The Art and Mathematics of Cycling: Using Old Bicycles to Draw Spirograph Patterns

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Abstract

This paper describes a series of drawing machines made from scrap bicycles by the author. The characteristics of these machines are discussed, as are the maths and geometry of the curves that can be drawn with them. These patterns include: *hypotrochoid* (*"spirograph"*), *harmonograph* (aka *Lissajous curve* or *"pintograph")*, and *epitrochoid* ("reverse *spirograph*").

Introduction

Since 2011, I have made a series of drawing machines from everyday materials, including old bicycles.

Like many children in the 1970-80s, I had a *Spirograph* set [14] (first marketed in 1965 by Denys Fisher). These were comprised of several plastic cogs and toothed wheels, which could be used to draw geometric patterns. Other geometric drawing toys marketed around the same era include *Sketch-a-Graph* [3] (the pantograph linkage of which I've used in many of my machines), *Harmonograph*, *Etch-a-Sketch, Hoot Nanny 'The Magic Designer'*, *Whirly Wizard*, and *Design-o-Graph*. My drawing machines project is also influenced by my love of bicycling and recycling, and kinetic sculptures by artists including Conrad Shawcross [13], Rowland Emett [1], and William Heath Robinson [4].

My bicycle drawing machines fit into my wider body of work [12], in which I explore connections between maths, science, and art. In my work I relate the abstract world of maths to the mass-produced detritus of human life. I make sculptures, functional items, and photographs – often from everyday or recycled materials. My other works include pinhole cameras, domes, and spherical sculptures.

Giant Pantograph

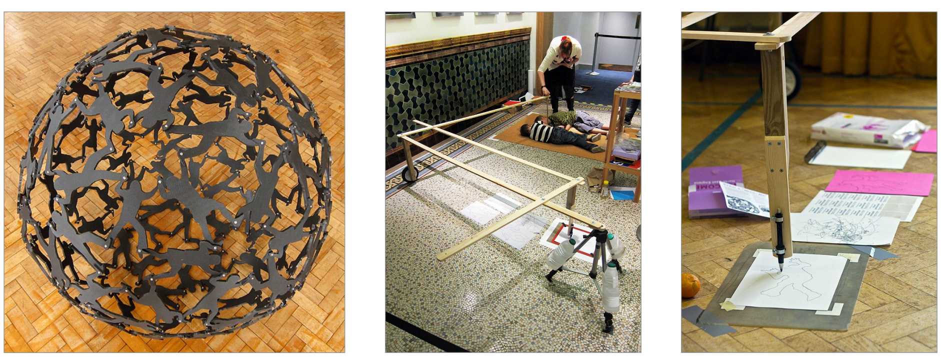

Figure 1: *Sphere Of Shadows, 2010, and Giant Pantograph, 2011.*

Before I made my bike drawing machines, in 2010 I was commissioned to make *Sphere Of Shadows* (Figure 1), a 120-piece spherical sculpture made from cut-out silhouettes of school children, for their school's outdoor classroom. I produced these outlines digitally, with a green screen, projector, and digital camera. However, this led to me wanting to make a machine that could trace bodies more directly. Thinking back to the *Sketch-a-Graph* toy I had as a child, I decided to make a 2-metre long version, *Giant Pantograph* (Figure 1), which could scale down outlines of bodies onto A4 art paper: a 6:1 reduction.

This is a pantograph linkage, which mechanically draws at scale whatever is traced with the stylus. It works through the principle of similar triangles – eg: whatever the stylus traces will be translated at 1/6th of the distance from the pivot. This artwork led to many of my subsequent bicycle machines, which have in common its wooden linkage and reference to children's geometry toys.

Bicycle Spirograph 1 (Sand)

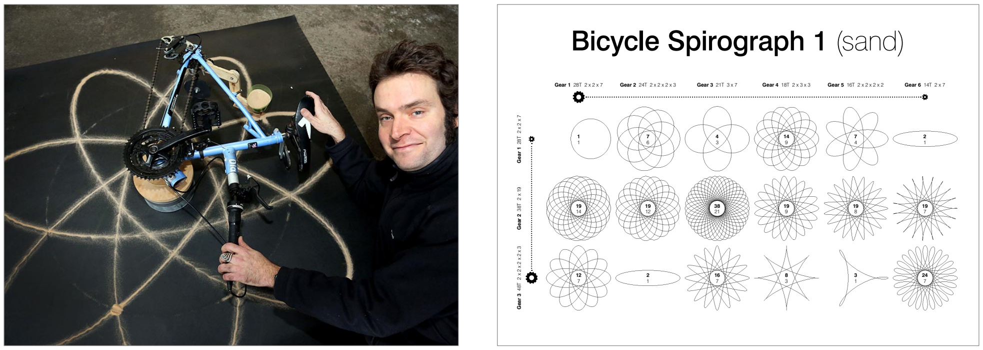

Figure 2: *Bicycle Spirograph 1 (Sand), 2012, and the 18 patterns that can be drawn with its 18 gears.*

I made the first of my bicycle drawing machines, *Bicycle Spirograph 1 (Sand)*, in 2012 (Figure 2). This came about partly because someone saw my *Giant Pantograph* body-drawing machine, and referred to it as a "Giant Spirograph." I wanted to see if the differential in speed between the crank and back wheel of a bike might produce Spirograph-like hypotrochoid [6] patterns.

The machine consists of the sawn-off back half of a bicycle, laid on its side, with stabilizer wheels beneath. I replaced the back wheel with a wooden drawing arm, at the end of which is a plastic bottle for sand or paint. I also made an optional pen/pencil attachment. Rotating the bicycle frame around the stationary crank produces 1.5-metre diameter patterns. Shifting between the bike's 18 gears (3 front chain-rings x 6 rear sprockets) produces 18 patterns (two of these patterns are the same – simple ellipses – as their cog teeth ratios, 48:24 and 28:14, simplify to 2:1). I initially attached a pen, to draw on large sheets of paper. However, I discovered it could also be used with a bottle of sand, to draw temporary patterns on a mat. These can be quickly cleared away and another pattern drawn in its place, which is useful for demonstrating at schools and science events. I also like the fact it can be used with water-based paint or pigment to draw on roads – a bicycle making a beautiful mark.

Construction of the machine involved finding a scrap bicycle from a local bike shop. The one I found had a cracked seat post, from where the previous owner had raised the saddle past the minimum insertion line. Despite cutting the bike in half, I wanted to keep its functional appearance, so I kept the saddle, pedal and even its bell. I made the drawing arm from wood and plastic, for contrast.

Figure 3: *Bike Cog Spirograph drawing activity, 2012.*

Bike Cog Spirograph Drawing Activity

I created this drawing activity (Figure 3) as a companion piece to *Bicycle Spirograph 1 (Sand)*. It consists of a set of separate bicycle gear sprockets (cogs) and specially laser-cut, internally-toothed acrylic wheels that I designed with Adobe Illustrator. The teeth of these wheels mesh with the bicycle cogs, which have a standard half-inch pitch, allowing hypotrochoid patterns to be drawn with coloured pens.

Bicycle Harmonograph

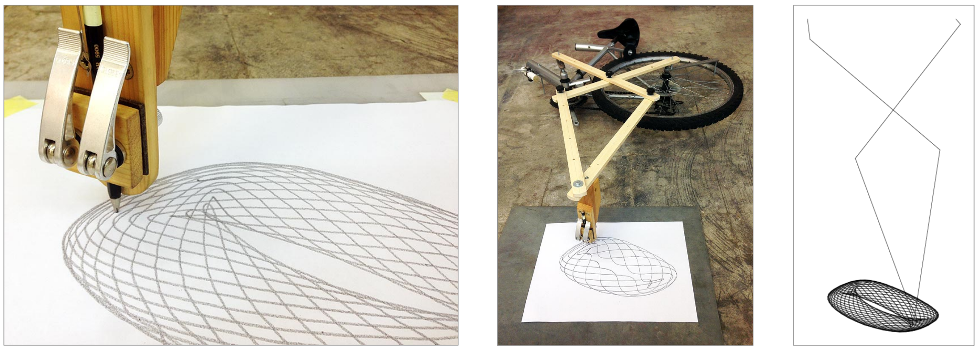

Figure 4: *Bicycle Harmonograph (2016), and simulated pattern using Processing language.*

Developing the theme set by *Bicycle Spirograph 1* – and inspired by pendulum- and turntable-based drawing machines I'd seen [5] – I built *Bicycle Harmonograph* in 2016 (Figure 4). With this machine, the bicycle frame stays stationary, and the user turns the crank arm. A wooden linkage is pivoted at the crank and back wheel, and scissors back and forth as the crank and wheel spin at different speeds. The resulting patterns are similar to a square-constrained Lissajous Curve [9] (the phase pattern between two sine waves of different wavelength) or harmonograph (the phase pattern between two pendulums swinging at different speeds). This sort of curve has been dubbed a "pintograph" [10] due to its use of a pantograph-like linkage to produce curvaceous distortion. The patterns resemble spirographs that have been warped out of shape, or "doughnuts with fishnet stockings on" (my comparison).

There are various parameters that affect the patterns drawn with this machine. I have locked a few of these parameters down for practical purposes (so that pattern switching is quick at science events, and so patterns fit on A3 art paper). However, it has been interesting to investigate how these factors affect pattern size, complexity, width, and "sway". I experimented with different parameters by writing a simulation app using the *Processing* language [11] (a Java-like language for quickly generating mathematical visualisations on various platforms):

Crank and back wheel radii (variable) – affects pattern width/sway

Gear ratio (variable, but kept at 34:21 for complex patterns) – affects pattern complexity

Distance to linkage crossover pivot (variable) – affects "scissoring" and pattern height

Lengths of the four linkage struts (fixed at 50cm each) – affects pattern width and height

Bicycle Spirograph 2 (Paper)

I made *Bicycle Spirograph 2 (Paper)* (Figure 5) following my previous sand-drawing machine, to allow science festival participants to take home smaller drawings on paper. I transformed the back wheel of the bike into a rotating drawing surface, to which 30cm square art paper can be attached. Turning the crank creates the patterns – it simultaneously powers both the back wheel (which turns at a different speed) and the circular drawing motion, which is scaled to 50% via a pantograph linkage, allowing the patterns to fit within the paper edges. The machine hypnotically draws epitrochoid [2] patterns. These can also be drawn with a conventional Spirograph toy, if the small cog is rolled around the outside of, rather than inside, the larger wheel (hence the epi- and hypo- in epitrochoid and hypotrochoid). Unlike the more common hypotrochoid flower patterns drawn with Spirograph, epitrochoid curves have their “petals” or lobes on the inside. This came as a surprise when making the machine: I expected the patterns to be hypotrochoids. The reason the machine draws epitrochoids results from the relative speeds of the inner and outer rotations: this is equivalent to a slow-turning inner circle moving fast around an outer circle.

Figure 5: *Bicycle Spirograph 2 (Paper), 2017, and the 15 patterns that can be drawn with its 15 gears.*

Bicycle Spirograph 3 (Paint)

In 2018 I was invited to a painting symposium in Egypt. I was very keen to see Cairo, Sharm El Sheikh, the Sinai desert, and work with other international artists, but I was secretly saying to myself, “…but I’m not a painter!” The answer I found was to make a suitcase-packable Bicycle Spirograph painting machine (Figure 6). I made this from a single-geared toddler bike, suspended from a collapsible wooden frame. I had to ensure that no parts were wider than my small suitcase: 50cm. The working principle is similar to my *Bicycle Spirograph 1 (Sand)* machine, but instead of having variable gearing (this was a single-speed bike), pattern variation is achieved by having different radii where the back wheel had been, ie: five slots for paint bottles.

I prototyped how the different hypotrochoid patterns would come out, and what colours to use, by writing a Processing app. For the painting symposium I dripped paint from the bottles to create five different patterns in a blue-to-red spectrum on top of each other, in a performance to a gathered and bemused crowd at our hotel. As an artist who likes control and order, I was interested to see how the geometric patterns came out more chaotically than my Processing prototype: curved lines wavered, paint pressure varied, and colours bled into each other.

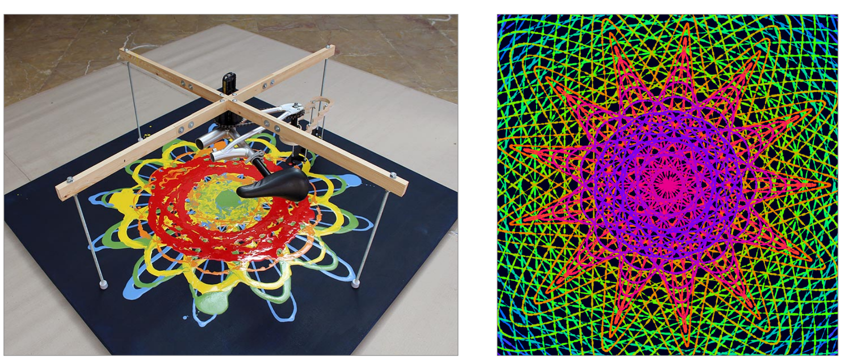

Figure 6: *Bicycle Spirograph 3 (Paint) and Artistic Spectrum painting, 2018. Pattern prototyping in Processing.*

Bicycle Spirograph 4 (Paper)

Figure 7: *Bicycle Spirograph 4 (Paper), 2019, in South Korea. Translation of circular motion to a more ovoid shape, for various drawing arm lengths, via a "piston" linkage, modelled in Processing.*

In 2019 I was invited to South Korea, for a painting/drawing exhibition in Eumseong, and a sculpture exhibition in Seoul [8]. I decided to make another travel-sized drawing machine, *Bicycle Spirograph 4* (Figure 7), which could be shown at both events. The principle of this machine was similar to my *Bicycle Spirograph 2 (Paper)* drawing machine, but with the motion of the back wheel being translated onto a drawing surface attached to the rotating crank (as opposed to the crank drawing onto the wheel). I again used a single-speed child's bike and allowed different radii of circles to be drawn to increase pattern variation. To reduce size and weight, I replaced the pantograph with a piston-like linkage for translating circular motion. The unexpected result of this was that the "circles" it drew were somewhat more egg-shaped, and the rotation of the pen at the drawing end of the linkage went in the same rotational direction as the drawing surface. It thus drew slightly distorted epitrochoids, like *Bike Spirograph 2*. I had hoped it would draw hypotrochoids (classic Spirograph patterns), so in future I might try making a pantograph linkage to do this (it would then draw circles in the opposite direction).

Common Theme: Mathematics and Geometry of Phase Patterns

All of these machines and drawing activities share a common theme: two wheels rotating at different speeds to create phase patterns. The number of petals/lobes of these patterns, and the number of turns of the wheel to create them, can be calculated from the numbers of teeth on the two cogs. Consider two Spirograph cogs: an inner disk with $28 = 2^2 \times 7$ teeth, and an outer wheel with $48 = 2^4 \times 3$ teeth. The lowest common multiple of 28 and 48 is $336 = 2^4 \times 3 \times 7$. The inner 28-tooth cog will thus rotate 12 times ($2^2 \times 3$) and create 12 petals, and will need to be rolled 7 times around the 48-tooth outer wheel to complete the pattern. How these phases translate into patterns is different for each of the machines, but the underlying principle is the same.

Cycling Through the Different Spirograph Patterns

Looking at the pattern tables I'd laid out for the Bicycle Spirograph machines, I was puzzled by the fact that certain patterns (eg: 2:1 ratio) repeated, despite being on different "speeds". I was also curious to see if there was a continuum between hypotrochoid curves and epitrochoids. I decided to write a Processing app to visualize this [7]. The program cycles between 45:10 and 10:45 ratio spirograph patterns, in descending order of values (eg: the 7th-11th ratios in the series are 43:11, 39:10, 42:11, 38:10, 45:12, 41:11). I weeded out repeating patterns (eg: 45:30 = 15:10). The result is quite hypnotic, and visually intriguing. In the animation, the first loop of the pattern arcs, goes pointy, doubles back on itself, and then loops in on itself.

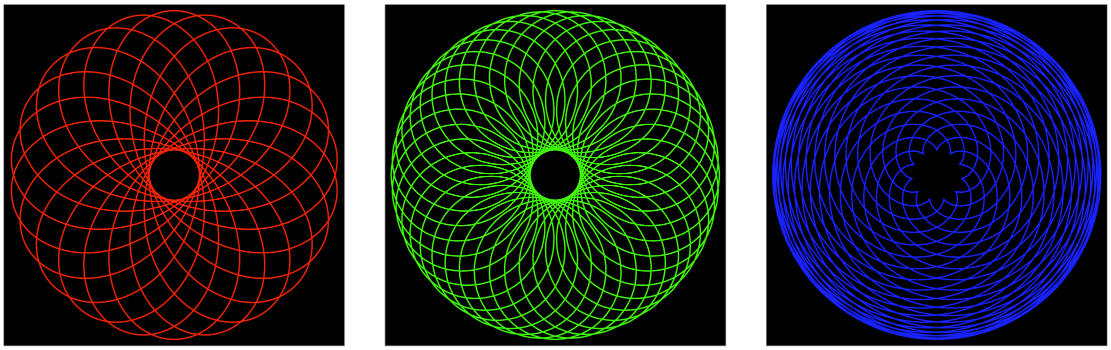

Figure 8: *Spirograph pattern variations, from hypotrochoid to epitrochoid, plotted using Processing.*

Summary and Conclusions

All of the machines I've made have been an education for me in both mathematics and engineering. It has given me a better understanding of prime number factorisation, introduced me to a world of mechanical bearings (thrust bearings for pivots, axle bearings for wheels, etc), and improved my engineering skills (working with component weighting, tolerances, counterbalancing, etc).

More importantly, I have taken them to school and art/science events (including Maker Faire, Abu Dhabi Science Festival, London Science Museum, TEDxBrighton, and WOMAD), where I have used them in an educational context to demonstrate mathematical and engineering concepts in a fun, quirky, and entertaining way. People enjoy making patterns without the use of computer, and can be quite hypnotised by the patterns being drawn.

I'm constantly making tweaks and improvements to these machines, mostly to make them child-safe when I take them to different events. I plan to make more machines, including a traditional pendulum harmonograph, camera obscura, giant *Pinscreen* board, etc.

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