The Surprising Symmetry Pairs of 24 Di-Oct Ochominoes Tiles

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Abstract

Ochominoes are a new type of polyform tiles: polyocts, specifically its subset of di-octs. Di-octs consist of pairs of octagons edge-joined to form a kind of domino, with from 0 to 6 squares attached on their diagonal edges in all possible combinations. The tiles are reversible, so mirror reflections are not included as separate pieces. We explore the different types of symmetries that pairs of Ochominoes tiles can form—orthogonally, diagonally, rotationally—as 2x2 or 1x4 pairs or even obliquely. Hand-derived solutions have been computer-verified. We also illustrate the aesthetics of such mathematical sets: math as art.

The Ochominoes Set

The unique Ochominoes set was created by Dan Klarskov of Denmark, developed with Kate Jones in 2016-2017, and produced by Kadon Enterprises, Inc. Named and styled by Kate Jones, the set is color-coded for the number of squares each piece has. The 0 and 6 share a color. Each shape is distinct (Figure 1). Many even have nicknames, like Teddybear, Cat, Duck, Penguin, Heart, etc.

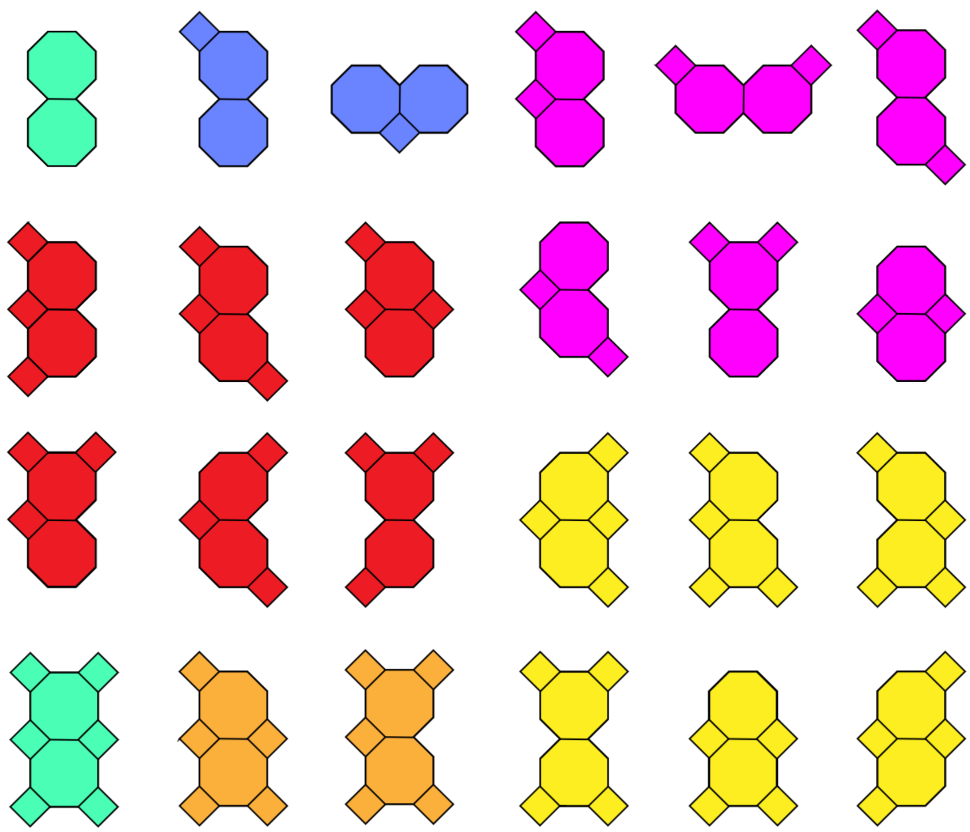

Figure 1: *The 24 Ochominoes tiles.*

Symmetry Types

Rectangles

The most compact symmetrical shape the entire set of 24 tiles can fit into is a 7x8 rectangle with 8 single octagonal spaces symmetrically enclosed. Figure 2 shows a few variations for embedding the free spaces:

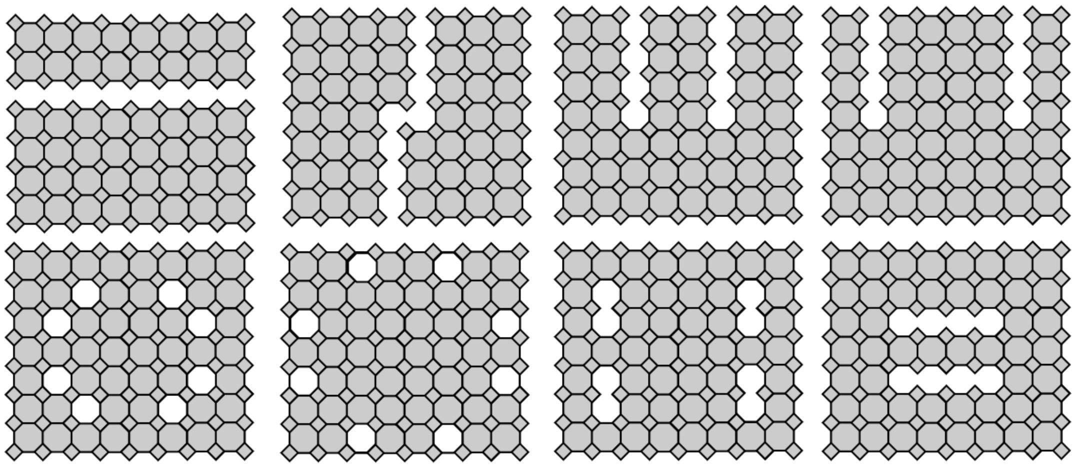

Figure 2: *Within the 7x8 rectangle, 8 octagonal free spaces can have many configurations.*

Mirror and Rotational Symmetries

Inexhaustible numbers of figures invite creativity and solving. Figure 3 shows just a few designs of vertical, horizontal, diagonal, and rotational symmetries, all proven solvable.

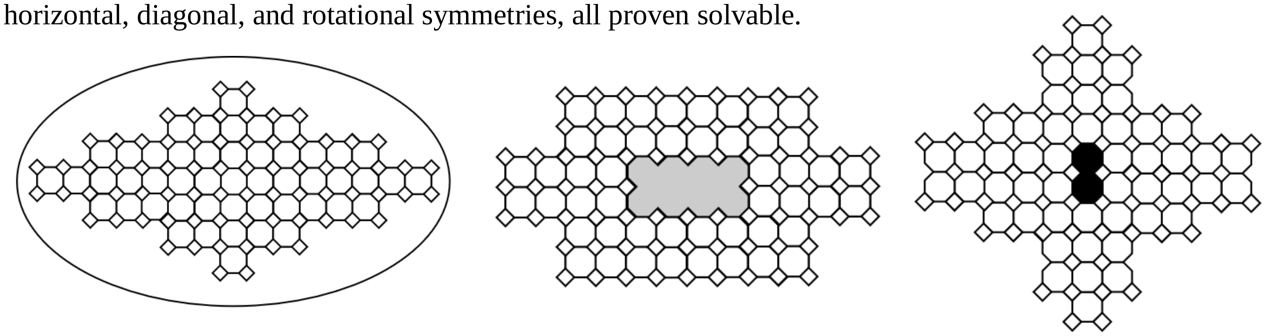

Figure 3: *Patterns with simultaneously horizontal, vertical, diagonal, and rotational symmetries.*

Symmetrical Pairs

The 24 all-different Ochominoes tiles have an amazing capacity for pairing up and forming symmetrical units. The total number of ways such pairings can form a symmetrical 2x2 has been identified as 122 [1]. Other pairings can be lengthwise or oblique. We can look for 12 simultaneous pairs with surprising characteristics. Figure 4 shows them joined end to end, as 1x4 units with all but one pair symmetrical on the short axis. The 12th has lengthwise symmetry. This grouping ranges from 3 to 9 edge squares.

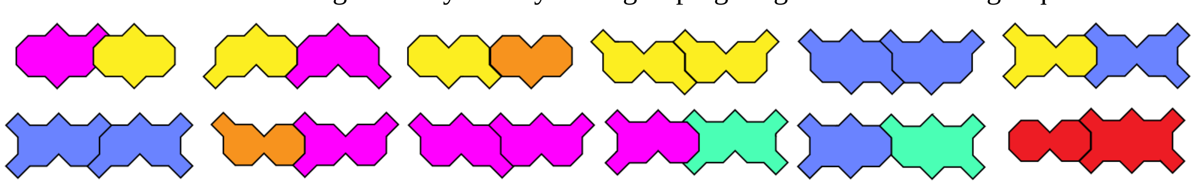

Figure 4: *Symmetrical pairs as 1x4 units, with all but one pair symmetrical on the short axis.*

Forming symmetrical pairs as 2x2 units (we call them "molecules") allows 12 different pairs, no two congruent, as shown in Figure 5. In other arrangements, up to 7 congruent pairs are possible. Again, the axes of symmetry can be vertical, horizontal, diagonal, and rotational, with 1 to 4 edge squares.

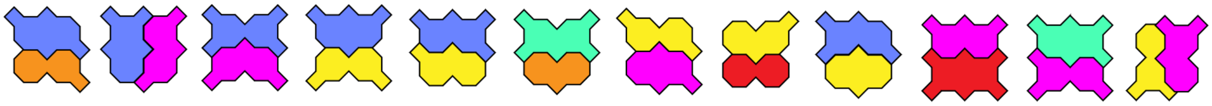

Figure 5: *12 all-different 2x2 symmetrical pairs. Note the one rotational and four diagonal pairs.*

Since 12 tiles are themselves symmetrical and the other 12 non-symmetrical, we investigated whether mixed pairs are possible: forming 12 pairs with one from each group. The results were surprising: one symmetrical tile would not pair with any non-symmetrical one. So of the 12 pairs, 10 are "mixed" and 2 are paired with one of their own, shown in Figure 6. The remarkable feature of the mixed pairs is that the joint line between their tiles is always diagonal while the symmetry axis is vertical, even the first one with rotational symmetry. Further, note the two congruent pairs at the end of the first row and the four congruent pairs on the second row. Even the two unmixed pairs are congruent with each other. These range from 1 to 7 edge squares.

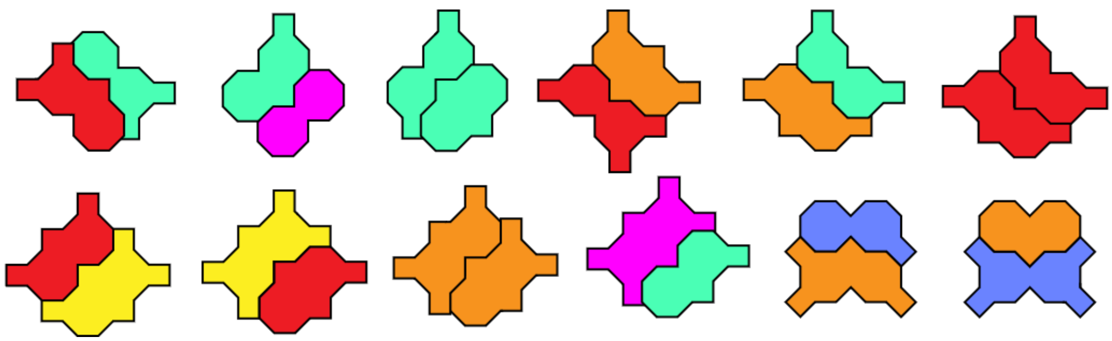

Figure 6: *The 10 symmetrical "mixed" pairs made of one symmetrical and one non-symmetrical tile, leaving two pairs with two non-symmetrical and two symmetrical tiles joined.*

With so much cooperation among the 24 tiles, the question arose: Can they form a complete string where every adjacent pair was a symmetry? They readily formed rows of 23 tiles and 22 symmetrical pairs, with some last piece not fitting anywhere. Figure 7 shows one such row, with one loner left out. Note the symmetries viewed horizontally, vertically, diagonally, and even rotationally. An exhaustive search showed that any one of 9 specific tiles (Figure 8) could be the loner.

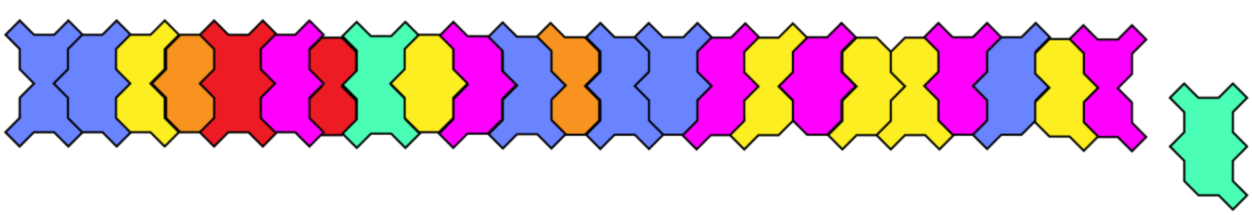

Figure 7: *A string of 23 Ochominoes tiles with every adjacent pair forming a symmetry.*

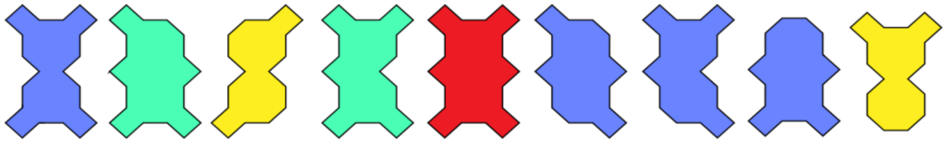

Figure 8: *The 9 Ochominoes tiles that can be the loner from a symmetry stream of 22 pairs.*

So far in all the symmetrical pairs, no interior holes were allowed. If we bend that rule to allow a single hole in a string of 24 tiles, as shown in Figure 9, we get the best surprise: All 24 tiles form only symmetrical pairs. This remarkable solution was found by George Sicherman [2] with his computer search program. It raises a new question: Can all 24 tiles form all symmetrical pairs so that, allowing one hole, the two ends also match, closing a loop?

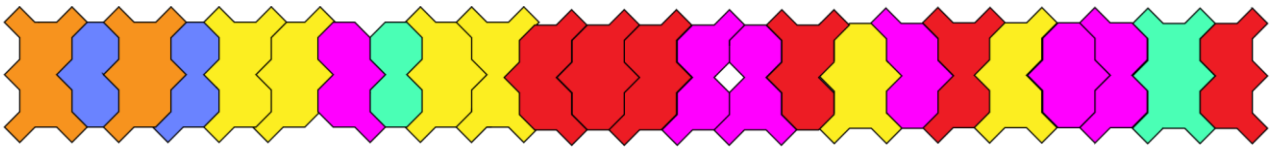

Figure 9: *With a single hole, all 24 tiles can form symmetries with all adjacent tiles.*

One further type of symmetrical pairs is still under investigation: *oblique*, where two tiles placed lengthwise join their end octagons side to side. Figure 10 shows the six that have been identified to date. The question is: Can 12 pairs be formed? Since many tiles require a copy of themselves as the other half, this is an elusive challenge. The first puzzler to send us an answer, or proves it unsolvable, will win a prize.

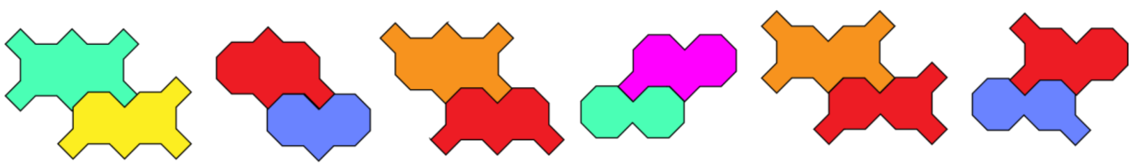

Figure 10: *Obliquely symmetrical pairs of Ochominoes tiles, a partial solution.*

Conclusion

The Ochominoes combinatorial puzzle set presents ever new research challenges in symmetries and tilings. In this paper we have described a few of the surprising findings made to date. In addition, this award-winning [3] set is a fine example of how simple math patterns can be used as a medium for creating surprisingly complex art. This short paper cannot do the Ochominoes set full justice for its artistic potential. Its accompanying handbook [4] illustrates over 100 attractive patterns to solve. I have named this original form of recreational mathematics "playable art".

References

[1] http://www.gamepuzzles.com/ochom.htm#sym
[2] https://userpages.monmouth.com/~colonel/polycur.html
[3] http://www.gamepuzzles.com/megistian2.htm
[4] http://www.gamepuzzles.com/ochbook.pdf