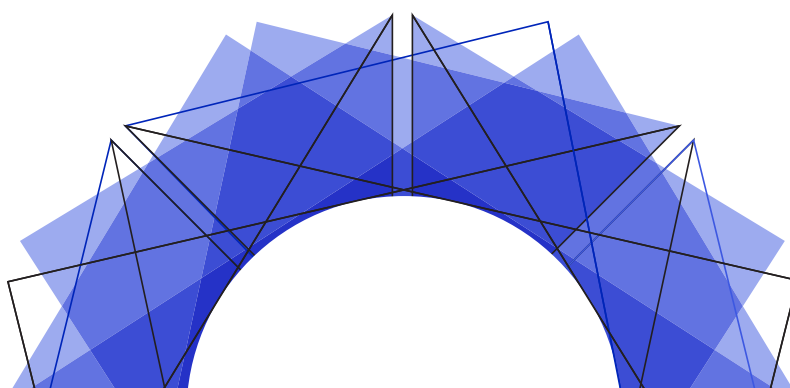


# Bridges Linz 2019

Mathematics | Art | Music | Architecture | Education | Culture

## 2019 Conference Proceedings



**BRIDGES LINZ 2019**

**JKU**

**JOHANNES KEPLER  
UNIVERSITY LINZ**



**ARS ELECTRONICA**

## **Editors**

### **Program Chair**

#### **Susan Goldstine**

Department of Mathematics and Computer Science  
St. Mary's College of Maryland  
St. Mary's City, Maryland, USA

### **Short Papers Chair**

#### **Douglas McKenna**

Mathemæsthetics, Inc.  
Boulder, Colorado, USA

### **Workshop Papers Chair**

#### **Kristóf Fenyvesi**

Finnish Institute for Educational Research  
University of Jyväskylä  
Jyväskylä, Finland

### **Production Chair**

#### **Craig S. Kaplan**

Cheriton School of Computer Science  
University of Waterloo  
Waterloo, Ontario, Canada

*Bridges Linz 2019 Conference Proceedings* ([www.bridgesmathart.org](http://www.bridgesmathart.org)). All rights reserved. General permission is granted to the public for non-commercial reproduction, in limited quantities, of individual articles, provided authorization is obtained from individual authors and a complete reference is given for the source. All copyrights and responsibilities for individual articles in the 2019 Conference Proceedings remain under the control of the original authors.

ISBN: 978-1-938664-30-4

ISSN: 1099-6702

Published by Tessellations Publishing, Phoenix, Arizona, USA (© 2019 Tessellations)  
Distributed by *MathArtFun.com* ([mathartfun.com](http://mathartfun.com)).

Cover design: David Chappell, University of La Verne, California, USA

## **Bridges Board of Directors**

### **Kristóf Fenyvesi**

Finnish Institute for Educational Research  
University of Jyväskylä  
Jyväskylä, Finland

### **George Hart**

Warton, Ontario, Canada

### **Craig S. Kaplan**

Cheriton School of Computer Science  
University of Waterloo  
Waterloo, Ontario, Canada

### **Carlo H. Séquin**

Computer Science Division  
University of California, Berkeley  
Berkeley, California, USA

### **Sujan Shrestha**

Science, Information Arts & Technologies  
University of Baltimore  
Baltimore, Maryland, USA

### **Eve Torrence**

Department of Mathematics  
Randolph-Macon College  
Ashland, Virginia, USA

## **Conference Organization**

### **Zsolt Lavicza**

Johannes Kepler University  
Linz, Austria

### **Barbara Lichtenegger**

Private University of Education, Diocese of  
Linz  
Linz, Austria

### **Sandra Reichenberger**

Johannes Kepler University  
Linz, Austria

## **Artistic and Scientific Committee Members and Coordinators**

### **Steve Abbott**

Department of Mathematics  
Middlebury College  
Vermont, USA  
*Theater Event*

### **Robert Fathauer**

Tessellations Company  
Phoenix, Arizona, USA  
*Art Exhibition*

**Kristóf Fenyvesi**

Finnish Institute for Educational Research  
University of Jyväskylä  
Jyväskylä, Finland  
*Family Day*

**Emily Grosholz**

Pennsylvania State University  
University Park, Pennsylvania, USA  
*Poetry Reading*

**Uyen Nguyen**

New York City, New York, USA  
*Art Exhibition*

**Sarah Glaz**

The University of Connecticut  
Storrs, Connecticut, USA  
*Poetry Reading*

**Tiffany Inglis**

D2L  
Waterloo, Ontario, Canada  
*Technical Support*

**Nathan Selikoff**

Digital Awakening Studios  
Orlando, Florida, USA  
*Technical Support*

**Bianca Violet**

IMAGINARY  
Berlin, Germany  
*Short Film Festival*

**Proceedings Program Committee**

**Steve Abbott**

Middlebury College  
Vermont, USA

**Kazushi Ahara**

Meiji University  
Tokyo, Japan

**Abdalla G. M. Ahmed**

Khartoum, Sudan

**Mara Alagic**

Wichita State University  
Kansas, USA

**Roger Antonsen**

University of Oslo  
Oslo, Norway

**Ellie Baker**

Lexington, Massachusetts, USA

**Javier Barallo**

The University of the Basque  
Country UPV/EHU  
Vizcaya, Spain

**Tom Bates**

Santa Barbara, California, USA

**Debra K. Borkovitz**

Wheelock College  
Boston, Massachusetts, USA

**Robert Bosch**

Oberlin College  
Ohio, USA

**Christopher Brownell**

Fresno Pacific University  
California, USA

**Doug Burkholder**

Lenoir-Rhyne University  
Hickory, North Carolina, USA

**Christopher Carlson**  
Wolfram Research  
Champaign, Illinois, USA

**David Chappell**  
University of La Verne  
California, USA

**Andrew Cooper**  
North Carolina State University  
Raleigh, North Carolina, USA

**Kelly Delp**  
Cornell University  
Ithaca, New York, USA

**Neil Dodgson**  
Victoria University of Wellington  
New Zealand

**Mircea Draghicescu**  
ITSPHUN LLC  
Portland, Oregon, USA

**Doug Dunham**  
University of Minnesota  
Duluth, USA

**Frank Farris**  
Santa Clara University  
California, USA

**Robert Fathauer**  
Tessellations Company  
Phoenix, Arizona, USA

**Loe Feijs**  
Eindhoven University of  
Technology  
The Netherlands

**Kristóf Fenyvesi**  
University of Jyväskylä  
Finland

**Paul Gailiunas**  
Newcastle, England

**Susan Gerofsky**  
University of British Columbia  
Vancouver, Canada

**Sarah Glaz**  
The University of Connecticut  
Storrs, Connecticut, USA

**Susan Goldstine**  
St. Mary's College of Maryland  
St. Mary's City, Maryland, USA

**Gary Greenfield**  
University of Richmond  
Virginia, USA

**Emily Grosholz**  
Pennsylvania State University  
University Park, Pennsylvania,  
USA

**Richard Hammack**  
Virginia Commonwealth  
University  
Richmond, Virginia, USA

**George Hart**  
Warton, Ontario, Canada

**Edmund Harris**  
University of Arkansas  
Fayetteville, Arkansas, USA

**Andrea Hawksley**  
eleVR, HARC, YCR  
San Francisco, California, USA

**Sara Hershkovitz**  
Center for Educational Technology  
Tel Aviv, Israel

**Judy Holdener**  
Kenyon College  
Ohio, USA

**Patrick Honner**  
Brooklyn Technical High School  
New York City, NY, USA

**Rachael Horsman**  
Cambridge Mathematics  
England

**Guy Inchbald**  
Worcestershire, England

**Tiffany Inglis**  
D2L  
Waterloo, Ontario, Canada

**Veronika Irvine**  
University of Waterloo  
Ontario, Canada

**Craig S. Kaplan**  
University of Waterloo  
Ontario, Canada

**Karl Kattchee**  
University of Wisconsin  
La Crosse, USA

**Margaret Kepner**  
Washington DC, USA

**Eva Knoll**  
Mount Saint Vincent University  
Halifax, Nova Scotia, Canada

**Lisa Lajeunesse**  
Capilano University  
North Vancouver, British  
Columbia, Canada

**Zsolt Lavicza**  
Johannes Kepler University  
Linz, Austria

**Penousal Machado**  
University of Coimbra  
Portugal

**Alice Major**  
Edmonton, Alberta, Canada

**Vincent J. Matsko**  
Spring Hill, Florida, USA

**Elisabetta Matsumoto**  
Georgia Institute of Technology  
Atlanta, Georgia, USA

**Dan May**  
Black Hills State University  
Spearfish, South Dakota, USA

**Douglas McKenna**  
Mathemæsthetics, Inc.  
Boulder, Colorado, USA

**Kerry Mitchell**  
Phoenix, Arizona, USA

**Mike Naylor**  
Matematikkbølgen  
Math Creativity and Competency  
Center  
Vanvikan, Norway

**Uyen Nguyen**  
New York City, NY, USA

**Doug Norton**  
Villanova University  
Pennsylvania, USA

**Werner Olivier**  
Nelson Mandela University  
Port Elizabeth, South Africa

**Osmo Pekonen**  
University of Jyväskylä  
Finland

**Kirsi Peltonen**  
Aalto University  
Helsinki, Finland

**Ulrich Reitebuch**  
Freie Universität Berlin  
Germany

**Rinus Roelofs**  
Hengelo, The Netherlands

**Radmila Sazdanovic**  
North Carolina State University  
Raleigh, North Carolina, USA

**Karl Schaffer**  
De Anza College and  
MoveSpeakSpin  
Scotts Valley, California

**Henry Segerman**  
Oklahoma State University  
Stillwater, Oklahoma, USA

**Carlo H. Séquin**  
University of California, Berkeley  
USA

**Sujan Shrestha**  
University of Baltimore  
Maryland, USA

**Donald Spector**  
Hobart & William Smith Colleges  
Geneva, New York, USA

**Catherina Steyn**  
Nelson Mandela University  
Port Elizabeth, South Africa

**John Sullivan**  
Technische Universität Berlin  
Germany

**David Swart**  
Waterloo, Ontario, Canada

**Felicia Tabing**  
University of Southern California  
Los Angeles, California, USA

**Tara Taylor**  
St. Francis Xavier University  
Antigonish, Nova Scotia, Canada

**Briony Thomas**  
University of Leeds  
England

**Bruce Torrence**  
Randolph-Macon College  
Ashland, Virginia, USA

**Eve Torrence**  
Randolph-Macon College  
Ashland, Virginia, USA

**Walt van Ballegooijen**  
Wijk en Aalburg, The Netherlands

**Tom Verhoeff**  
Eindhoven University of  
Technology  
The Netherlands

**Charles Wampler**  
General Motors Research and  
Development  
Warren, Michigan, USA

**Ryan Webb**  
Maggie L. Walker Governor's  
School  
Richmond, Virginia, USA

**Phil Webster**  
Phil Webster Design  
Scotts Valley, California, USA

**Jiangmei Wu**  
Indiana University Bloomington  
Indiana, USA

**Carolyn Yackel**  
Mercer University  
Macon, Georgia, USA

### **Art Exhibition and Catalog Program Committee**

**Robert Fathauer**  
Tessellations Company  
Phoenix, Arizona, USA  
*Co-curator*

**Uyen Nguyen**  
New York City, New York, USA  
*Co-curator*

**Conan Chadbourne**  
San Antonio, Texas, USA  
*Catalog*

**Nathan Selikoff**  
Digital Awakening Studios  
Orlando, Florida, USA  
*Technical Support*

**Barbara Lichtenegger**  
Private University of Education, Diocese Linz  
Linz, Austria  
*Local coordinator*

**Short Film Festival Program Committee**

**Susan Gerofsky**  
University of British Columbia  
Vancouver, Canada

**Elisabetta Matsumoto**  
Georgia Institute of Technology  
Atlanta, Georgia, USA

**Henry Segerman**  
Oklahoma State University  
Stillwater, Oklahoma, USA

**Bianca Violet**  
IMAGINARY  
Berlin, Germany  
*Chair*

**Math + Fashion Show Committee and Jurors**

**Susan Goldstine**  
St. Mary's College of Maryland  
St. Mary's City, Maryland, USA

**Elisabeth Heathfield**  
Warton, Ontario, Canada

**Uyen Nguyen**  
New York City, New York, USA

**Frank Farris**  
Santa Clara University  
California, USA

**Loe Feijs**  
Eindhoven University of Technology  
The Netherlands



# Contents

*Preface* ..... *xix*

## Invited Papers

---

*That Is Not Art, It Is a Puzzle!* ..... 1  
Oskar van Deventer

*Discrete and Computational Differential Geometry for Functional Pattern Design* ..... 9  
Helmut Pottmann

*The Foundations of Mathematics and Art: Form, Logic, Intuition* ..... 11  
Lynn Gamwell

*The Seduction of Curves* ..... 12  
Allan McRobie

## Regular Papers

---

*Walkable Curves and Knots* ..... 13  
Robert W. Fathauer

*Exploring Szpakowski's Linear Ideas* ..... 21  
Robert Bosch, Abagael Cheng, and Ari Smith

*Making A Mystic Dream of 4* ..... 29  
Iggy McGovern

*Graeco-Latin Square Poems* ..... 35  
Lisa Lajeunesse

*Folding Functions: Origami Corrugations from Equations* ..... 43  
Uyen Nguyen, Ben Fritzon, and Marcus Michelen

*Impossible Folding Font* ..... 51  
Erik D. Demaine, Martin L. Demaine, Tomoko Taniguchi, and Ryuhei Uehara

*Max Brückner's Wunderkammer of Paper Polyhedra* ..... 59  
George W. Hart

<i>Helixation</i> .....	67
Rinus Roelofs	
<i>Bridges and Artists</i> .....	75
Paul Gailiunas	
<i>Hyperbolization on a Squircular Continuum</i> .....	83
Chamberlain Fong and Douglas Dunham	
<i>Conformal Mappings of the Hyperbolic Plane to Arbitrary Shapes</i> .....	91
Eryk Kopczynski and Dorota Celińska-Kopczyńska	
<i>Animated Isohedral Tilings</i> .....	99
Craig S. Kaplan	
<i>Exploring a Taxicab-based Mandelbrot-like Set</i> .....	107
Loe Feijs and Marina Toeters	
<i>Generative Art from One-Dimensional Chip-Firing Automata</i> .....	115
Gary R. Greenfield	
<i>Portraits from the Family Tree of Plane-filling Curves</i> .....	123
Jeffrey Ventrella	
<i>Puzzling plane-filling curves</i> .....	131
Jörg Arndt and Julia Handl	
<i>A Generalization of the Chaos Game</i> .....	139
Tom Bates	
<i>A Family of Fern-like Ternary Complex Trees</i> .....	147
Bernat Espigulé	
<i>Which Flower is It?</i> .....	155
Andrew J. Simoson	
<i>Reconstructing Early Islamic Geometries Applied to Surface Designs</i> .....	163
Roger Burrows	
<i>Shape-Partitions: New Elements, New Artworks</i> .....	171
James Mai	
<i>From Seifert Surfaces to Star Cinders</i> .....	179
Carlo H. Séquin	

<i>Stiefel Manifolds and Polygons</i> .....	187
Clayton Shonkwiler	
<i>Tuti-Like Interweaving</i> .....	195
Abdalla G. M. Ahmed	
<i>Symmetries of Intermeshed Crochet Designs</i> .....	203
D. Jacob Wildstrom	
<i>Embroidery of a Hyperbolic Fish Pattern</i> .....	211
Douglas Dunham and Lisa Shier	
<i>Deriving Baskets</i> .....	217
James Mallos	
<i>Maths Craft New Zealand: An Unexpected Journey</i> .....	223
Jeanette C. McLeod and Phillip L. Wilson	
<i>Interlacing Mathematics and Art: Hands-on Non-Euclidean Geometry</i> .....	231
Andreia Hall, Isabel Brás, and Sónia Pais	
<i>Dancing Euclidean Proofs: Experiments and Observations in Embodied Mathematics Learning and Choreography</i> .....	239
Samuel J. Milner, Carolina Azul Duque, and Susan Gerofsky	
<i>Three-Dimensional Symmetries in Dance and Other Movement Arts</i> .....	247
Karl Schaffer	
<i>Animated Motions of Exceptional Flexible Instances of Generically Rigid Graphs</i> .....	255
Georg Grasegger, Jan Legerský, and Josef Schicho	
<i>Fractal Images from Multiple Inversion in Circles</i> .....	263
Peter Stampfli	
<i>Fun with Newton's Method</i> .....	271
Kerry Mitchell	
<i>The Secret behind the Squiggles: Guitars with Optimally Curved Frets</i> .....	279
Mitchell Chavarria and Jordan Schettler	
<i>Sound Signature Detection by Probability Density Function of Normalized Amplitudes</i> .....	287
Ion Bica, Zhichun Zhai, Rui Hu, and Mickey H. Melnyk	

<i>Mapping Diagrams and a New Visualization of Complex Functions with GeoGebra</i> .....	295
Martin Flashman	
<i>The Planar Crystallography Groups as an Iconographic Analysis Tool in Islamic Art</i> .....	303
María Antonieta Emparán	
<i>Ceramics Inspired by String Theory</i> .....	311
Nadav Drukker	
<i>Dissecting a Cube as a Teaching Strategy for Enhancing Students' Spatial Reasoning: Combining Physical and Digital Resources</i> .....	319
Diego Lieban and Zsolt Lavicza	
<i>Impossible Pictures: When Art Helps Math Education</i> .....	327
Matteo Torre	
 Short Papers	
<hr/>	
<i>Divisible Skylines: Exploring Least Common Multiples and Divisibility through Visual Art</i> .....	335
Saara Lehto, Anne-Maria Ernvall-Hytönen, and Tommi Sottinen	
<i>A Tribute to M.C. Escher in the Crop Fields of Fryslân</i> .....	339
Hans Kuiper and Henk Rusman	
<i>A Minimal Art Object with Six Horses in a Carousel</i> .....	343
Walt van Ballegooijen and Hans Kuiper	
<i>Escher's Polyhedral Models</i> .....	347
Doris Schattschneider	
<i>Curve Stitching Density Plots</i> .....	351
John Nicholson	
<i>Citizen Art – Collective Mathematical Art to Raise the Public Awareness of Mathematics</i> .....	355
Anna M. Hartkopf	
<i>Sombrero Vueltiao – Weaving Mathematics</i> .....	359
Milena Damrau	
<i>Hendecagonal Near-miss Polyhedral Cages</i> .....	363
Jonathan G. Heddle, Agnieszka Kowalczyk, and Bernard M. A. G. Piette	

<i>Images Produced via Modular Multiplicative Inverse Filters</i> .....	367
Donald Spector	
<i>Generalized Julia Sets: from Cantor Bouquet to Cantor Cheese</i> .....	371
Stefanie Hittmeyer, Bernd Krauskopf, and Hinke M. Osinga	
<i>Lehmer's Dance – A Lecture Performance</i> .....	375
Roos van Berkel and Tom Verhoeff	
<i>Eliciting Students' Visual-Spatial Thinking Processes in an Art Studio</i> .....	379
Mehtap Kuş and Erdiñ Çakırođlu	
<i>The Magnified Pixel: Digitally Fabricated Prototypes at the Intersection of Art, Mathematics and Architecture</i> .....	383
Ioanna Symeonidou	
<i>Teaching Advanced Mathematical Concepts with Origami and GeoGebra Augmented Reality</i> .....	387
Natalija Budinski and Zsolt Lavicza	
<i>The Hairy Klein Bottle</i> .....	391
Daniel Cohen and Shai Gul	
<i>Creating Deltahedra with Unfolded Net of Tetrahedron</i> .....	395
Liang-Chen Peng	
<i>Geared Jitterbugs</i> .....	399
Elisabetta A. Matsumoto and Henry Segerman	
<i>Exploring Raaga Improvisations of Carnatic Music with Mathematical Proof Writing</i> .....	403
Srividhya Balaji	
<i>The Obtetrahedrilla as a Modular Building Block for 3D Mathematical Art</i> .....	407
Tom Verhoeff and Koos Verhoeff	
<i>Symmetry in Koloman Moser's Flächenschmuck</i> .....	411
John M. Sullivan	
<i>Extensible Structures of Interlinking SL Strands</i> .....	415
Shen-Guan Shih, Chih-Hung Yen, and Long-Wei Chou	
<i>Combining Two Pictures on a Miura Fold</i> .....	419
Alewyn P. Burger	

<i>Curved Crease Folds of Spherical Polyhedra with Regular Faces</i> .....	423
Klara Mundilova	
<i>Light and Geometry: An Artistic-Mathematical Experience in Primary School</i> .....	427
Valeria González Roel, Teresa F. Blanco, Raquel Valuja Framiñán, and José Manuel Diego-Mantecón	
<i>Symmetry Patterns from Multiple Identically Patterned Cubes</i> .....	431
David A. Reimann	
<i>Grid-Based Circuits from de Bruijn Sequences</i> .....	433
Karl Kattchee	
<i>Mandelshapes: Thinking Outside the Mandelbox</i> .....	437
Gregg Helt	
<i>New Generalizations of Quadratic Julia Sets to 3D</i> .....	441
Václav Kučera	
<i>Braids Formed by the Impression of Knots</i> .....	445
Alexander Åström and Christoffer Åström	
<i>The Pythagorean Forest</i> .....	449
Josef Weese, Joshua Martinez, Christian Caraveo, and Jesse Atkinson	
<i>Sandpiles and Synthesizers: Listening to the Discrete Laplacian</i> .....	453
John A. Lind	
<i>Perception of Students in an Arts-integrated Mathematics Performance Task</i> .....	457
Kit Lee, Rachel Lim, and Oi Mei Teo	
<i>Discrete Gyroid Surface</i> .....	461
Ulrich Reitebuch, Martin Skrodzki, and Konrad Polthier	
<i>Polyhedral Music</i> .....	465
Jouko Koskinen and Petteri Mäkinieni	
<i>Maths Craft in Class</i> .....	469
Jeanette C. McLeod, Phillip L. Wilson, David Pomeroy, and Erik Brogt	
<i>Visually Synthesizing the Genotype-Phenotype Distinction in Cannabis sativa</i> .....	473
Martin D. Pham	
<i>Art and Math Using an Equally Linked Tetrahedron</i> .....	477
Josep Tarrés, Luis Sanchez Cuenca, Manuel Moreno, and Toni Vinyes	

<i>Fractal Analysis of a Sequence of LSD-Influenced Self-Portraits</i> .....	481
Debanuj Chatterjee	
<i>Regular Whitney Decompositions</i> .....	485
Krystian Kazaniecki and Michał Wojciechowski	
<i>Setting a Creative Math Task with SET 3D: Modeling Physical Pieces through Digital Resources</i> .....	489
Diego Lieban, Bjarnheiður Kristinsdóttir, and Zsolt Lavicza	
<i>3D Printing to Address Solids of Revolution at School</i> .....	493
Sandra Reichenberger, Diego Lieban, Cecilia Russo, and Barbara Lichtenegger	
<i>Light Painting: Visualization with and through Light</i> .....	497
Barbara Lichtenegger	
<i>Rhythm Domes: Applications of Euclid’s Algorithm to Architecture</i> .....	501
Eric Worcester	
<i>The Sum of Odd Integers Quilt</i> .....	505
Elaine Krajenke Ellison	
<i>Quest for the Golden Egg</i> .....	509
Hedy Hempe	
<i>Double-layered Weaving of Infinite Bi-foldable Polyhedral Complexes</i> .....	513
Jiangmei Wu and Matthias Weber	
<i>Tessellated Seven-Color Tori</i> .....	517
Ellie Baker and Kevin Lee	
<i>Mindful Geometries: Making and Moving Inside the Icosahedron</i> .....	521
Daniela Bertol	
<i>Search for a Perfect Unequal Trapezohedron with Similar Faces</i> .....	525
Chirag Mehta	
<i>The Children’s Congress at the JKU Linz: Young Researchers and Interdisciplinary Projects</i> .....	529
Sara Hinterplattner, Barbara Sabitzer, and Jakob Skogø	
<i>Floraautomata   Faunaautomata</i> .....	533
Anna M. Chupa and Michael A. Chupa	

<i>Artistic Excursions with the Sierpinski Triangle</i> .....	537
Roza Aceska and Ben O’Brien	
<i>The Surprising Symmetry Pairs of 24 Di-Oct Ochominoes Tiles</i> .....	541
Kate Jones	
<i>Experiencing Mathematics through Astonishment</i> .....	545
Jean Francois Maheux	
<i>Magritte Meets Matisse Meets Mathematics</i> .....	549
Demian Nahuel Goos	
<i>Bead-Chain Construction Set and Interlocking Puzzle Inspired by Polyhedranes</i> .....	553
Bih-Yaw Jin	
<i>Exploring Simple Geometries Using Beaded Jewelry</i> .....	557
Petra Surynková	
<i>Looking at Mathematics through the Lens of a Camera</i> .....	559
Karina Amalia Rizzo, Laura Sombra del Río, and Mónica Ester Manceñido	
 Workshop Papers	
<hr/>	
<i>Hungry Birds – a STEAM experience</i> .....	561
Melissa Silk, Corey Stewart, and Annette Mauer	
<i>Exploring Randomness in Digital Art</i> .....	569
Lali Barrière	
<i>Active Learning ArtMath Project in College Algebra Classes</i> .....	575
Elizabeth Lugosi	
<i>Yo Math Is So Arty: Inspiring Creative Learning with Mathematical Internet Memes</i> .....	583
Giulia Bini and Ornella Robutti	
<i>A Shape-based Approach to Creativity and Connection Making</i> .....	587
Briony Thomas, Azael Capetillo, Alejandra Díaz de León, Fabio López, and Rafael Machado	
<i>Exploring Kaleidocycles with LUX</i> .....	595
Michael Acerra, Matias Kaukolinna, Alexey Ivchenko, and Kristóf Fenyvesi	
<i>Weaving Mondrian with GeoGebra</i> .....	599
Marina Menna Barreto and Keri Howard	



<i>A Synthesis of Sectors</i> .....	605
Annette Mauer, Melissa Silk, and Lisa Giles	
<i>Using GeoGebra and 3D Printing for Introducing Voronoi Diagrams in School</i> .....	611
Edith Lindenbauer and Sandra Reichenberger	
<i>Mathematics and Maypole Dancing</i> .....	615
Christine von Renesse and Julianna Campbell	
<i>Maths in Motion: Exploring Rotational Symmetries and Triangles through Dance and Body Movement</i> .....	621
Lena Nasiakou, Saara Lehto, Svetlana Goranova, Kerry Osborne, and Kristóf Fenyvesi	
<i>Building Polyhedra Models for Mathematical Art Projects and Teaching Geometry</i> .....	629
Mircea Draghicescu	
<i>Environmental Problem-solving and Hands-on Geometry Learning through Storytelling inside a Geodesic Dome: Ice, Honey and Stardust</i> .....	635
Kristóf Fenyvesi, Christopher S. Brownell, Hannu Salmi, Ho Gul Park, Adela Muntean, Matias Kaukolinna, Helena Thuneberg, Franz Bogner, and Zsolt Lavicza	
<i>How to Use Prime Numbers and Periodicity to Write a Poem</i> .....	643
Emily R. Grosholz and Sarah Glaz	
<i>Make Music Visible, Play Mathematics</i> .....	647
Andrea Capozucca and Marco Fermani	
<i>The Organ Base in Origami: Teaching Mathematics and Creativity in the Elementary Classroom</i> .....	651
R. Alan Russell, William DeMichele, Nicole Holland, Caroline Lanzarone, and Nettie Leepson	
<i>A Fisheye Gyrograph: Taking Spherical Perspective for a Spin</i> .....	659
António Bandeira Araújo	
<hr/>	
<i>Author Index</i> .....	665



## Preface

Welcome to the 22nd annual Bridges Conference! This year we travel to Linz, the beautiful capital of the state of Upper Austria, on the banks of the Danube River. In the city where Johannes Kepler discovered his third law of planetary motion, mathematicians, scientists, artists, musicians, architects, dancers, designers, writers, educators, and other explorers of the deep connections between mathematics and the arts come together in a lively atmosphere of intellectual exchange. Our host institutions this year are Johannes Kepler University and Ars Electronica Centre.

Johannes Kepler University (JKU), the largest university in the region of Upper Austria, will host Bridges 2019 in its STEM Education Centre. Here, Professors Markus Hohenwarter and Zsolt Lavicza have established a STEM Education Research Group which is integrating the arts to become a STEAM education center in the near future. The STEM Education Research Group runs a Masters and PhD program in mathematics, science and computer science education. Currently, there are 12 PhD students and 20 masters students enrolled in its program. The group also works in close connection with more than 160 international GeoGebra Institutes, a network of teacher education and research groups at universities in 75 countries that has its headquarters in Linz. The STEM Education Research Group holds regular seminars, organizes conferences, and hosts international visitors, all to create and support an environment for international research. The mathematics education team has many years of experience in research and development projects and a profound knowledge of all aspects of teaching and education.

The Ars Electronica Festival began in Linz in 1979, bringing together artists and scientists from around the world every two years for an interdisciplinary exploration of a particular theme. In 1986, the festival became an annual event, and ten years later, the original Ars Electronica Center opened. When Linz was designated a European Capital of Culture in 2009, the center was expanded into the vibrant facility we see today. The Ars Electronica Center is a museum of the future, devoted to the themes of Art, Technology, and Society. In addition to the annual festival, the center runs the Prix Ars Electronica competition and the Futurelab research and development facility. The exhibitions in the Ars Electronica Center are built around interactivity and participation, allowing visitors to immerse themselves into visions of future society and explore how we communicate with our environment. Here, biotechnology, genetic engineering, neurology, robotics, prosthetics, virtual reality, media art, and all of their intersections are on display, and there are no “Do Not Touch” signs. The integration of art and science, the focus on innovation and education, and the emphasis on interactivity are all themes that resonate with the mission of the Bridges Organization, and we are grateful that Ars Electronica is joining JKU in opening their doors to our community.

This year’s Bridges Program Chair is Susan Goldstine. She coordinated an international Program Committee of over 75 experts who provided extensive reviews and editorial comments on the roughly 200 submissions we received this year. She also served as chair of the regular paper track, and as chair of the Math + Fashion Show committee. Douglas McKenna chaired the short

paper track, and Kristóf Fenyvesi chaired the workshop paper track, each handling a record number of submissions. David Chappell designed the Kepler-inspired planetary orbit cover for the proceedings and the complementary cover for the art catalog. Special thanks go to Bianca Violet for heading the Short Film Festival committee, to Sarah Glaz and Emily Grosholz for organizing the Poetry Reading, to Uyen Nyugen for proposing and editing the Math + Fashion Lookbook, to Stephen Abbott for running the Theater Event, to Mike Naylor for leading Informal Music Night, and to Kristóf Fenyvesi for organizing the public Family Day.

The 2019 edition of the Bridges proceedings includes 4 invited papers, 41 regular papers, 59 short papers, and 17 workshop papers. Among these pages, you will find new discussions of the aesthetics of mechanical puzzles, freeform surfaces in architecture, fractals and generative art, paper folding, polyhedral models, fiber arts, poetry construction, mathematical music, large-scale crop designs, art in the mathematics classroom, and many other topics. We have workshops on music, perspective hand-drawing, mathematical connections in dance and body movement, creative methods in STEAM and technology for education, paper folding, creative writing, creating digital art, and hands-on geometric constructions. It is a testament to the vibrancy of the Bridges community that we have so many excellent papers on such a wide variety of topics. We are especially grateful to this year's Program Committee for their core service in the increasingly difficult task of selecting papers from our ever-growing pool of submissions, and for helping our authors to make their work shine. Extra thanks to Eve Torrence, George Hart, and Craig Kaplan for their advice and support throughout the process.

An exhibition of mathematical art has been an annual feature of Bridges since 2001. Artists participate from around the world, representing diverse cultural backgrounds. A wide variety of artistic media are represented in the exhibition, including 2D and 3D digital prints, drawing, painting, beadwork, weaving, ceramics, woodwork, stained glass, metalwork, quilting, and paper cutting and folding. Artists drew inspiration from the mathematics of fractals, polyhedra, non-Euclidean and four-dimensional geometry, tiling, knot theory, number theory, and more. This year Uyen Nguyen and Robert Fathauer served as co-curators of the exhibition. The jury considering the artworks consisted of Karl Kattchee, Barbara Lichtenegger, Uyen Nguyen, and Robert Fathauer. The print catalog was prepared by Conan Chadbourne with cover art by David Chappell, and the art submission website was created and administered by Nathan Selikoff. Barbara Lichtenegger was the local art exhibition coordinator in Linz.

We are grateful for the contributions to the organization of the Bridges 2019 conference from Johannes Kepler University and Ars Electronica Centre in Linz. First of all, we would like to thank the local organizing committee Zsolt Lavicza and Sandra Reichenberger from JKU, Barbara Lichtenegger from the Private University of Education, Diocese of Linz, and Andreas Bauer from Ars Electronica Centre for their efforts in the organization and making this event happen. We are also thankful for the continuous support of Professor Markus Hohenwarter, the Head of the Linz School of Education, in making connections within the University and in the City as well as in securing substantial administrative and financial support for this event. JKU has greatly supported our efforts by offering the University facilities for the conference and

assisting us with technical and administrative issues; we are particularly grateful to Rektor Meinhard Lukas, who made the conference possible at JKU and granted financial support for the event. Also, we are thankful for the contributions of the PR office at JKU for helping with the design of our conference cards, posters and video. The promotional video was professionally produced by Chris Noelle and was viewed by thousands of interested people around the world. For administrative support in dealing with financial and booking arrangements, Barbara Fröhlich and Günter Sageder were incredibly helpful in making the conference run smoothly. We also appreciate the assistance and flexibility of the staff of the Sommerhaus Hotel. Similarly to JKU, the Ars Electronica Centre with the lead of Andreas Bauer offered incredible help in the organization and offered us their support and facilities for the conference. The Arts University in Linz was an active member of the organization and we appreciate that we can host events in its buildings. We are grateful to the Tabakfabrik for hosting us, and to Andreas Weixler and Se-Lien Chuang from Bruckner University for offering their facilities and expertise. Furthermore, the city of Linz and Upper Austria as well as the Linz City Tourist Office offered great help for supporting participants and organizing excursions. In addition, we are thankful for our student helpers and the long list of volunteers who signed up to assist at the Bridges conference. Finally, we would like to acknowledge the continuous assistance of Kristóf Fenyvesi as a liaison, both online and in person in his visits to Linz.

This year we were able to offer student travel scholarships to seven students from around the world who authored accepted papers and created mathematical artworks. We are very grateful to the Bridges community for supporting the Reza Sarhangi Memorial Fund through the rezafund.org online art auction to make this program possible. This is a testament to the enduring, generous spirit of Bridges founder Reza Sarhangi. We still feel his absence, but we take comfort in the strong and nurturing community he built.

We welcome you to join us in our celebration of mathematics and the arts, whether in person in Linz, through the pages of our printed publications, or in our online archive that covers two decades of magnificent work from conferences past. Bridges serves as a beacon for all those fascinated by the connections between mathematics and the arts. With your support, it will continue to light the way for many years to come.

The Bridges Organization Board of Directors and Bridges 2019 Chairs  
[www.bridgesmathart.org](http://www.bridgesmathart.org)

