# The Organ Base in Origami: Teaching Mathematics and Creativity in the Elementary Classroom 

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#### Abstract

Origami, the Japanese art of paper folding, tends to arise organically from the intersection of mathematics and art. This workshop explores the organ base in origami as both an item for creative play and a manipulative for the elementary mathematics classroom. Participants will learn to fold several origami models starting from the organ base. We will then explore the ways in which we used the model for elementary mathematics. Aside from the printable materials, workshop participants will use notions of Williams' taxonomy of Creative Thinking as brainstorming techniques for more ways to use the model in a classroom setting.


## Introduction

Much of our research focuses on the use of simple origami models that we convert into useful classroom manipulatives for teaching mathematics. Kathy Brady [1] suggests that paper folding in the elementary classroom improves students' engagement in several ways. Students have affective engagement because they enjoy the paper folding. Behavioral engagement is seen by student persistence when folds become difficult and by their deep concentration on the task. Brady also documents sustained cognitive engagement as students explain their reasoning over the models they have created. While we have witnessed each of these levels of engagement, we claim that paper folding also enhances creative development in both students and teachers. This article explores the organ base as a simple origami model, a tool for teaching mathematical content, and a vehicle for enhancing the creativity of both students and teachers.

## Williams' Taxonomy of Divergent Thinking

In an effort to define what creativity means and in hopes that we might "develop the creative potential of school children", Frank Williams [3] created a list of four ways creative people think and four ways that creative people act. This list has become Williams Taxonomy of Divergent Thinking.

## Four Ways to Act

Williams noted that creative people exhibit curiosity, risk-taking, complexity, and imagination. Curiosity represents a willingness to engage in learning while risk-taking reflects the courage to try new things without a fear of failure. Complexity, for Williams, reflected a person's ability to find order in chaos, while imagination was particularly an intuitive ability to visualize new ideas. These four affective behaviors may be quite difficult to teach.

## Four Ways to Think

Williams noted that creative people think with fluency, flexibility, originality, and elaboration. Fluency is the ability to think of many solutions to a problem. Flexible thinking is adaptive, often working around limitations to find new alternatives. Originality, for Williams, seemed generally linked to one's ability to blend two disparate ideas together while elaboration was a person's ability to give more information through
embellishment and expansion. These four cognitive behaviors are much easier to teach. Once we have explored the organ base as an origami model, we will revisit fluency and flexibility in more depth.

## The Organ Base

Bases in origami refer to an intermediate stage of folding that is shared by many models. The organ base, Figure 1, then becomes the starting point for a variety of models including the piano or organ which gives the base its name.


Figure 1: The organ base is the starting point for the piano (organ), fox puppet, GI hat, and the house.
This model is simple to fold and easily accessible to folders as early as kindergarten which makes this a great model for the elementary classroom. Figure 2 shows the folding instructions.


Figure 2: Folding the organ base.

Step 1 - Fold the square in half top to bottom.
Step 2 - Fold the rectangle in half edge to edge and unfold.
Step 3 - Fold the outer edges to the center crease.
Step 4 - Squash fold the left side. That is, separate the two layers of paper as you press down from the top.
Step 5 - Squash fold the right side to get the completed organ base.

## The Activities

I brought the organ base to my group of research students who happen to be pre-service elementary and middle school teachers. In an effort to work on our own fluency, each created twenty-five uses for the organ base in the elementary classroom. Note that if I had asked for one to three ideas, the students might have succumb to self-editing. The point of fluency is to generate as many possibilities as you can. Asking for twenty-five uses then opens the floodgates with no idea discarded prematurely. What follows are four of their one hundred ideas. Each activity will share the particular manipulative for that activity as well as an overview of the procedures used in the classroom.

## Activity 1: Fraction House

Time: This activity should take approximately 30 minutes.
Materials: Start with a square of paper.
Common Core Math Standard [2]: CCSS.3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

## Activity Instructions:

Start with a square sheet of paper and reference this a one whole unit for the entire activity. As students progress through folding the organ base, they are asked what fraction they have created at each step. After folding the paper in half through Step 1, they have, of course created the fraction $1 / 2$. You may choose to unfold the paper and reinforce the fact that 1 is equivalent to $2 / 2$. After refolding Step 1 and proceeding through Step 2, students have a sheet that has an area $1 / 4$ the area of the original square. If you open this paper back to the previous step, you can reinforce student understanding that the $1 / 2$ from the previous step is equivalent to $2 / 4$. You may even choose to unfold this step entirely and make note that 1 is equivalent to $4 / 4$ or $8 / 8$. The challenge in this activity comes by determining the fractional area of the completed model. With enough time to process, third graders will eventually arrive at the correct answer of $7 / 16$ and be able to share their reasoning logically with other students as shown in Figure 3.


Figure 3: Reasoning the fractional area using the full square in sixteenths for scale.

## Activity 2: Number Patterns and Sequences

Time: This activity should take approximately 30 minutes.
Materials: Start with a square printed as shown in Figure 4.


Figure 4: Number Pattern template for Activity 2.
Common Core Math Standard [2]: CCSS.Math.Content.3.OA.D.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

## Activity Instructions:

This manipulative has the numbers one through twelve printed on it. The teacher can make several of these and fill in one specific operation (multiplication or addition) per sheet. For the purposes of illustration, we have chosen multiplication by 4 . Once students have folded the paper into an organ base, they can begin to solve the manipulative by filling in the answers to each of the individual multiplication problems. Once completed, the model will stand and students can begin to investigate the patterns they see. Students must formalize the pattern and then prove to other students that their pattern is correct. In the example of multiplying by 4 , students could illustrate that each number is even by actually calculating the two equal addends for each number as shown in Figure 5.


Figure 5: Proof that multiples of 4 are even.

Students can also open the entire model flat and formally write their findings in the middle of the manipulative as shown in Figure 6．Notice that the findings are hidden when the model is folded．

| $\begin{gathered} \text { の亡+の亡= } \\ 乙 \varrho \end{gathered}$ |  | $て+て=$ |
| :---: | :---: | :---: |
|  |  | $\downarrow$ |
|  | Multiplying by 4 always gives an answer which is even．We showed the answers were even by showing they each have equal addends． |  |
| 28 |  | 8 |
| $=14+14$ |  | $=4+4$ |

Figure 6：The findings are then written inside the unfolded model．

Other students or groups of students can then pick up the model and investigate the work they see on the outside．If they think they know the pattern，they can open the model flat for confirmation of their findings．

## Activity 3：STOP！and Practice Addition

Time：This activity should take approximately $30-45$ minutes．
Materials：Start with two printed rectangles shown in Figure 7.


Figure 7：Printables for Activity 3.

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*Note that we have two $2 \times 1$ rectangles printed. If you fold these in half, you will have a square that is printed on both sides. This is required for the finished model to work correctly. The double-thickness has not caused difficulty when folding the model.

Common Core Math Standard [2]: CCSS.1.OA.9: Demonstrate fluency with addition within ten.
Activity Instructions:
Half the students are given an "Addition" sheet while the other half are given a "Practice" sheet. Students fold the organ base but hold it so that the words and numbers can be read correctly. Students pair up addition and practice to form a stop sign as shown in Figure 8. Now they STOP! to practice addition.


Figure 8: Making the Stop Sign for a pair of students.

This student pair can change the flaps to run through thirty-two different problems or, once they have solved the four visible problems, they can manipulate the flaps as they move to find a different partner for a new set of four problems. Since all of the numbers are between zero and five, the sums are guaranteed to stay between zero and ten.

## Activity 4: Build a Neighborhood in Math Town

Time: This activity should take approximately 20 minutes.
Materials: Start with a set of 10 houses with objects inside as shown in Figure 9.


Figure 9: Ten house blanks with items inside each house.

Common Core Math Standard [2]: CCSS.K.CC.6: Compare numbers- Identify whether the number of objects in one group is more, less, or equal to the number of objects in another group, when the quantities are clearly different.
Activity Instructions:
You may fold the set of ten houses and give them to a group of students or have the students help with folding if you have an additional 10 minutes. Each house has a different number of objects printed on the inside. The objects include one bed, two bathtubs, three televisions, four lamps, five people, six telephones, seven bowls, eight chairs, nine spoons, and ten hangers. Students count the number of objects inside of each house and write the answer on the sides of the house as in Figure 8.


Figure 8: The eight house.
This will act as the house address. Students then use the answers (the new house addresses) to arrange the houses in numerical order. This is how all the streets in Math Town are organized. Next, have students arrange the houses with even numbers on one side of the street and odd numbers on the other.

## Creative Investigation

To begin your own creative investigation, it may be fruitful to note patterns in the overall brainstorming we have done so far. As with Activity 1, about a fourth of the uses we found during brainstorming involved observation of the physical model. We measured angles that naturally arise from the folding procedure, identified shapes such as triangles and squares that lay within the model, and compared the physical dimensions of the finished model with the starting dimensions of the square. Activity 2 and 3 fell into a classification we called flap manipulation. Activity 2 allowed us to look at various calculations side by side so the elementary students might notice patterns among those calculations. Once they had a conjecture in mind, they could manipulate the flaps to bring a new set of four calculations into view. Once the conjecture held for all views of the manipulative, the students should feel they have a solid finding. Activity 3 used flap manipulation to create a randomly generated set of thirty-two problems. Flap manipulations represented about half of the brainstorming ideas we created. Activity 4 fell into a class of uses we called piecewise. Piecewise is distinct from the others because many models are created and used collectively in activities that feature sorting, classifying, and organizing. For us, thinking about observation, flap manipulation, and piecewise models helped us brainstorm even further.
Note that all of the results we have reported came from Williams' [3] notion of fluency. He concluded that one of the key ways that creative people think is their urge to create many responses to a prompt. If we shift gears slightly and think about Williams' notion of flexibility or the ability to adapt and work around limitations, we can brainstorm even more ideas. To do this, one simply has to create limitations. For
example, what if we do not have a square piece of paper? A long rectangle folded into the house shape would create a long, short object reminiscent of a "net" for table tennis or a name card that would accommodate a student's full name. What if we have no paper? We have made these organ bases from waxed paper, cardboard, and even carpet remnants. Readers of this article are encouraged to create their own limitations to explore.

## Why Limit to Mathematics?

The creative journey we began by bringing Williams' Taxonomy, origami, and the elementary mathematics classroom together has led us to many interesting explorations and classroom activities. It doesn't take very long when asking yourself "What else can we do?" to figure out that the entire elementary curriculum can be explored.
Literacy and language arts ask students to learn word families. If those families live in houses, you can easily imagine using this model to create an "-at" house. In the "-at" house, you'll find a cat, a bat, a hat, and a rat as in Figure 9. An art lesson might ask us to draw the items ourselves. The reader is encouraged to think about science, history, music, and beyond.


Figure 9: The -at house.

## Summary and Conclusions

The organ base is a model that is simple enough for all students to fold successfully in the elementary classroom from kindergarten upward. Play is a natural part of creativity and a natural part of Williams' notion of fluency. If creativity is going to be one of the hallmarks of education in the future, then giving students a solid foundation for fluency is a crucial first step.
To satisfy Williams' Taxonomy of Divergent Thinking, you should now create twenty-five models on your own using fluency. Please let us know if you come up with a category that differs greatly from observation, flap manipulation, or piecewise. When you have completed that, try refining your flexibility by altering shape, size, and source material. It would be better still to find an elementary classroom full of young minds unspoiled by the education system and set them free on this creative journey.

## References

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