

Making A Mystic Dream of 4

Iggy McGovern

School of Physics, Trinity College, Dublin; iggy.mcgovern@tcd.ie

Abstract

A Mystic Dream of 4 is the title of my verse biography of the 19th century Irish mathematician, William Rowan Hamilton. This paper presents the thinking behind the making of the book, illustrated by a brief selection of the sonnets therein, hopefully finding some resonance with the themes of the Bridges Conference.

William Rowan Hamilton (1805-65) is Ireland's most celebrated mathematician; indeed, he is arguably the greatest mathematician of his time; in evidence of that, when the National Academy of Sciences elected its first foreign members in 1864, Hamilton's name was top of the list! He had made significant contributions in a number of areas, many of which bore greater fruit in the following century (for example, ask any physical sciences graduate about the 'H' in the Schrödinger wave equation!). Equally significant, at least for our purposes here, he was also a poet!

In this paper, I will discuss the making of a book: A title like *A Mystic Dream of 4* [5] might suggest elements of spirituality, Freudian psychology and number theory; while all of these are present to some degree, it is in fact a verse-biography, as the subtitle reveals: *A sonnet sequence based on the life of William Rowan Hamilton*. But the first word of the title of this paper, 'Making', is no mere description of the business at hand, 'to make' deriving from ancient Greek as *poiesis*, the root of the word poetry. In the tradition of 'bad poets borrow, good poets steal', *A Mystic Dream of 4* is lifted from one of Hamilton's own poems, called *The Tetractys* [2]:

THE TETRACTYS

Or high Mathésis, with its "charm severe
Of line and number," was our theme; and we
Sought to behold its unborn progeny,
And thrones reserved in Truth's celestial sphere;
While views before attained became more clear;
And how the One of Time, the Space of Three,
Might in the chain of symbol girdled be:
And when my eager and reverted ear
Caught some faint echoes of an ancient strain,
Some shadowy outline of old thoughts sublime,
Gently He smiled to mark revive again,
In later age, and occidental clime,
A dimly traced Pythagorean lore;
A westward floating, mystic dream of FOUR.

William Rowan Hamilton 1846

My title then truncates the last line. *The Tetractys* celebrates in his own words Hamilton's significant discovery of what he termed Quaternions three years earlier. Hamilton had been trying for many years to

extend the idea of complex numbers (then known as ‘couples’) from two to three dimensions. The multiplication of two complex numbers yields rotation in two dimensions; Hamilton hoped that the multiplication of two ‘triples’ would yield rotation in three dimensions, but he was unable to achieve multiplication. But on the 16th of October in 1843, while walking with his wife from his home in Dunsink Observatory to Dublin, he suddenly realized that he needed a fourth dimension. Hamilton recounts [4] the discovery as follows:

An electric circuit seemed to close; and a spark flashed forth, the herald (as I foresaw, immediately) of many long years to come of definitely directed thought and work, by myself if spared, and at all events on the part of others [...] nor could I resist the impulse - unphilosophical as it may have been - to cut with a knife on a stone of Brougham Bridge, as we passed it, the fundamental formula.”

A replica of this formula of quaternion multiplication, $i^2 = j^2 = k^2 = ijk = -1$, forms part of a commemorative plaque attached to the present-day bridge. Quaternion algebra did not gain widespread application in Hamilton’s lifetime, being largely displaced by the related but simpler vector algebra; a quaternion can be viewed as the sum of a scalar and a vector, so it is something of an irony that these terms were first named by Hamilton! But, like many discoveries, quaternions came to prominence a century later with applications both in video-gaming and in spacecraft guidance. Buzz Aldrin, the pioneering moon landing navigator paid homage to Hamilton’s bust in Trinity College’s Long Room Library saying “this is the guy that got us home!”

Quaternions are, at least on the surface, the simplest of Hamilton’s mathematical successes, suggesting that they should be reflected in the making of the book. In that regard I must be grateful for Hamilton’s remark that the quaternion was the offspring of a quaternion of parents, namely, geometry, algebra, metaphysics, and poetry. Hamilton’s sixty years lifespan divides neatly into four parts, which I have named for these four ‘parents’. The opening sonnet in each part is in the voice of one of these, Part I in the voice of *Geometry*:

GEOMETRY

Once, any pupil could define me best:
“points, lines, angles and figures”, could amuse
The table with the Christmas cracker jest
About ‘the squaw’ on the hypotenuse!
I was the Lord of Space, the one in three
Dimensions where you lived each mortal day,
Coordinates describing pointedly
A final resting place in graveyard clay.
But that’s to come; for now reserve your pity,
Observe the longitude and latitude
Of Dominick Street, the kingdom’s second city,
A multigravida in plenitude
And my coy mistress, Time, deploys her power
To act precisely on the midnight hour.

Although this sonnet is relatively understandable – the “squaw” for “square” notwithstanding - it was necessary to include brief explanatory notes with each of the remaining sonnets. As to the number of sonnets, these had to be a total of some power of four: four squared being too few and four to the fourth power too many, I quickly settled on four cubed or sixty-four sonnets.

Following the opening sonnet, each part then has fourteen of what I call ‘person-sonnets’, sonnets in the voices of people who knew Hamilton and who tell some part of the story of his life. These include family

members, scientific colleagues, poets, revolutionaries, clergymen, lovers and rivals. That leaves the four sonnets that close each of the four parts. These are in the voice of Death, an ironic voice, who tells us who *inter alia* has died in that quarter of Hamilton's life.

Part I covers Hamilton's childhood, which is chiefly recalled by family members. A significant interloper is a near contemporary called Zerah Colburn, also known as *The Calculating Boy*. Colburn was an American with a gift for mental arithmetic, who came to Dublin on a performance tour; Hamilton competed with him - and lost - which may have piqued an early interest in mathematics:

ZERAH COLBURN (Performer)

How many minutes since Christ went to Heaven?
 What are the two prime factors of, say, four,
 Two nine four, nine six seven, two nine seven?
 All ere the second's hand will mark a score.
 They billed me as *The Calculating Boy*:
 What cogs and wheels were whirring 'neath my crown
 To entertain street trader and Viceroy?
 I almost met my match in that drab town...
 I cared naught if I never understood
 Exactly how I did it, whereas he
 Was interested less in magnitude
 And more in finding methodology.
 I see him living well into his pension,
 Computing Christ's velocity of Ascension!

Part II covers Hamilton's early career, beginning with his outstanding performance as an undergraduate student, winning major prizes in both Classics and Science; this performance saw him appointed to the Chair of Astronomy while still technically a student! In this regard he was much in debt to his College Tutor, Charles Boyton, who knowingly exaggerated his pupil's talent in star-gazing. But Hamilton's mathematical ability more than justified the appointment, as when five years later he predicted an optical effect known as *Conical Refraction* (in which a beam of light is transformed into a cone in biaxial crystals). The mathematics of this phenomenon is complex. Moreover, the confirming demonstration by Hamilton's colleague Humphrey Lloyd using a beam of sunlight is itself a major achievement.

Hamilton's friend and fellow poet, Aubrey de Vere, called conical refraction 'The Radiant Stranger' and poetic interactions were also a significant feature of this part of Hamilton's life. Shortly after his appointment to the Chair of Astronomy, Hamilton travelled in the Lake District of England, where he encountered the Romantic poet, William Wordsworth. They became lifelong friends and correspondents; when Hamilton sent his poems to Wordsworth, the poet was forthright in his comments [1, p. 266]:

"You will have no pain to suffer from my sincerity. With a safe conscience I can assure you that, in my judgment, your verses are animated with true poetic spirit, as they are evidently the product of strong feeling [...] Now for the per contra. You will not, I am sure, be hurt, when I tell you that the workmanship (what else could be expected from so young a writer?) is not what it ought to be."

Moreover, it was Wordsworth who told Hamilton that he would achieve more fame as a mathematician than as a poet, and that he should not try to be both! This was sound advice, given Hamilton's extraordinary career as a mathematician.

Part III finds Hamilton at the peak of his mathematical productivity. Key papers such as *On a General Method in Dynamics* and *Algebra as the Science of Pure Time* were followed by the discovery of quaternion algebra. He was now married but there was a shadow on the marriage: as an undergraduate he had fallen in love with Catherine Disney, the sister of a fellow student. But Catherine was already promised elsewhere and the severance would affect both of their lives, most seriously in Catherine's attempt at suicide some years later.

Hamilton's wife Helen was not unaware that her husband still had feelings for Catherine. Moreover, she was not highly regarded by Hamilton's friends and colleagues, largely because she was almost invisible (by virtue of ill-health and personality). So, Lady Hamilton sets out to claim her place, if only because she was there for the 'Eureka' moment at Brougham Bridge:

LADY HELEN HAMILTON (Spouse)

A Lady, yes, but still without a carriage,
 Long treks to Dublin at a walking pace
 And there were always three souls in our marriage
 Or four, if you count Missy Curraghchase!*

I knew about the whispers behind-backs
 That I was just a phantom of a wife,
 My absences the focus of attacks;
 As if my presence could enlarge his life?
 But I was witness to his darker days,
 A genius, yes, but still a child half-grown;
 I weathered his precocious wants and ways
 And gave him three strong children of his own
 And I was midwife when, against the odds,
 He brought forth his canal-bank set of quads.

**Curraghchase was the home of Ellen de Vere, a possible bride for Hamilton*

Part IV finds Hamilton beset by many issues. I have suggested above that this book might have elements of spirituality, Freudian psychology and number theory; all of these feature to some degree. The Oxford Movement induced many Anglicans to convert to Roman Catholicism, including two of Hamilton's closest friends, the poet Aubrey de Vere and Viscount Adare (a former pupil). Hamilton suffered from depression and attempted to medicate his condition with alcohol; at one stage he took a pledge of abstinence but lapsed shortly after. But he continued to work, even overwork, on his mathematics, largely on quaternions. In 1853 he published a 700-page book *Lectures on Quaternions* which sold very few copies; he then began work on what was meant to be a shorter primer *Elements of Quaternions*; this was published posthumously and ran to 500 pages. Despite their championing by the mathematician Peter Tait, interest in quaternions waned after Hamilton's death.

However, Hamilton's other work found a new champion in the early 20th century; Erwin Schrödinger had received a thorough grounding in Hamiltonian dynamics from his professor, Friedrich Hasenöhr, who, in turn, was a student of Arnold Sommerfeld. Schrödinger gave the Hamiltonian formulation a central role in his construction of wave mechanics. On that basis, Schrödinger is included in the list of sonnet-makers.

Schrödinger was also the first Director of the Dublin Institute of Advanced Studies, and the first home of the Institute was labeled *Teach Hamilton*. 'Teach' being the Irish Gaelic word for house, this translates as *Hamilton House* or *Hamilton Building*. However, the equivalent spelling of the English word 'teach' (to educate) yields a macaronic pun. Indeed, this led one guest lecturer (also called Hamilton) to complain

peevishly that he was there to instruct and not to be taught! Moreover, Schrödinger's frequent extramarital affairs are in marked contrast to Hamilton's Victorian ethics.

ERWIN SCHRÖDINGER (Mathematical Physicist)

Teach Hamilton - our happiest of days
 Among a people wonderfully odd,
 As when O'Nolan linked our first forays
 To write of "Two St Patricks and no God!"*
 He is, perhaps, the ghost in the machine
 Of Quantum Physics, with his clanking chain
 Announcing the analogy between
 Mechanics and optics; first to distain
 Commutativity; re-formulation
 Of energy in systems large and small;
 He is the "H" in Schrödinger's Equation
 And hence the Doktorvater of us all.
 Teach Hamilton? As well the stars above
 Unless, perhaps, in Elements of Love.

* *conflating Schrödinger's lecture on 'Causality' with that of a fellow professor on 'Patrick and Paladius'*

The Schrödinger connection prompts some further thoughts on Science versus Poetry; In his book *Erwin Schrödinger and the Quantum Revolution* the Popular Science writer John Gribben describes Schrödinger's poetry as "a pastiche of the kind of poetry you would expect a physicist to write" [3]. There is a companion piece from Schrödinger's fellow Nobel Prize-winner, Paul Dirac. Writing to fellow theoretical physicist Robert Oppenheimer, Dirac [7] criticized the latter for writing poetry:

"The aim of science is to make difficult things understandable in a simpler way; the aim of poetry is to state simple things in an incomprehensible way. The two are incompatible."

At the very least these statements invite further discussion. But, personally, I find comfort in Hamilton's remarks in a letter [1, p. 486] to Wordsworth:

My dear Wordsworth

As Keats exclaimed "O for ten years that I may overwhelm myself in Poesy", so you will perhaps exclaim "O for some Pause that Mr Hamilton may not overwhelm me with his verses! *Occiditque Legendo!* What makes the matter worse, and your case more desperate, is that this is far from being my idlest time; on the contrary it is my busiest, and I am in the midst of a course of lectures, of which I am delivering two (a physical and a mathematical) every second day, in our university. The only hope is that I am rather perverse and often go by contraries, as soon as Science may leave me comparatively at leisure I may cease to versify too. (Nov 11, 1831)

In other words, creativity in science and literature is not a case of either/or but of both or none.

Finally, it seems appropriate to finish with one of the four sonnets in the voice of Death. Although Hamilton was born more than 200 years ago, many of his life experiences seem entirely modern. When he died he was almost bankrupt (largely through supporting his eldest son's enterprises), there were periods of his life where alcohol and depression combined to his detriment, and there was the lifelong obsession with Catherine Disney. He also suffered the antagonism of a Trinity College colleague; James MacCullagh,

Professor of Mathematics, claimed priority with regard to much of Hamilton's research. This man subsequently committed suicide, an event that weighed heavily on Hamilton. Although Hamilton was orphaned at the age of fourteen, he was fortunate in a succession of father-figures, including his Uncle James Hamilton who raised him, his cousin Arthur Hamilton who supported him financially in college, his Trinity College tutor Charles Boyton who hustled to get him the Chair of Astronomy and, of course, Wordsworth. These five individuals all died in the third quarter of Hamilton's life, as recorded by the sonnet Death in Part III. That period also witnessed the disaster of the Great Famine in Ireland.

DEATH

A feast or famine? – famine *is* my feast!
 Who lives or dies is in the penny's toss.
 He kept his head down at his sums; at least
 He sought no profit from another's loss.
 He coined me five across the River Styx:
 First, Cousin Arthur, fountain of goodwill,
 Then Boyton, star of College politics
 And Uncle James, the lowly curate still.
 He mourned these and moved on, as if by rote;
 The fourth, though, haunts him like Old Marley's ghost:
 The vision of MacCullagh's bloodied throat,
 So much alike, affecting him the most.
 And Wordsworth, in the poet's own words 'bound
 Within the sonnet's scanty plot of ground'.

Summary and Conclusions

There is a rough symmetry here: a poet who is a physicist writing about a mathematician who is poet. In making this book I have worked primarily as a poet, and the mathematics therein is little more than a glance at the four-ness of Hamilton's quaternions. Rather, the overall aim is to honour an extraordinary life in mathematics and poetry.

Acknowledgements

This paper is a condensed version of a paper first read at the conference 'Physics and Literature' held in Erlangen University in 2014 [6]. I am grateful to Anne van Weerden for notification of essays that make the case for a more positive view of Hamilton's private life, his marriage and alcohol issues [8].

References

- [1] Robert P Graves. *Life of Sir William Rowan Hamilton*. vol. 1. Hodges, Figgis & Co., 1882
- [2] Robert P Graves. *Life of Sir William Rowan Hamilton*. vol. 2. Hodges, Figgis & Co., 1882, p. 525
- [3] John Gribben. *Erwin Schrödinger and the Quantum Revolution*. Bantam Press, 2012, p. 217
- [4] Thomas L Hankins. *Sir William Rowan Hamilton*. The Johns Hopkins University Press, 1980, p. 293
- [5] Iggy McGovern. *A Mystic Dream of 4*. Quaternia Press, 2013
- [6] Iggy McGovern. "The Making of a Mystic Dream of 4." In Aura Heydenreich and Klaus Mecke, Eds. *Physics and Literature*. De Gruyter, to appear in 2019
- [7] Jagdish Mehra. In Eugene Paul Wigner and Abdus Salam, Eds. *Aspects of Quantum Theory*. Cambridge University Press, 1972, pp. 17–59
- [8] Anne van Weerden. *A Victorian Marriage - Sir William Rowan Hamilton*. BoekenGilde, 2017