Metaphors at the Crossing of Mathematics and the Literary Arts

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Abstract

As one of the prime vehicles for creative expression in the literary arts, metaphors are frequently discussed in the fields of literary studies and linguistics. Although less talked about, metaphorical thinking is nevertheless often encountered in mathematics as well. In the present paper we examine the uses and roles of metaphors in mathematics and propose a taxonomy of mathematical metaphors.

Introduction

The first steps in crossing the bridge between mathematics and the literary arts took place, in the case of the present authors, a mathematician and a literary scholar, in a very homey way: during a casual conversation over a cup of coffee. Lorelei, who was teaching e.e.cummings that week, praised poetry for its ability to fill abstract words with diverse and unexpected images. Sânziana then pointed out that something similar happens in mathematics, but in the reversed order: starting from concrete problems, one arrives at abstract concepts. This first casual conversation ignited further explorations (at first over breakfasts, then soon spilling into dinners and finally occupying whole days) which eventually became the foundation for a larger project which we called "Strange Encounters between Mathematics and Poetry".

In the present paper, we focus on how metaphors lie at the intersection of both mathematics and the literary arts. Why metaphors? Because at the heart of metaphor is analogy and the understanding of analogy is an essential part of the study of literature and maths.

Metaphors in Mathematics

It is well known that metaphors are the preferred instrument for poetical expression, but, even if less visible than in the literary arts, they are employed in mathematics as well. A metaphor is defined as "a figure of speech in which a word or phrase literally denoting one kind of object is used in place of another, to suggest likeness or analogy between them" [4]. In other words, a metaphor states that something is something else.

Briefly and mathematically speaking, there are two terms: A (the symboliser) and B (the symbol) which are said to be equal (A=B), when obviously they are not (A≠B). If one says that "life is a tale told by an idiot", then one has produced a metaphor. However, according to mathematical rules, the only correct statement is A=A, since the equality relation is reflexive. But "life is life" (A=A), while a great musical hit from the 1980s, no longer functions as a metaphor. It is due to its paradoxical nature that poets use metaphor for a variety of purposes: to suggest, to intrigue, to hide or to astonish. At the same time, mathematicians also find it useful in spite of its contradictory nature. Why is that, one may ask? Because of the "unreasonable effectiveness" [6] that metaphors have in fulfilling a diversity of functions, depending on the context in which they are placed.

A Taxonomy of Mathematical Metaphors

With respect to their main attributes which are: to transmit, transform and create knowledge, we have carved out three types of metaphors encountered in mathematics which we designated as “homey”, “discovery” and “creative” (or “special”) metaphors.

The “Homey” Metaphor

Its main function is to transmit knowledge, making it easier to grasp. It may be the best, if not the only way, to translate the beauty of mathematics for the non-mathematician. As Demian Nahuel Goos said at Bridges 2019: “it is impossible for the layman to fully comprehend the far-reaching power of a theorem due to its stern and austere nature. It is the duty of the mathematical community to make it visible” [2].

In homey metaphors, the first term (A), the symboliser is a mathematical object and the second term (B), the symbol, may be: either a more familiar mathematical object like a graph, a diagram or a geometrical figure or a non-mathematical object like a drawing, a structure or an object. In many cases, (B) is a verbal construction: a couple of words, a sentence or phrase, a story, a riddle.

We are going to illustrate how homey metaphors work in mathematics by choosing as symboliser (A) a famous theorem from topology, Brouwer’s fixed point theorem.

(A) : *Any continuous function f, that maps a closed ball from an n-dimensional Euclidian space into itself, has at least a fixed point, that is $f(x) = x$.*

An appropriate metaphor may be imagined for different dimensions of the Euclidian space.
For $n = 1$, the ball is a closed bounded interval $[a, b]$ and for (B), a suitable choice may be a graph.

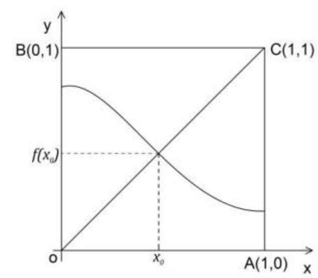

Figure 1: *Visual representation of Brouwer’s Theorem for $n = 1$, using a graph.*

(B): Any continuous curve, lying inside the square OABC that unites a point from the side OB with a point from the opposite side AC, crosses the diagonal OC, in at least one point ($f(x_0)$,x_0).

For $n = 2$, the ball is a closed disk and to represent this, an imagistic metaphor like a diagram may be used.

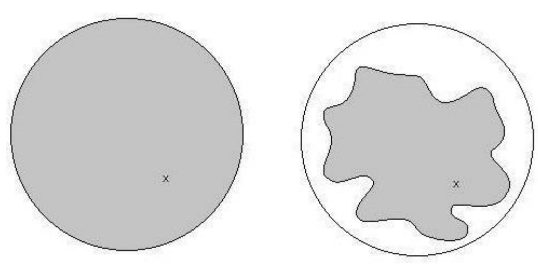

Figure 2: *Visual Representation of Brouwer’s Theorem for $n = 2$, using a diagram.*

*(B): If a disk suffers a distortion, without tearing, and it is left within the same space it previously occupied, then there is a point that did not move*For $n = 3$, a metaphoric “imagistic domestication” [5] was created by Brouwer himself when, for the symbol (B), he used a non-mathematical object from the realm of human activities.

(B): No matter how much we stir the coffee in our cup, finally there will be a point in the same place as before the stirring.

The Discovery (or Eureka) Metaphor

Its function is to transform knowledge. This time, both (A) and (B) are mathematical objects and the metaphor is a conceptual one, which means that a certain idea is replaced by a similar one. Discovery metaphors are, in many cases, at the turning points of demonstrations and because they can be used in a variety of ways, we have further identified the following subtypes: "as if" metaphors, "billiard shot" metaphors, "hidden" metaphors and "two faced" metaphors:

As if Discovery Metaphors. (A) → (B) => (A)

First one finds an analogy for (A) which is (B) and then one returns from (B) to (A), but enriched with the knowledge found in (B), from which (A) is going to benefit.

One of the most beautiful mathematical theorems, Cantor's theorem, which states that "the size of the real set is strictly greater than the size of the set of natural numbers" constitutes an illustration of the "as if" discovery metaphor. The proof uses the "reduction at absurdum" strategy and supposes that this is not true. Cantor's metaphor starts by fixing the symboliser (A) as being the real set ℝ, with the property that the set is countable. Then Cantor imagined all the real numbers, arranged one below the other, forming a giant square. The symbol (B) is thus a square with infinite sizes formed by an infinite, but countable number of rows. And because a square has a diagonal, it is possible to walk on this path starting with the first element on the first row, picking one by one (in the order of the rows) all the digits lying on the diagonal. Then, like in a fairy-tale, Cantor changed the value of every digit, not by a spell, but by a simple algorithm. This way, he created a number which cannot be on any row of the square.

Billiard Shot Discovery Metaphors (B) => (A)

This type of metaphor starts with the symbol (B) and, using analogy, one arrives to the symboliser (A), where interesting properties of (B) are used. An example of the "billiard shot" metaphor in mathematics is the way Archimedes computed the area of a circle. He started from (B), a right triangle, with sides equal to the radius r and to the circumference C of the circle. In order to show that the area of the triangle ($T = \frac{1}{2}rC$) is equal to the area of the circle, he used a double *reduction ad absurdum* method involving two other metaphorical constructions, the inscribed and the circumscribed regular polygons. Finally, he returned to the circle (A), after three billiard-shot metaphors.

Hidden Discovery Metaphors (A) → (?) => (A)

Mathematicians use metaphorical thinking as cognitive process quite often, but sometimes this process is hidden from plain view. "Hidden" mathematical metaphors are (as somebody said about how Gauss presented his proofs) like foxes cleaning up their tracks using their tail. Sometimes, techniques from (B) are used in (A) because there is an analogy between (A) and (B), but (B) is not overtly mentioned and remains invisible. For instance, when one performs a polynomial decomposition into factors one has as a model the decomposition of the integers in prime factors. And since ℤ, the set of integers may play the role of symbol also for other sets of mathematical objects (matrices, functions, series), mathematicians created an abstract concept by emptying ℤ of its elements, while keeping its shell (the algebraical structure) that they called "ring". This way, the analogy with ℤ was completely hidden under the new notion of "ring". The process involved here is the reverse of a "homey" metaphor. So, one may even playfully call them "yemoh" ("homey" spelled backwards) metaphors.

Two Faced Discovery Metaphors (A) <=> (B)

In this category of metaphors, not only does the symbol (B) have an influence over (A), but we could say that (B) receives new possible meanings from (A). The roles of symbol and symbolizer switch places. For instance, in the Euclidian plane, vectors can be seen as pairs of real numbers, but also inversely. That is, if $P(a,b)$ is a point in xy- plane, then it may be identified with a vector $\overrightarrow{OP}$, having the origin at $O(0,0)$ and the endpoint at point $P(a, b)$. Inversely, the vector $\overrightarrow{OP}$, is identified with a pair (a, b) , where a and b are the scalar projections of the vector on the axes of the coordinates system.

Special or Creative Metaphors

While metaphors are created to solve or clarify concrete problems, sometimes a metaphoric manoeuvre surpasses the boundaries of the particular problem which inspired it and proves to be useful in other cases as well. We have named this type of metaphor possessing a flavour of universality "special" or "creative" metaphors. For this type, we have further identified various exotic subtypes, but for brevity's sake, we shall stop only at the subtype we called "bird-metaphors" because these are the type of metaphors with the ability to open gates of new worlds. We have used the term "bird" as a reference to Freeman J. Dyson's characterisation of mathematicians [3]. He considered that some mathematicians are like birds and others are like frogs. The bird ones are those who fly so high that they can catch, in their view, different domains and find the metaphors linking them. This is the case with Descartes, who connected geometry and algebra when he imagined a pair of numbers as a point in the plane. This is how a new domain emerged: that of analytic geometry. And Descartes is not the only one. Boole had the idea of describing logical operations (conjunction, disjunction and negation) in the same way elementary algebra describes numerical operations. He thus linked logic and algebra and nowadays, the two-valued Boolean logic is to be found in all modern computers. Riemann is another example. He realised that the fundamental ingredients for geometry are a space of points that need not even be Euclidian space. These spaces, called Riemann surfaces are, so to speak, an "unearthly" encounter between geometry and complex analysis ("unearthly" since complex analysis is based on a mathematical object that does not belong to our reality, the imaginary number i). Riemann's metaphors provide the mathematical foundation for the four-dimensional geometry of space-time in Einstein's theory of general relativity.

Conclusions

We have outlined a taxonomy of mathematical metaphors which shows that, even if less employed than in the literary arts, they are nevertheless at the crossing of these two domains. Besides, the important aspect regarding metaphors is not their frequency, but their ability to create beauty. Even if defining aesthetical norms for mathematics is impossible, some features of beauty such as unexpectedness, enlightenment or profundity are unanimously recognized. These particular traits also describe metaphors.

Paul Gailiunas lists among some of the purposes of having Bridges conferences the "desire to come together from a diverse set of apparently separated disciplines, to share and recognize abstract similarities, common patterns and underlying characteristics" [1]. These are also main attributes of metaphors. As Yuri Manin asserts, that "metaphor helps human beings breathe in this rarefied atmosphere of God" [3], we may therefore conclude that, under the metaphoric arch of this conference, we all share a divine atmosphere.

References

[1] P. Gailiunas. "Bridges and Artists." *Bridges Conference Proceedings*, Linz, Austria, Jul.16-20, 2019, pp. 75-82.

[2] D.N. Goos. "Magritte Meets Matisse Meets Mathematics." *Bridges Conference Proceedings*, Linz, Austria, Jul.16-20, 2019, pp. 549-552.

[3] Y.I. Manin. *Mathematics as Metaphor: Selected Essays of Yuri I. Manin*. American Mathematical Society, 2007.

[4] Merriam-Webster Online Dictionary. https://www.merriam-webster.com/dictionary/metaphor.

[5] A. Wall. "The Janus Face of Metaphor." The Fortnightly Review. https://fortnightlyreview.co.uk/2012/05/wall-metaphor/

[6] E.P. Wigner "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." *Communications on Pure and Applied Mathematics*, vol.13, no.1, pp.1-14.