3D Printing to Address Solids of Revolution at School

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Abstract

The use of technology has evolved over the last decades offering new opportunities for mathematics education. In particular, integrating dynamic mathematics software with 3D printing becomes an alternative to transform digital representation into concrete manipulatives. In this paper we aim to offer examples in which the use of 3D printing can support the investigation of solids of revolution.

Introduction

In recent years, 3D printing technology has become more affordable and commercially accessible [1]. Some mathematicians and artists have been using algebraic description and geometric construction to create 3D printing Euclidean or non-Euclidean models, opening new possibilities to visualize and understand mathematics [1][7]. For instance, Henry Segerman [7] published a beautiful book through which he exhibits 3D structures and offers readers access to the 3D printing files and to hands on exploration of the printed objects. Symmetry, tiling or knots are some of the mathematical ideas that can be investigated through his models. We believe that 3D printing technology can also be used to trigger opportunities for teaching and learning of mathematics at school level, opening students' and teachers' minds towards creative thinking. In this paper we aim to address one of these possibilities.

3D Printing as a Motivational Resource

There are several possibilities to implement 3D prints in schools outlined in different publications [e.g. [2], [3], [7], [10]]. Teachers and students can experience how to design mathematical objects and how mathematical models can be brought to reality. Witzke, Hoffart [10] and Dilling [3] attempted to classify the use of 3D printing with students and addressed some important didactic concepts ("EIS-Principle" of Jerome Bruner, "Three Worlds of Mathematics" of David Tall, "subjective realms of experience" of Heinrich Bauersfeld). On the one hand, the potential of the 3D printing technology for the mathematics classroom was emphasized in the literature. On the other hand, several authors such as Pielsticker and Witzke [5] highlighted in their research that most teachers consider 3D printing a time-consuming extra gadget, and not useful for their everyday classroom practice. For this reason, it is necessary to offer teachers prime examples that are connected to the curricula and highlight the benefit of the 3D printing technology.
In this paper, we outline some perspectives on using the dynamic mathematics software GeoGebra and 3D printing as a motivational resource to introduce some exploratory modelling activities. We focus on one specific topic: solids of revolution and the insights to investigate their volumes.

From Mental to Digital Visualization

Transformations in the 3D space such as symmetry and rotation usually are not emphasized enough during the school years. When this is done, it focuses mostly on exploring rotations of simple planar figures about the coordinate axes. Shriki, Barkai and Patkin [8] in their paper discuss how they use this idea to foster students' mental rotation skills as shown in Figure 1.

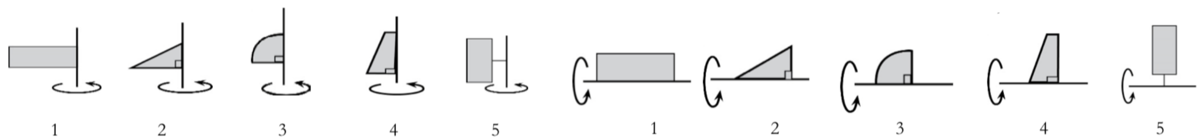
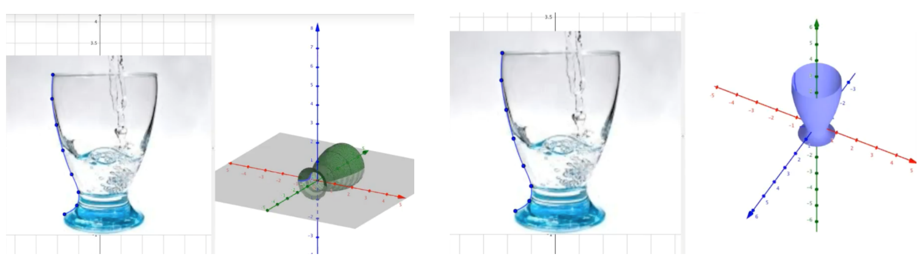


Figure 1: *Assignment II – Drawing representations of solids of revolution of 2-dimensional shapes [8].*

The benefit of 3D representation using dynamic mathematics software such as GeoGebra is that it allows the users to interact with these rotational representations. Some examples of solids of revolution in which students can investigate a variety of functions and quickly visualize the virtual outcome generated by their rotation are already available online (e.g. [4] and [9]) and have been used by some educators.

Virtual Models Becoming Hands-on Materials

A newly implemented feature on GeoGebra enables the user to create a 3D digital model and export it to an STL file (the extension of printable files). This implementation encourages teachers and researchers to create new task designs and inquiry approaches. Here we show how the rotational concept could be introduced in a meaningful and funny task, generating solids of revolution from objects of daily-life based on their images. Both of them apply the same principle of using a guideline that is easily adjustable with dragging features and then rotating this line around of one particular axis. The example below (Figure 2) shows a glass modeled from a set of points carefully arranged on the boundary of the object and turned around its axis of symmetry. The 3D representation turns out then in the right side and it can be seen from different perspectives and mesh styles ((a) and (b)).



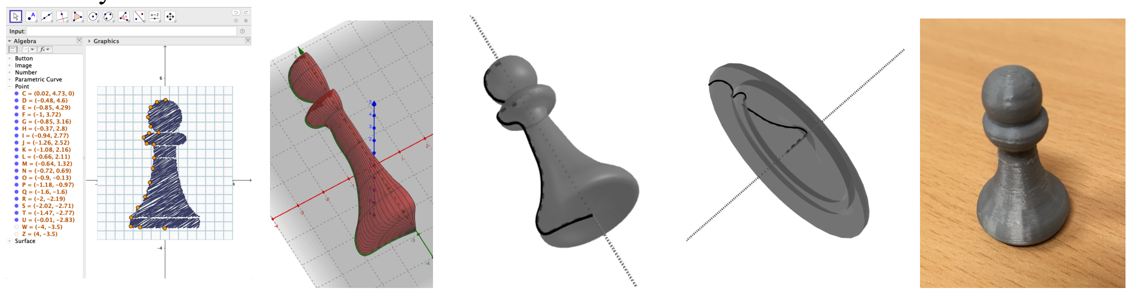
(a) (b)

Figure 2: *Glasses or other object with radial symmetry can be easily generated and printed. (https://www.geogebra.org/m/sy22mmth)*

The mathematics behind this construction involves the Spline Command. This command uses a piecewise polynomial function in a single variable to adjust an appropriate curve to a set of given points. This practice of adjusting points, combined with other GeoGebra features, enhances the visualization of how the layers change and affect the shape of the rotational solid. This can support the understanding of adding up infinitely thin cross sections to obtain the volume of the solid, opening a discussion about the Cavalieri's Principle and central idea of the Integral.

The next example (Figure 3) illustrates the different steps of the process: (a) adjustment of the points; (b)-(c) generation of the target solid by the curve previously defined; (e) final 3D printed model.

Moreover, this kind of task serves to easily highlight the different outcomes regarding the orientation of the rotation. For example, as shown in Figure 3 ((c) and (d)), we have to be careful in deciding the axial line of symmetry to properly obtain the final pawn. That is a common gap observed by the authors in introductory courses of Calculus.

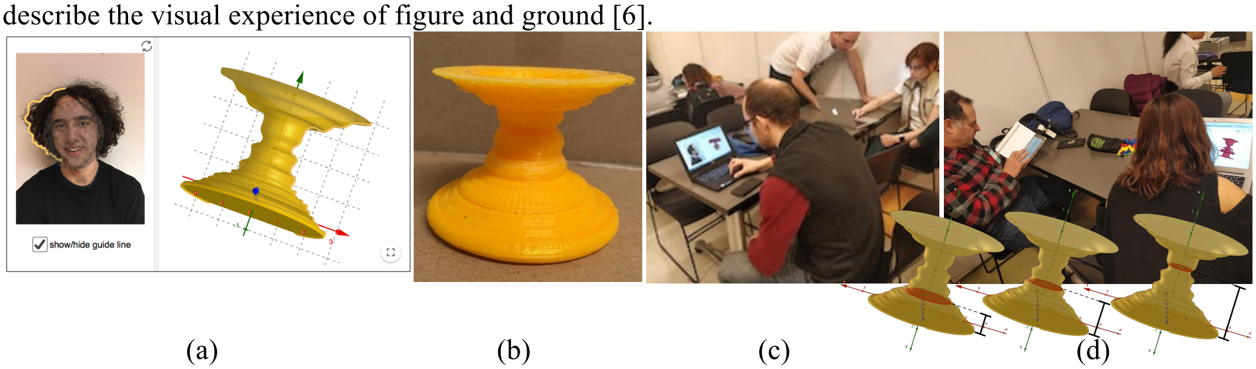


(a) (b) (c) (d) (e)

Figure 3: *Silhouette of a pawn generating different solids regarding its rotational orientation. (https://www.geogebra.org/m/vavzafan)*

Additional resources can be implemented in order to orchestrate the discussion according to specific goals. For instance, in the previous constructions, the integral feature on GeoGebra could be used to define the associated volume of the models.

We also believe that this approach will encourage students to develop personal and creative projects. One example could be having students designing and printing their own game pawn like the chessmen illustrated in Figure 3. Furthermore, we outline an enjoyable idea to integrate students' own faces to create artwork pieces, as shown in Figure 4 (a) and (b). This example is inspired by the classical Rubin's vase, which is widely circulated as an optical trick to test different viewers' perspectives and their different interpretations regarding specific stimuli. In this investigation, Rubin's major aim was to describe the visual experience of figure and ground [6].



(a) (b) (c) (d)

Figure 4: *Having fun can be a good motivation for working with personalized projects. (https://www.geogebra.org/m/fyhqg7ze)*

Taking this ambiguous representation to the 3D space, we can also enhance spatial reasoning and connect with the prior discussion of obtaining the volume of solids of revolution. In a recent workshop (Figure 4 (c) and (d)) developed at the National Museum of Mathematics (MoMath, in New York, USA), we applied some of these concepts and conducted a brainstorm session afterwards with mathematics teachers related to the use of GeoGebra for new approaches with technology. Among other aspects, a particular idea came out and it has already been implemented: the use of dynamic slices might be helpful to better represent the sum of the infinite sections. The overlapped Figure 4 (d) represents three different moments of such interaction. While the 3D printing enables a tactile exploration and physical comparison with real-life objects, the dynamic aspect allows observing a variety of cases with a simple drag movement and a valuable inside representation thanks to the possibility of controlling the transparency/opacity of the models.

Discussion

In this paper we presented selected examples to highlight the potential of 3D printing for teaching mathematics. At the moment, we have only included some of the aforementioned concepts at a workshop at the National Museum of Mathematics, but we intend to further apply it to students in different school contexts. We aim to connect 3D printing technology with GeoGebra to share some insights into how students can explore both physical and digital resources combined. Further tasks might follow such as estimating and comparing volumes of these rotational solids with or without GeoGebra to assess the lesson's effectiveness and students' mathematical understanding. For instance, after 3D printing their models, students could fill them with salt or liquid to be able to estimate their volumes as a practical approach. Students could also be assigned to discuss strategies to model other solids having the same volume as their rotational profile vase. That could be an experiential learning task to address Cavalieri's Principle and promoting critical and creative thinking.

In addition to the discussed examples related to the mathematical concept of volume and solids of revolution, 3D printing can also address other topics. For instance, we also highlight the benefit for solving day-to-day problems. In one further project we repaired personal items using 3D printing technology. For instance, the wheels of a suitcase were broken and needed to be replaced. For this reason, new pieces were modelled and 3D printed.

In our projects, we mostly use GeoGebra and Tinkercad due to their intuitive interface for educational purposes, and friendly connections with 3D printers. For instance, while for boolean operations (union and subtraction) Tinkercad is more suitable, GeoGebra is a powerful tool for more precise geometrical constructions and also to model algebraically, in a way, that the parameters of the equations can be adjusted through the slider. In the future, we will plan further projects addressing even more topics to take advantage of 3D printing technology in many different ways.

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