aMazing Mathematical 3D Modeling

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Abstract

We use the fun of the experience and challenge of solving mazes as motivation in a workshop that teaches 3D modeling, augmented reality and 3D printing skills. The workshop was tested both on-line and offline at the ArsElectronica outside in Kepler's garden. Participants experience the geometrical attributes of mazes to develop an understanding of mathematical concepts such as mirroring, rotation, translation and certain basics of algorithms. The mazes created in this workshop can be solved, shared and experienced using mobile devices.

Labyrinths and Mazes

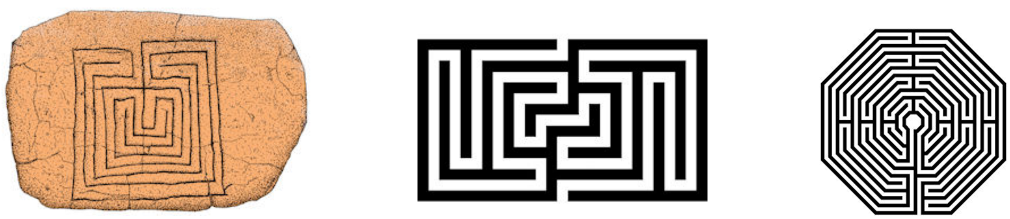
Mazes and labyrinths are architecture, art, as decorations, and gardening features, and can be motivational fun experiences in mathematics. We developed a workshop around this idea which was tested with participants aged five to 65 and above at the *ArsElectronica* festival 2020. The workshop was developed as an online as well as offline experience for the festival 2020 during the COVID-19 pandemic. Participants are invited to create and solve mazes using 3D modeling and augmented reality (AR) to explore their logical, mathematical, and design attributes. This workshop provides the opportunity to create mazes and labyrinths based on mathematical features and solve them at home. *GeoGebra* is used as a basis to share and experience the created work.

Classical labyrinths date back about 4000 years and usually have one path, in contrast to mazes, which have several paths from which to choose. Labyrinths also found their way into mythology, serving as a challenge that is very hard to solve. Roman labyrinths and mazes started to appear from 200 BC and can be found throughout the roman empire in buildings and on floors as artful decoration. Labyrinthic decorations sometimes grace houses in Greece and mazes are part of gardening motifs such as in Schönbrunn in Austria [2]. Labyrinths and mazes are usually created based on mathematical principles and algorithmic rules. Previous Bridges papers indicate that the creation of labyrinths and mazes can contain many opportunities to experience computational thinking, combinatorics, reasoning and explore results of geometrical operations [5]. It can be educational to experience various geometrical features such as mirroring, rotations, multiplications of a feature, and scaling. This aims to train skills of abstract and logical thinking, computational thinking, and observing mathematical concepts found in mazes. In addition, a new experience such as solving a maze can also be added [1].

Steps to Create Mazes

Exploring mazes in artwork can help understand mathematical concepts and can also inspire a participant's own ideas about labyrinths and mazes. Mazes can be created by principles such as mirroring, translation, symmetries, or rules like ‘always turning right’. This can train algebraic thinking as it can be used to assemble fractured parts and deductive and spatial reasoning by addressing the challenge in two as well as in three dimensions [3]. The workshop consists of four steps on how to explore the mathematical features of a maze using various materials and perspectives. It needs pen and paper, *GeoGebra* and a mobile device.

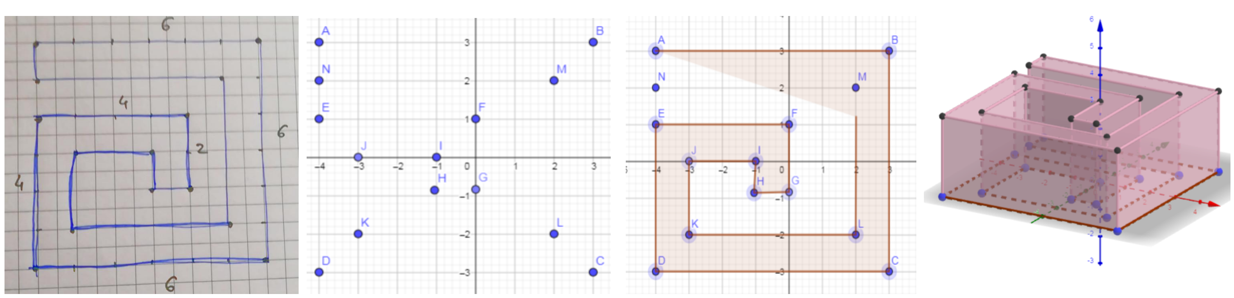
Step one is to invite participants to observe several representations of mazes that can be found on art and on architecture. A great source of inspiration is the collection of pictures and descriptions found throughout the internet or at *labyrinthos.net* [4]. The participants should observe, investigate, and formulate what they believe the mathematical basis of the mazes could be. For example, participants could see the principle ‘cross and four corners’ as described by Thompson and Cheng [5] in Figure 1(a) and could observe that the pattern in Figure 1(b) is symmetrical, while the one in Figure 1(c) is not entirely symmetrical but has a short way, then a long way, then a short again and so on. Based on these observations, participants should think about their personal labyrinth or maze and on which principles they would like to base it on.



(a) (b) (c)

Figure 1: *(a) Prehistoric labyrinth from Greece (b), pattern on the Belfast Cathedral, (c) labyrinth on the Arras Cathedral for step one for participants creating their personal maze.*

Step two involves drawing a maze or labyrinth using pen and paper as can be seen in Figure 2(a). Using a squared sheet of paper and a pen, the geometrical attributes of a maze or ‘maze-like architecture’, a geometrical idea or a repeating element can form the basis of a drawn maze a participant chooses to create. Physically drawing the maze idea can allow participants to explore their imagination and play with the impressions from art they observed. In our example, the maze has the shape of an incomplete square by concatenating a wall by a 90° angle counter-clockwise, adding 2 units for every second wall placed and subtracting two units at the entrance wall, see Figure 2(a). The drafts should follow the rule that a maze has one entry and should consist of one single object. This object will be turned into a three-dimensional object. As all 3D objects have a thickness, it needs an outer shell and can not be just a line. While in AR this might not be necessary due to its virtual nature, this produces important information when modeling something for 3D printing. Another important purpose of drawing the maze idea is the prototypical nature of the activity. A participant can try out whether the idea is working and can test their geometrical ideas without technology by drawing a line or following the path with a finger.



(a) (b) (c) (d)

Figure 2: *(a) A drawing of a labyrinth following certain rules, (b) dots in GeoGebra connected by (c) the prism tool and (d) the prism extruded to 3D as step two and three.*

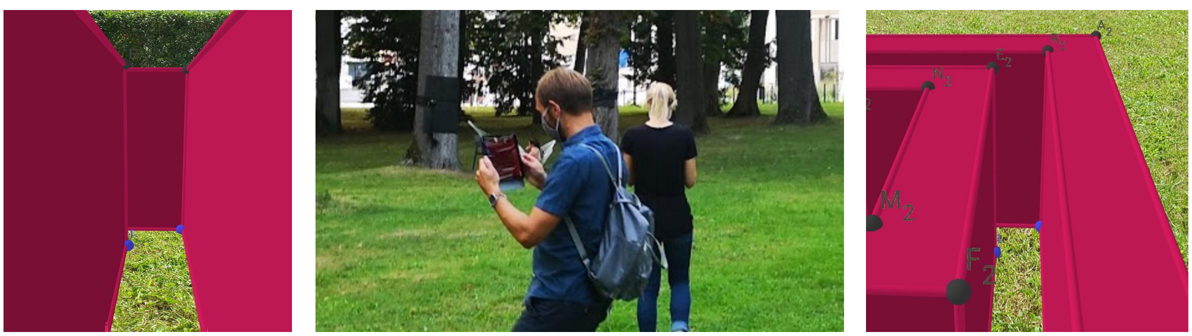
The third step is to create the maze idea as a 3D model in *GeoGebra* requires drawing all corners and then creating a polygon that connects all corners by edges as shown in Figures 2(b) and 2(c) and then extruding the model as seen in Figure 2(d). The participants are asked to reproduce the steps described in a *GeoGebra* resource [6] to create the 3D model of their labyrinth or maze as seen in Figure 2(b)-2(d).

GeoGebra has to be opened in the classical view and the points need to be drawn using the point tool. Depending on the maze's complexity, these dots need to be carefully added as still seeing a participant's maze in a cloud of dots can be challenging. Next, the polygon tool is selected to connect these dots. To create an edge that can work as an outer border of a 3D model, the corners have to be clicked one by one until the first corner completes the polygon in the order that resembles the maze which is described step by step at the *GeoGebra* resource.

After creating a 2D polygon maze, the polygon can be extruded to a three-dimensional model. After completing the 3D modeling, the model can be saved and the privacy settings can be set to public. The resource can then be shared and used by all other *GeoGebra* users and, thus, the workshop participants. The maze can be experienced on most smartphones after having installed the *GeoGebra 3D* app.

Going 3D: Solving the Maze in AR Mode and More

As a final step in the workshop, participants that installed the *GeoGebra 3D* app on their mobile device as described in a *GeoGebra* resource [7] can open their created *GeoGebra* labyrinth or maze, start AR mode, and walk through their virtual maze (Figure 3). Tablets provide a bigger screen but smartphones can also be used. The height and width of the walls can be adapted by manipulating the size of the walls. This can directly show the impact of manipulating vectors while scaling such that the path is big enough to walk through. This way, the participants can observe the influence of the choice of vector lengths and thus wall sizes on the resulting object.



(a) (b) (c)

Figure 3: *(a) and (c) a maze through a mobile device, (b) participants solving an AR maze as step four.*

Using the tablet and walking through the AR maze can serve for testing the physical impact of virtual ideas which helps to identify areas that could be improved. Participants can then alter their maze in *GeoGebra* and test it using AR again, determining whether they appreciate the changes. This can be a motivational aspect for a participant to engage directly in the mathematics behind the mazes. Moreover, it will help the participants to understand possible weaknesses of a 3D printable model. It moves the perspective of a participant directly into the math, making geometry experienceable.

As an optional step for live workshops, the created 3D model can also be downloaded from *GeoGebra* for 3D printing and can be created with a 3D printer. Changing the height of the maze offers the opportunity to investigate the differences in the maze's production time and experiencing the implications of volume changes towards or against the printing time exploring vectors and scaling, connected to the scaling for the AR exploration of the maze. The 3D print can add several more attributes for exploring the mazes such as height, weight, texture, and more. Moreover, 3D geometry attributes can be experienced using this approach, which is an addition to the geometrical and virtual mathematical attributes. Senses such as touch or smell can also be helpful in building up an understanding of the mathematics responsible for the maze's geometry. Later, it can be used as a memento from the experience or as for example personal jewellery.

Case Study of a 'five-year-old' Egyptian Explorer

As this activity seems to be very intriguing for young participants as discovered during the workshops, we highlight the work of a 'five-year-old' explorer. He created the 2D maze in *GeoGebra* after some picture inspiration about mazes. Second, he extruded the 2D maze into a 3D version and downloaded the model with the help of his mother. Since he did not have a 3D printer, we created the maze remotely and he was able to observe the printing process as well as the finished object that is shown in Figure 4.

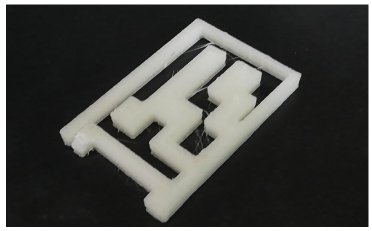


Figure 4: *The printed maze of a five year old explorer from Egypt.*

Third, he solved the maze himself alone using AR mode on the smartphone of his mother. He was exceedingly pleased during the experience of solving it and excited when he managed to solve the maze himself. His mother observed that he also had grasped the concept of mazes since he started to create mazes out of building blocks days after the workshop.

Summary and Conclusions

Participants of the workshop investigate geometrical and other attributes of labyrinths and mazes by 3D modeling them and using several materials in 2D as well as 3D. These models can be a possible tool for investigating the mathematical aspects of mazes and as a quick step into 3D printing.

Following the activities of the 'five-year-old' explorer, being able to create mazes using building blocks adds another perspective for younger participants since no technical knowledge is needed to create the maze. As the workshop seems to be a fun activity especially in early childhood, a next step could be to test the setting with this age group in a task based design environment to investigate in depth. It would be interesting to see which conclusions a student can draw from creating, investigating and solving a maze using AR and 3D printing. It would be interesting to compare drawing a maze with pen and paper and creating it using building blocks at the start of the workshop.

References

[1] K. Fenyvesi, S. Jablan, L. Radovic. "Following the Footsteps of Daedalus: Labyrinth Studies Meets Visual Mathematics." *Bridges Conference Proceedings*, Enschede, The Netherlands, 2013, pp 361-484. http://archive.bridgesmathart.org/2013/bridges2013-361.pdf.

[2] H. Kern. "Through the Labyrinth: Design and Meanings over 5,000 years." English Edition by editors Robert Ferré & Jeff Saward, *Prestel*, 2000.

[3] D. Lieban and Z. Lavicza. "Dissecting a Cube as a Teaching Strategy for Enhancing Students' Spatial Reasoning: Combining Physical and Digital Resources." *Bridges Conference Proceedings*, July 16-20, 2019, pp 319-334. https://archive.bridgesmathart.org/2019/bridges2019-319.pdf.

[4] J. Saward. "Mazes or Labyrinths… What's the difference & what types are there?" http://www.labyrinthos.net, 2017.

[5] D. Thompson and D. Cheng. "Square Seeds and Round Paths: Exploring Patterns within the Art of Classical Labyrinths" *Bridges Conference Proceedings*, Phoenix, USA, July 29-Aug 1, 2015, pp. 555–548. http://archive.bridgesmathart.org/2015/bridges2015-555.html.

[6] E. Ulbrich: "Creating a Maze". https://www.geogebra.org/m/s3v96xcz, 2020.

[7] E. Ulbrich: "Walk through the maze using AR!". https://www.geogebra.org/m/gcnr6nwk, 2020.