Ygrography, Creating Artworks by means of Hele-Shaw's Fluxes

Amada Navarrete[1] and Regina Bittencourt[2]

[1]MuArt Mathematical Engineer and Painter, Santiago, Chile; amada.navarrete@gmail.com
[2]APECh Programmer and Mathematical Artist, Santiago, Chile; Art.RBittencourt@gmail.com

Abstract

As we explore nature, we observe similarities underlying among its many different forms. Driven by the desire of finding what is it that associates them, we searched for representation methods that would capture their essence, first graphically, and later on by means of mathematical models able to describe them. Flowing fluids ranging different densities running over surfaces with different kinds of porosity, bearing compressing sheets can create the type of patterns, "fingering", we find on vegetation, corals, rocks and body capillaries. On the other hand, the description of fluids mechanics' processes and their equations can enlighten us in the attempt of depicting the shape of flows. The physical phenomenon of fingering is a non-linear mathematical representation that is at the same time independent from scale: the same patterns can be seen in the delta of a river, on the blood system or in the images of our own work. The irregular boundaries produced by fingering may also make us think of a fractal nature, which is independent form scale and has self-similarity. We called this painting method, "Ygrography".

Introduction

Even though corals and river deltas are different entities, they present similar morphological developments. This can be clearly seen in Figure1 below. This similarity led us to look for the common origin in these shapes in order to represent them graphically, using a physical or mathematical model that describes them.

(a) (b) (c)

Figure 1: *(a) Venus fan octocoral [1], (b) blood vessels [7], (c) Lena delta river in Russia [3].*

The morphology of the beautiful formations of plants, trees, ferns, river deltas and other elements of nature can be represented with fluids of different densities pressed against sheets of varied porosities, using Hele-Shaw's flow model. With Darcy's law [4], fundamental to hydrogeology, it was possible to start quantifying flows in natural aqueducts. Darcy's law establishes that the flow going through a porous media is proportional to the constant of permeability of that media, to the area it goes through and to the difference of the height between both points and inversely proportional to the distance separating those two points.

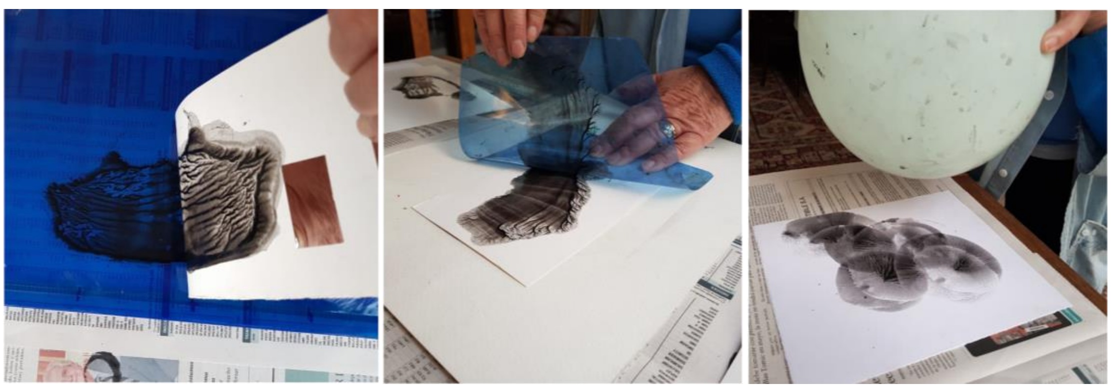
From 1897 to 1900 Hele-Shaw developed a technique whereby laminar flow between millimeter-spaced parallel plates simulated potential flow when viewed from above the plates [8]. The article [2] studies the specific movements of fluids on planes and the equations that govern them, which corresponds to weakly non-linear functions.

Our Method

Our work dealt with obtaining shapes and structures using inks and paints of different densities and viscosities, applied to supports with more or less porosity. The beauty and consistency of the shapes going in different directions led us to study and try to understand the physical phenomenon behind them. Not surprisingly, mathematics does have an explanation for it. It is the instability of Saffman and Taylor, which arises when a fluid with a lower viscosity (air) displaces another with a higher viscosity (paint). S. Obernauer [6] shows experimental studies and its graphic, numerical and theoretical results allowing us to understand Saffman and Taylor's fluid dispersion instability.

The use of this painting method came up when creating engraving monotypes, (simple and quickly executed single works, whose outcome are to a great extent haphazard).

We carried out several testing on which we used materials of different kinds for the fluids (inks, watercolors, acrylics, oils), for the supports (paper, poster board, thin poster board, coated cardboard and canvas) along with three pressing techniques to exert pressure between the fluid and the support. Figure 2 shows all the used techniques.



(a) (b) (c)

Figure 2: *Illustration of used techniques: (a) T1, (b) T2, (c) T3.*

Technique 1 (T1): The fluid is distributed on a very smooth non-absorbent surface, like an acrylic or melanin sheet. The support is placed on the surface, pressed manually for a few seconds and then lifted. The direction in which the support is detached is determinant to the outcomes.

Technique 2 (T2): The fluid is distributed on the support and pressed manually with the help of a 0.2 mm transparent acetate sheet and allows to observe the formation of spots and branching. The direction towards which the acetate sheet is lifted defines the orientation of the branching. The operation may be repeated once dry or even slightly humid. Superposition of coats enriches the depth of the work to the point of being able to visualize it in three dimensions.

Technique 3 (T3): The fluid is applied to the support in the same way as in Technique 2, but here using an inflated balloon instead. The vertical lifting of the balloon creates radial images similar to those obtained in the Hele-Shaw's cells experimentation.

The diversity of the resulting images with any of the techniques relies on the particular properties of viscosity, superficial tension and moving speed of the fluids themselves. Air, always existent, is another fluid joining the interaction. As to experiment with the effects of different viscosities we organized two groups of fluids. Group 1) distilled water for watercolors (Van Gogh), acrylics (Amsterdam, Winsor &Newton) and water-based inks and Group 2) turpentine for oil (Van Gogh) and inks for engraving (Sun Lit).

(a) (b) (c)

Figure 3: *Paintings resulting from applied techniques using acrylic stain over cardboard treated with three layers of primer: (a) T1, 11x11 cm, (b) T2, 10x17 cm, (c) T3, 15x20 cm.*

Figure 3 shows testing with one single acrylic stain. Branching was wider and longer in the centre of the cardboard where we concentrated the paint, opposed to thinner and shorter branching on the borders with less paint. It is true then, that a constant (k) – which directly depends on the viscosity of the fluid- defines the relation $k= A/L$ on which A is the width and L the length of each branch.

(a) (b) (c)

Figure 4: *Artworks made with different materials and techniques: (a) T1, oil on poster board 13x17cm, (b) T2, acrylics on canvas 70x70cm (detail), (c) T1, oil on coated poster board 15x11 cm (detail).*

The physical phenomenon of fingering is, in relation to space and time, a non-linear mathematical representation [6]. It is as well independent from scale: the same patterns can be seen in the delta of a river, the blood system or in the images of our own work. The irregular boundaries produced by fingering make us think of a fractal nature as it responds to the features of independence from scale and self-similitude, as we can observe in Figure 4.

The work of some surrealist painters like Oscar Dominguez, Marcel Jean, Ives Tanguy and Max Ernst, using a technique called *decalcomania* [5], may look visually similar, but is not the same technique and was not used with the intention of referring to distinctive natural features.

So, now we know of the existence of a mathematical and physical knowledge [6, 8] that may be integrated to an artistic technique for the applications of paints on porous supports, we propose to give life to a coral reef image in a two-dimensional artwork.

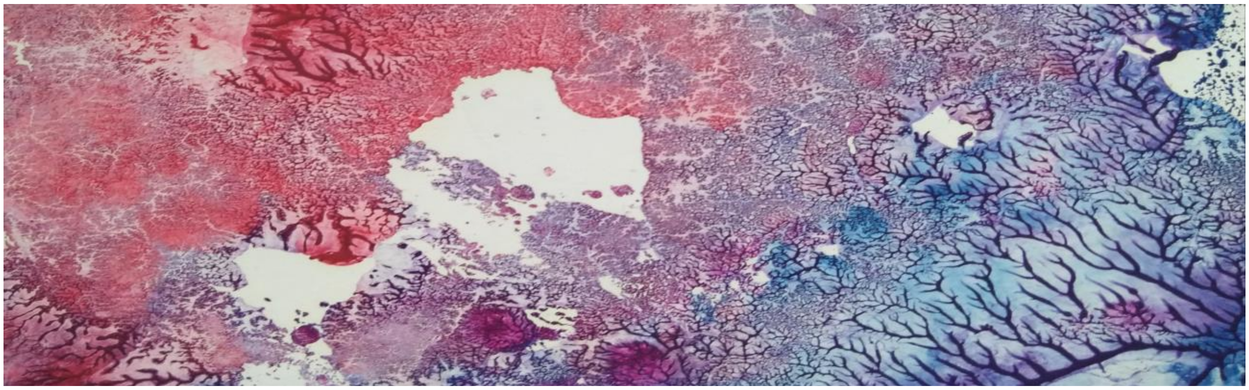


Figure 5: *T1, oil on coated poster board, 15x11 cm (detail).*

Figure 6: *T2, acrylics on cardboard treated with three layers of primer, 33x12cm.*

Summary and Conclusions

This work achieves an aesthetical integration of the Hele-Shaw method of fluids dispersion with different viscosities, on dry and humid surfaces with low porosity, and the so-called Saffman and Taylor instability of viscous fingering. Our method uses axial pressures in different orientations and central pressures, to generate the shapes of the corals in a reef, Figure 5 and 6. We will call this method "Ygrography".

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