Reconstructing Early Islamic Geometries Applied To Surface Designs

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Abstract

An introduction to six early Islamic geometric methodologies that were used to create many two-dimensional geometric surface designs that appear in windows, and on doors, walls, domes and minbars. These are reconstructions based on analysis and research of methodologies developed during the early years of Islam - particularly during Abbasid Caliphate (750–1258CE in Baghdad and 1261-1517 CE in Cairo) and also during corresponding periods in Persia, Morocco, Syria and Moorish Spain. The six methodologies presented are: (i) Grids, (ii) Tessellating Polygon Subdivisions, (iii) Rays, (iv) Close-Packed Circles, (v) Nesting Polygons and (vi) Modular Tiles. This paper also puts forward the idea that surface designs generated by these methods were not always intended to be purely decorational—they may have had meaning according to the logic of their design methodologies, their symbolic values, their 'perceptual' qualities and, possibly, their numeric values.

Introduction

The various types of visual logic that we use today are very much formed by choices that were made in the past creating a pathway of concepts from Euclid and Descartes to that of Einstein's 'space-time.' Many ideas of the past do not fit neatly into our accepted pathways of understanding and of our current geometries and concepts of space. When studying ideas of the past it is the prospect of finding ancient concepts that might have created very different pathways that is intriguing.

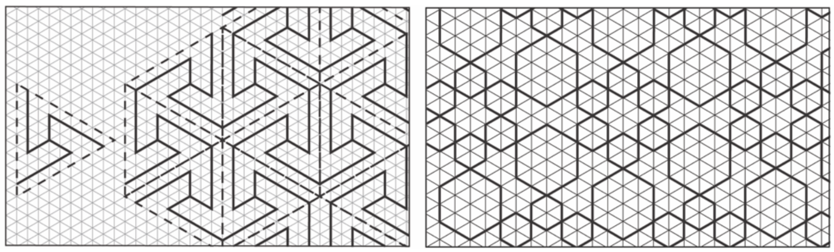
This is a presentation of six methodologies used to order two-dimensional space and the types of surface design generated from these methodologies; developed during the Abbasid Caliphate (750-1258CE in Baghdad and 1261-1517 CE in Cairo) and during corresponding periods in Persia, Syria, Morocco and Moorish Spain. Early Islamic designers ordered two-dimensional space using classic Euclidean points, lines, angles, lengths, grids, and polygons. They also appear to have developed new ways of ordering space using close-packed circles of different sizes and other methods that anticipated fractals and vectors.

There is little that allows us to truly understand early Islamic geometric methods with regard to establishing the relative positions of figures and properties of space and nothing to say that all designers used the same methods and concepts to create any one type of surface design. So, identifying possible early Islamic ways of ordering two-dimensional space required an analysis of hundreds of designs, a reconstruction of the logic used and a grouping of the reconstructed methodologies, see [2]. Research included studying the works of Jules Bourgoin, 1879 CE, see [1], Albert Calvert, 1905 CE, see [6], Owen Jones, 1856 CE, see [9], the Topkapi Scroll, 15th Century CE, see [11] and studying actual designs in Egypt, Iran, Spain, Morocco, and Syria.

The author has assumed that the works of Jules Bourgoin and Owen Jones are based on reconstructions of methodologies as no sources are mentioned in their works. Calvert's work is mostly that of a composite of the work of others, particularly Bourgoin and Jones, without acknowledgement—but is, nonetheless, of interest. The Topkapi Scroll shows the logic of constructing muqarnas (honeycomb like structures positioned between, for example, domes and arches), and must be taken as a true representation of actual methodologies used during the 15th Century CE, but muqarnas first appeared some four hundred years before the Topkapi Scrolls, in the 10th century CE, so the concepts revealed in the Topkapi Scrolls may, or may not, represent methodologies used earlier—certainly more ancient muqarnas were simpler.

Geometric Grids

The first and simplest early Islamic means of ordering space from which designs were extracted is a geometric grid methodology used well before the first years of Islam—in ancient Rome, for example—a methodology based on ordering space with square and equilateral triangle grids from which design forms were extracted. The design forms that were extracted were typically contained within the bounding mirror lines of "unit" 60º-60º, 30º-60º and 45º-45º right-triangle tiles of regular p6m hexagon tilings and p4m square tilings. Typically, "unit" triangles were rotated, reflected, and translated to create surface designs. See Figure 1 (a) from the Alhambra, Spain, 1238 CE., also from the Kharaghan twin towers, Qazvin province, Iran, 1067 CE; and Figure 1 (b) from the Ibn Tulun Mosque, Cairo, Egypt, 884 CE.

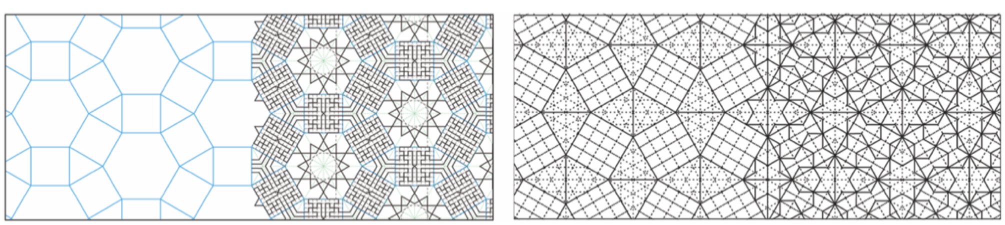


(a) (b)

Figure 1: *(a) Hexagonal arrow design, (b) Star design.*

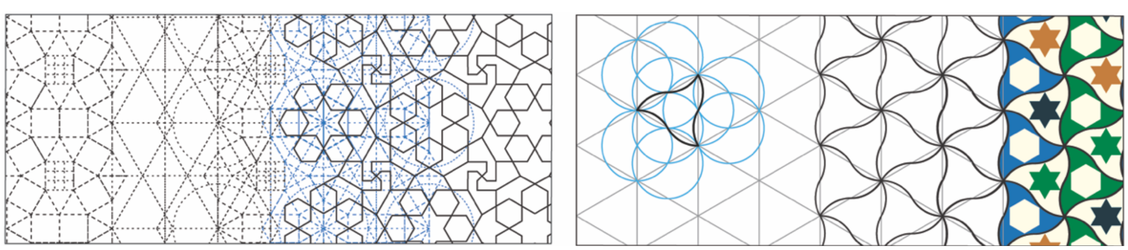
Tessellating Polygon Subdivisions

A second means of ordering space from which designs were extracted—and more unique to Islam by way of its variety and scale of subdivision—is that of various methodical approaches used to subdivide the internal spaces of regular and semi-regular tessellations, and other polygonal arrangements, including subdivisions with grids. Many of the subdivision methods were designed to generate specific forms, rosettes in particular. Some subdivisions revealed higher or lower levels of tessellation over that of the initial tessellations. Many methods involved drawing curved or segmented lines on and about the sides of tessellating polygons to create new shapes with rotational, reflective, and translational symmetry. See Figures 2(a) Bourgoin plate 36 see [1], (b)) Calvert page 469 see [6] also Selimye Mosque, Edirne, Turkey, 1568CE and Figure 3(a) Calvert page 469 see [6], (b) the Alhambra, Spain, 1238 CE.



(a) (b)

Figure 2: *(a) Key design, (b) Star design.*



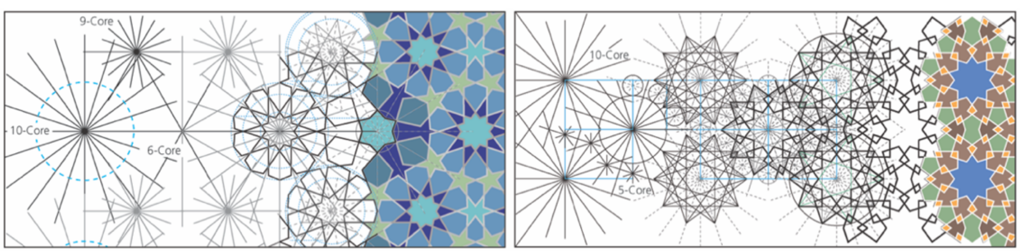
(a) (b)

Figure 3: *(a)* Rosette design, (b) *Curved triangles.*

The Ray Method

A third means of ordering space—a reconstruction, that if correct, would be unique to the Abbasids—can be called the 'Ray' method. This is a means of ordering space based on lines that extend out from different points in two-dimensional space—lines that order space according to the properties of their intersections. In this respect the method appears to be very different from anything else. Taken into three-dimensions, and further developed, the method could be used as a means to fabricate space in a new way.

The 'ray' method appears to have been used by the Abbasids to create designs composed of polygonal rosettes of different numbers of sides—possibly to communicate groups of numbers. A 'ray' is a line radiating out from the center point of a regular polygon to symmetrically divide a vertex or side of the polygon. A 'ray core' is composed of the rays that divide all the sides and vertices of a regular polygon. A decagon, for example, has twenty such lines but the numerical value of the ray core is based on the polygon's 'lines of symmetry.' So, the ray core number value of a decagon is '10-core.' The ray methodology starts with aligning ray lines from two ray cores and determining the angles of intersection of the other rays—where intersection points, with angular correspondences with other regular polygons, can serve as center points for new ray cores. An example, Figure 4(a), from the mosque of the Sultan of Qa'it Bay, Cairo, 1474CE starts with a 10-core ray that has lines angled at 18-degress, the angles of a decagon. A second ray core, 6-core, is added on one of the 10-core rays with lines angled at 30-degrees, the angles of a regular hexagon. Looking at the angles of the intersection points between the rays of the 10-core and the 6-core, we find that a 9-core, with angles of 20-degrees, can be approximately added. The intersections of a 9-core, 10-core, and 6-core ray determine the size of the 10-core, 6-core and 9-core rosettes within a rectangular tile. The end result is a design of regular polygonal rosettes of 10, 6 and 9 sides. Given that the 'ray' method generates groups of numbers within a design then there is a possibility that the numeric values of some 'ray' designs were used to communicate meanings, see [12]. If a meaning was intended for Figure 4(a) design then it might be from a translation using the Arabic Abjad-Number correspondence as follows, 10 ى(yaa) + 6 و (waw) + 9 ط (ta) = طوى . This is a consonantal 'word-root' that has an added vowel meaning of, 'twice sanctified' referring to the sacred valley of Tuwa as in the Koran 20:12, Lane [10], Vol 5, page 183. Another possibility is that of the numeric total correspondence with the Koran's 25th Sura and its message. Figure 4(b) shows a design created with 10 and 5 ray cores; Mosque of the Sultan of Qa'it Bay, Cairo, 1474CE.



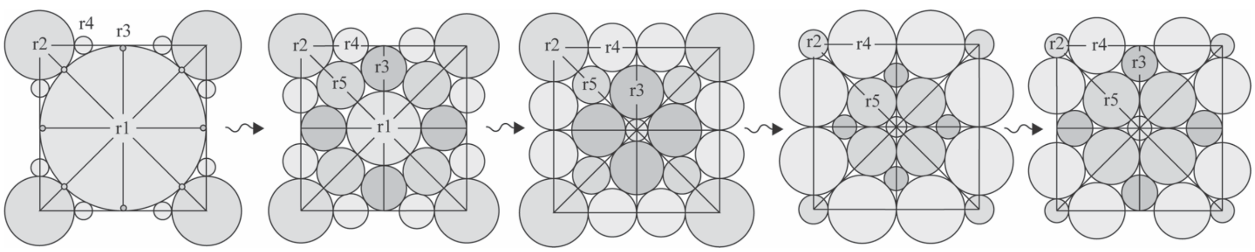
(a) (b)

Figure 4: *(a) 10, 6, and 9 core ray design, (b) 5 and 10 core ray design.*

Close-Packed Circles

This fourth reconstructed Islamic methodology provides a means by which two-dimensional space can be organized and triangulated and, in that, it is unique. The method generates arrangements of 'close-packed' circles, of same and different sizes, positioned on and within the bounding symmetries of tessellating tiles (tiles that fit together, edge to edge, without leaving gaps)—generally regular polygons—on mostly flat, but in some cases projected onto curved surfaces, such as on the external surface of a dome. "Close-packed" circles of equal or varying sizes are circles that touch each other in tangential arrangements, and in triples, without overlapping on a two-dimensional Euclidean plane. The arrangements are typically contained within the bounding mirror lines of "unit" triangular tiles of regular polygons such as the 45º right triangles of a regular p4m square tiling or the 60º-30º degree right triangles of a regular p6m hexagon tiling, see [2]. Such close-packing arrangements are precise—a slight change in circle size or position will negate the close-packing, see Figures 5, 6 and 7. There are early Islamic cases within regular polygons that do not regularly tessellate - as with regular pentagons - where certain circles are allowed to either overlap or not touch to complete a tessellated tiling, see Figure 8.

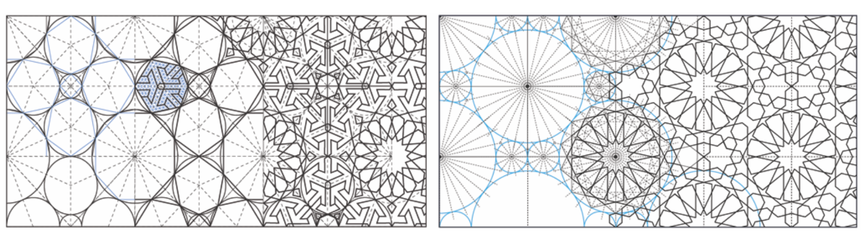
Close packing arrangements and their associated line segments (lines that connect the circle centers, circle contact points and the spaces between close-packing circles of same and different sizes) appear in nature from beehives and a fly's eye to the packing of atoms. A limited number of close-packing arrangements have been known for millennia to scientists, geometricians, and engineers; from Plato and DaVinci to modern day chemists and molecular physicists—whereas many of the Abbasid close-packing circle arrangements, of different sized circles, are generally unknown. A method by which these more complex arrangements might have been generated is to incrementally change the size and position of a starting set of circles within unit triangles or tessellating polygons, see Figure 5. This logic is presented in [5] and extended into close-packing spheres where line segments are used to create architectural structures.



(a) (b) (c) (d) (e)

Figure 5: *Generating close-packing by incrementally changing circle size and position.*

Connecting the contact points between close-packed circles creates arrangements of internal polygons; connecting clipped tangents at the contact points of close-packed circles creates arrangements of external polygons; connecting circle centers creates radii to internal polygons; polygonal stars or rosettes can be drawn within internal or external polygons. Connecting the circle centers of the close-packing circle arrangement of Figure 6(a) creates the semi-regular square-and-octagon tessellation. Internal polygons are an octagon, a square, and an irregular hexagon, Calvert page 467 [6]. The design shown in Figure 6(b) combines 14-sided and 7-sided rosettes and is from the caravanserai Wakala al-Ghuri, Cairo, Egypt, 1504 CE.



(a) (b)

Figure 6: *(a) Regular octagons, squares, irregular hexagons, (b) 14-sided and 7-sided rosettes.*

The Figure 7 close-packing is the same as Figure 5(c) with regular octagonal and square rosettes and 'almost' regular hexagonal, heptagonal, and pentagonal rosettes. The design is from a window of the 1356CE Madrasa of Amir Salf al-din Sargatmish, Cairo, Egypt. The circle arrangement generated the patterns used in the first book of Altair Designs, see [7]. One possible Abjad numeric communication through the numbers, 50 ن (nun) + 7 ز (za) + 600 خ (kha) = خ ز ن , means 'storehouse' or 'treasure chest,' see [10] part 2, page 734. The numeric total of 5, 6 and 7 might be a reference to Sura 18 of the Koran.

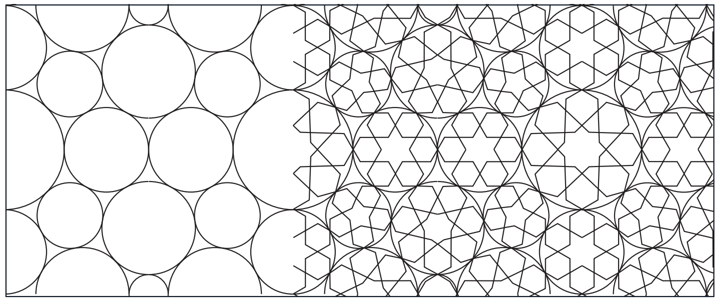


Figure 7: *Rosettes of regular octagons, squares and 'almost' regular hexagons, heptagons, pentagons.*

Figure 8 shows a rosette close-packing circle design, with number values of 5 and 10, of a 13th Century CE door of the Seljuk period, see [2]. The door is in the Ince Minaret Medrese (school) in Konya, Turkey. The door was to a Tekkia (a large room often with an octagonal floor plan) where 'Whirling Dervishes,' danced. Numbers associated with this design are 5, 6, 10.

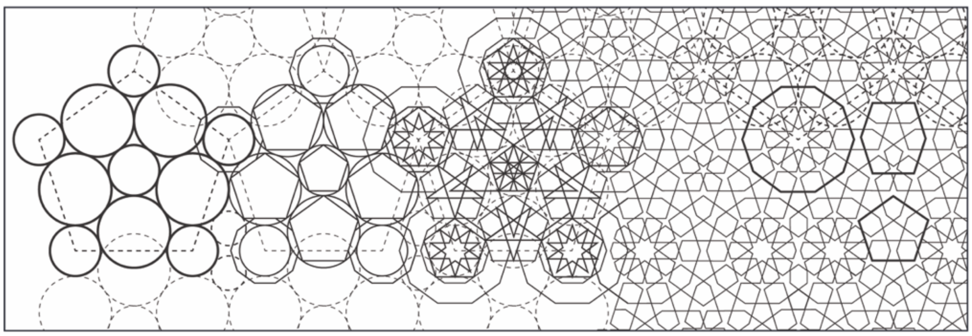


Figure 8: *13th Century CE Seljuk Door Design Construction.*

It appears that a close-packing circle arrangement, Figure 9, was used to create the surface design on the dome of the Sultan Al-Ashraf Qa'it Bay mausoleum in Cairo, Egypt. The rosettes have 16, 9 and 10 sides. It is possible, as with the Seljuk period door, that the circle-packing, and the generated rosettes, were selected to have a meaning. The meaning might have been symbolic, but the numbers might also have been selected to communicate a word through the Abjad number-word method. Applying an Abjad-number-word methodology the first level of letters corresponding to 16, 9, and 10 are as before: 10 ى(ya) + 6 و(waw), + 9ط (ta) = طوى (reading right to left) with one vowel meaning of "twice sacred" referring to the sacred valley of Tuwa, Lane [10], Vol 5, page 183. The next order would be qaf ق(100) + wawو(6) + Ta ط(9) = طوق: (a) A ring around the neck. (b) A difficult burden like a ring around the neck. (c) A badge of favor conferred. (d) God strengthened me to handle the burden. Lane [10], volume 5, page 178.

Al-Ashraf Sayf ad-Din Qa'it Bay (1416/18 - 1496 CE) was a slave who was taken to Cairo and purchased by the reigning sultan. The classic Arabian image of a slave is one with a ring around the neck. Al-Ashraf became a member of the palace guard, was later freed, and then was promoted through the Mamluk military hierarchy to become a Field Marshal - and later the Sultan. He amassed a fortune during his time with the military that enabled him, as Sultan, to exercise many acts of beneficence without draining the royal treasury. He is best remembered as a great patron of art and architecture. In fact, although Qa'it Bay fought sixteen military campaigns, he is best remembered for the spectacular building projects that he sponsored, leaving his mark as an architectural patron on Mecca, Medina, Jerusalem, Damascus, Alexandria, and Cairo.

If an Abjad meaning was intended through numbers-word associations, then the above meaning is perfect for Qa'it Bay's final resting place in Cairo.

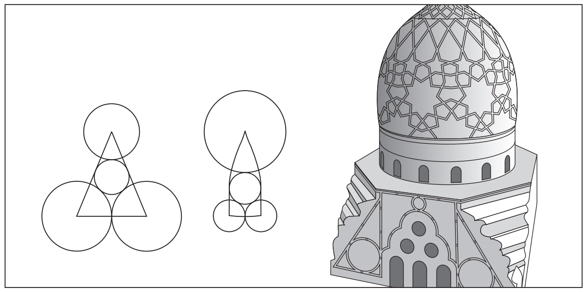
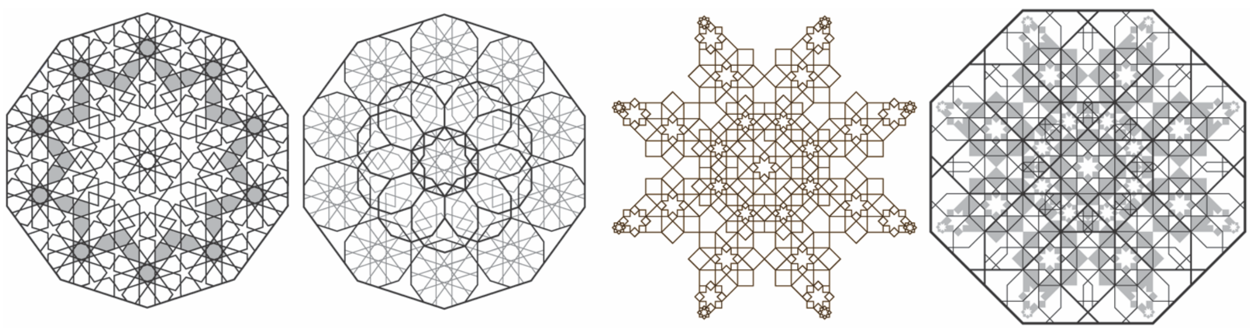


Figure 9: *Close-Packing Circles Projected onto Domes.*

Nesting Polygons

A fifth reconstructed means of ordering space was the use of 'nesting polygons' to position design elements or to create base line grids from which designs could be extracted. These were arrangements of same-size regular polygons created by connecting duplicates at vertices, or at edges, to create arrangements that could be contained within a larger version of the same regular polygon. An example is that of a zellige (shaped stones) mosaic positioned on the side of a doorway into the courtyard of the Al-Attarine Madrasa in Fez, Morocco, 1325CE, see Figure 10 (a). The rosettes of the design can be seen to be surrounded by regular decagons that touch and overlap within a containing decagon. Such arrangements, of polygons within polygons, creates the possibility of 'infinite progression' by scaling a first level of an arrangement to create a second level that fits within each polygon of the first level arrangement - a process that can be repeated.



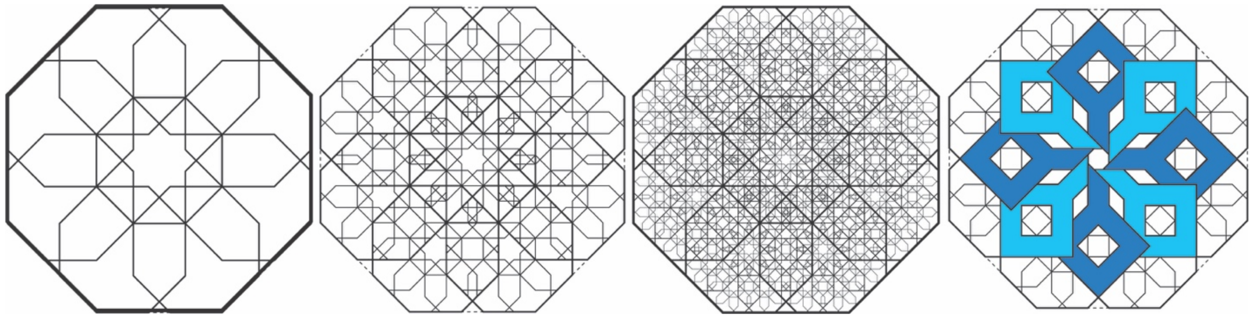
(a) (b)

Figure 10: *(a) Nesting decagons, (b) Nesting octagons.*

Figure 10(b) shows a design inlaid into the surface of a copper tray in the collection of Ensor Holliday, 10^{th} Century CE, Cairo, see [7]. The design is that of an arrangement of scaled 'stars,' a design concept often seen in Islamic art. Applying a nesting polygon approach to the design the author found that it neatly fits within a scaled repeat of a regular octagon surrounded by eight regular octagons of the same size—where the surrounding octagons each connect to the center octagon at every other vertex, see Figure 11 (a). The octagonal nesting arrangements shown in Figure 11(a), (b) and (c) follow a scale ratio of $1/(\sqrt{2}+1)$. The sizes of the octagons reduce in size according to the stage of the progression (n) and the scale ratio $(1/(\sqrt{2}+1))^n$, where a single octagon has an n value of zero and the first stage, Figure 11 (a) has an 'n' value of 1.

Once a nesting arrangement of polygons has been created the lines of it can serve as a sort of base grid upon which images can be positioned, or 'seen,' or created; all in proportion and all following the lines of the grid. The more lines in a grid, such as with Figure 11(c), the greater the possibilities and the greater the diversity of images that can be positioned, 'seen,' or extracted – as many as the imagination will allow.

Figure 11(d) shows how a traditional Islamic design can be extracted from a nesting grid, but here it is colored creatively.

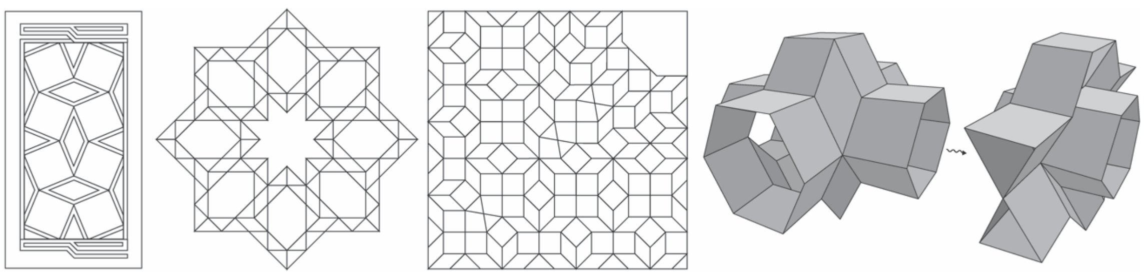


(a) (b) (c) (d)

Figure 11: *(a) n=1 Octagons, (b) n=2 Octagons, (c) n=3 Octagons, (d) Traditional Islamic design.*

Modular Tiles

A sixth pre-Islamic means of ordering space, not unique to Islam, from which designs can be composed or extracted is based on the fact that many regular and symmetrical polygonal tiles will combine, edge to edge, in a multiplicity of ways. For example, squares with 45º-135º degree rhombi. A fairly common Arabian design technique was to combine this type of polygon in creative ways—in solid colors or with internal design structures. For example, see Figures 12(a), (b) and (c). Figure 12(d) Shows a shape-changing polyhedra made with the same two polygons, see[4].



(a) (b) (c) (d)

Figure 12: *(a) Dome of the Rock, (b)Islamic Star, (c) Takht-e Soleyman muqarnas (d) Shape-changer.*

Rhombi and regular polygon combinations feature in many early Murqanas (transitional 2D to 3D design structures) and in the Shape-Changing Polyhedral Geometry developed by the author, see [4].

The Persian Topkapi Scroll, of the 15th century CE, includes designs that use a variation of the modular design system and uses five different polygons (The author has added a 36-degree rhombus to make 6) each with an internal design structure that creates complex pattern variations when the polygons are combined, see Figure 13. The tiles can be combined in an indefinite number of ways and subdivisions can create quasicrystal tilings similar to those of Roger Penrose.

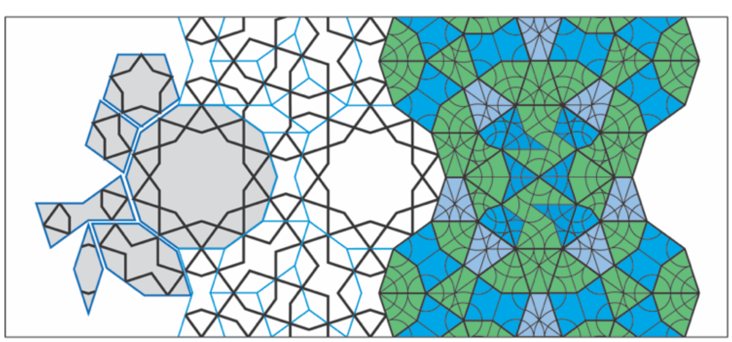


Figure 13: *Topkapi Tiles.*

Conclusion

Many Islamic designs create an effect where images seem to change before the "mind's eye," and I wonder if this was intended to stimulate perception. The various types of geometrical concept explored in this paper can certainly exercise one's imagination as well as one's logic. If, indeed, number-word associations were used then another level of communication might have been woven into Islamic designs communicating ideas to ponder. In addition to all of this we have symbolic shapes, the use of symbolic colors and even ceremonies and dances associated with some of the designs – so, truly, and depending upon the eye of the beholder, the geometric designs of early Islam communicate in a multiplicity of ways. A wonderful example of this is that of a Bokhara prayer rug, 18thCenturyCE, Figure 14, in my collection. The design is arranged around a distorted tessellation of octagons and squares with images of a garden, pools of water and trees.

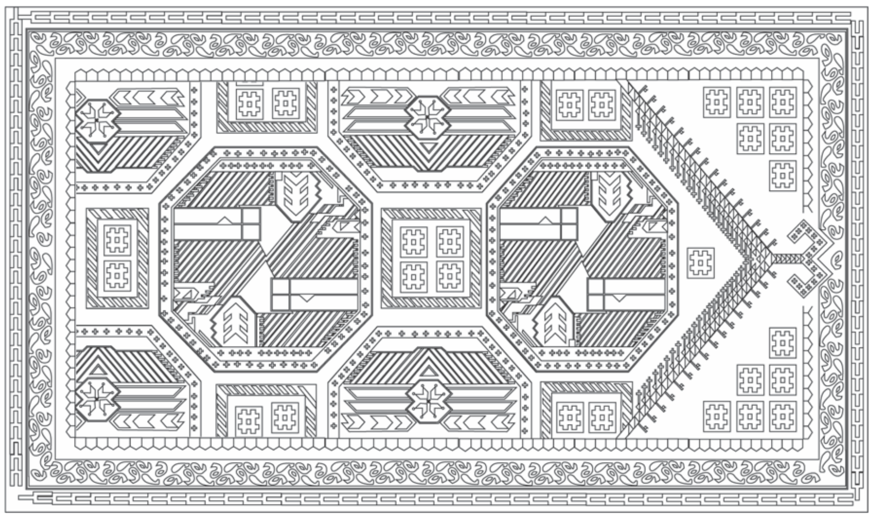


Figure 14: *Bokhara Prayer Rug, 18th Century.*

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