon & Spreng Co. Los Angeles in 1903.
- [7] C. Westfall, Plaiting with Recycled Materials. *Shuttle Spindle & Dyepot*. Issue VIII No. 4. West Hartford CT: Handweavers Guild of America, Inc., pp. 87-89. 1977.
- [8] C. Westfall, Hexagonal Plaiting. *Shuttle Spindle & Dyepot*. Issue IX No. 4. West Hartford CT: Handweavers Guild of America, Inc., pp. 40-41. 1978.

**Illustration credits.** All photographs and illustrations are the author's originals. The two-dimensional representations of three-dimensional objects were produced using Google Sketch-up.