A Minimal Art Object with Four Famous Fabulous Faces

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Abstract

This paper describes our next step in the development of math objects with hidden graphics. It is a follow up on our minimal art object with the three faces of Gödel, Escher and Bach (GEB-object), again using a low resolution optical minimal art technique. Now we make an object with four faces: the famous faces of the fabulous four Beatles: John, Paul, George and Ringo. We selected a suitable unit cell to build the object. We describe the shape of the unit cells and the shape of the objects composed with these unit cells. It was necessary to developed a new method to display optical minimal art within hexagons. Finally we explain several ways to show the objects.

Starting Point

Our earlier contribution in creating minimal art objects with hidden graphics is described in [1,2,3,4]. We developed the GEB-object consisting of 256 cubelets which together outlined the contours of one cube. Each contour is a square containing one of the three faces. That was our starting point. The question was: Is it possible to increase the number of faces? And how?

Our first thought was to increase the number of dimensions. In a four dimensional world we could develop a hypercube with the possibility to provide space for 6 faces. But this object could only exist in a math world with its projections on a flat screen, not in the real world. So the question became: Can we develop a minimal art object with four faces that we can hold in our hands? We did [see Figure 1]!

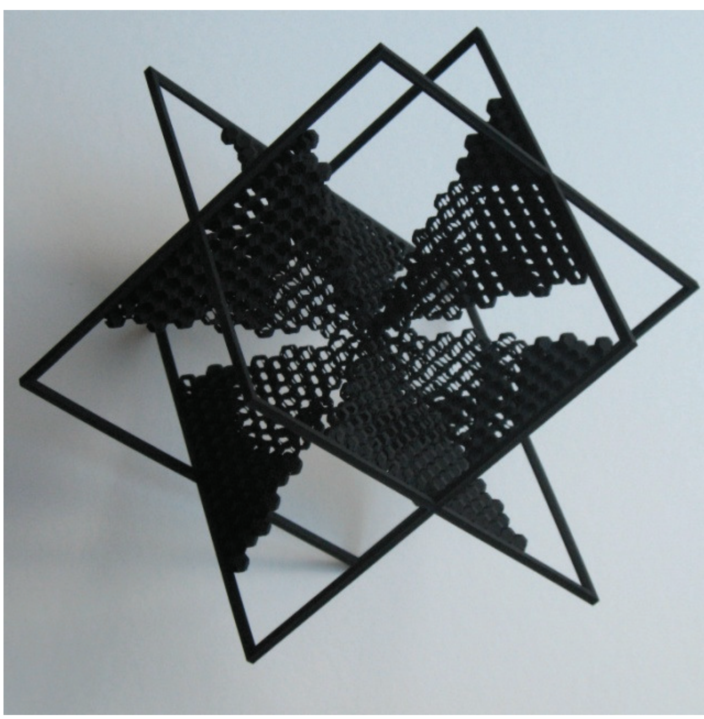


Figure 1: Minimal art object with 4 hidden faces

We did not follow a straightforward path in creating our new object but it was not a trial and error process either. That is why we describe the results and not the step by step process. To speak with the Beatles the process was like "*A long and winding road*" and we cannot remember all steps.

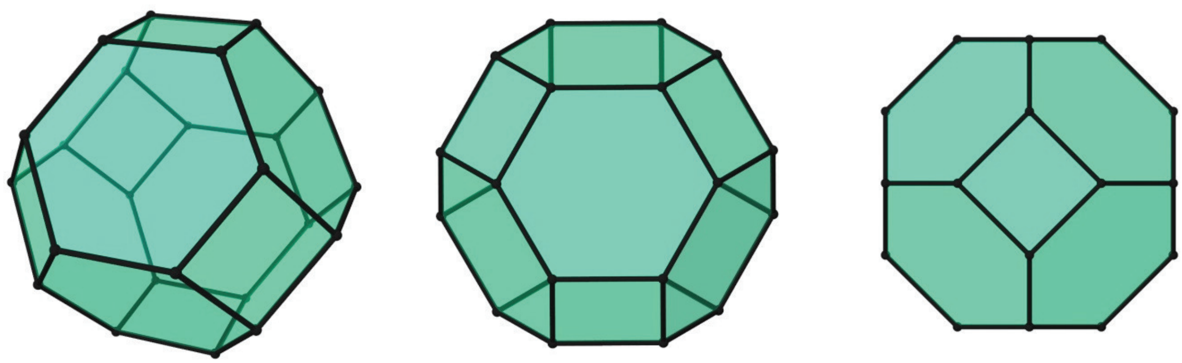
Our new challenge was to develop a cell with structural and optical properties. Each cell must contribute one tile to four different images and moreover all these cells must be connected in one object.

The object we created has 577 cells. The four images we created have 469 tiles each. That means that 108 cells have a projection beyond the image border.

We designed our object in Rhino [5]. It was printed by Shapeways [6].

Exploring Cell Shape Possibilities

Our GEB-object was built up with cubelet cells with 3 perpendicular viewing directions. Now we were looking for cells with 4 viewing directions, which of course could not be perpendicular. One thought was: is there another Platonic or Archimedean solid with more viewing directions? The octahedron was the first Platonic solid to consider. One could distinguish four main (non perpendicular) directions but the triangles on the opposite sides of the solid have a different orientation. Then we tried truncating the octahedron into an Archimedean solid: the truncated octahedron. This appeared to be a proper building cell for our goal. We call these cells in this paper: unit cells. The truncated octahedron has 14 surfaces: 8 hexagons and 6 squares [see Figure 2a].



(a) (b) (c)

Figure 2: *Truncated Octahedron as Unit Cell: (a) in perspective, (b) the hexagons provide in optical properties, (c) the squares provide in connectivity and structural properties*

Our unit cell has 4 different viewing directions and surfaces oriented the same way on opposite sides: hexagons. We use these hexagons to create the images [see Figure 2b]. The unit cell has the property to built objects with by connecting the squares in 3 perpendicular directions [see Figure 2c]. We use these squares to connect the unit cells to each other in order to build the minimal art object.

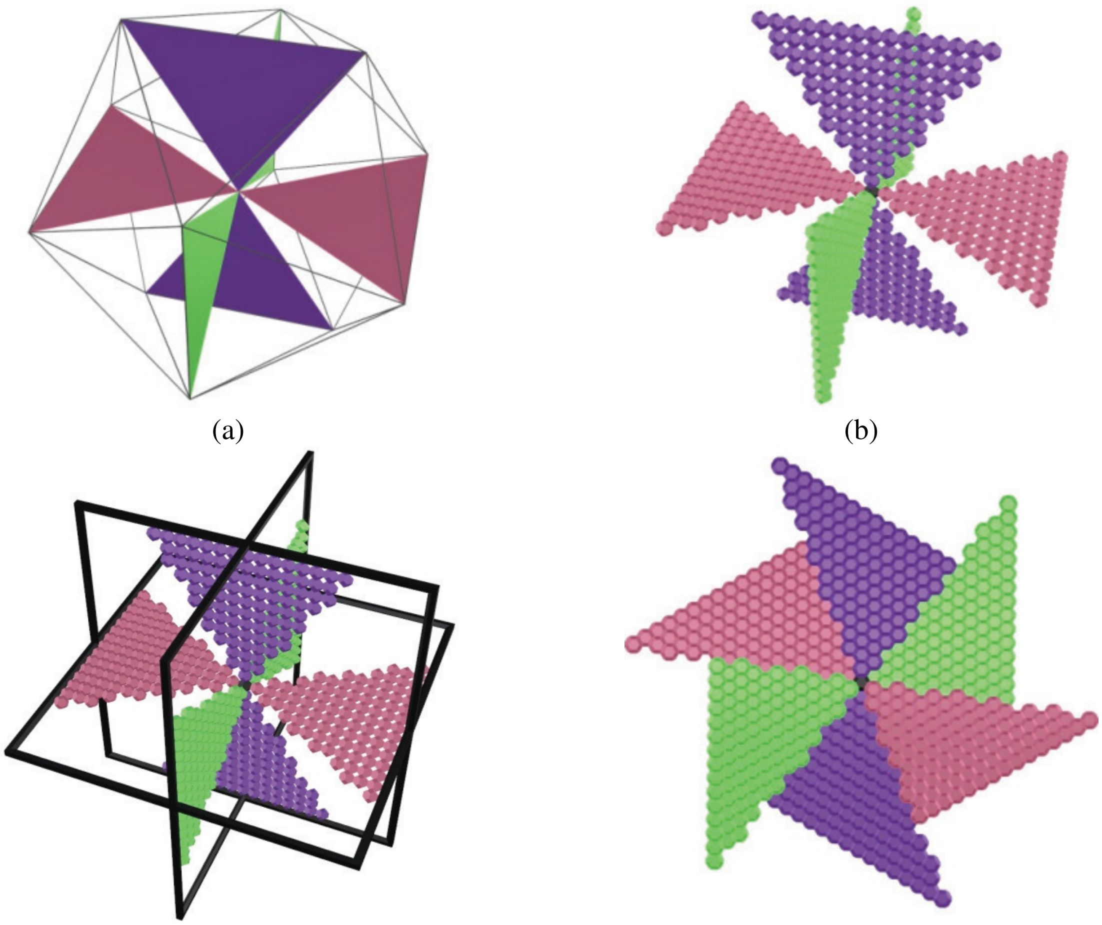
Building a Minimal Art Object

We consider our object as minimal if and only if every hole in the image is created by only one unit cell in the object. That means that in the viewing directions every single unit cell is visible so no unit cell is hidden behind another. Each position of a cell in the object, determined by 3 spatial coordinates, appears to be corresponding to a position in each of the four images, determined by 2 flat coordinates [see Table 1].

Table 1: *Image coordinates h and v due to spatial coordinates x, y and z*

Image coordinates	Image 1	Image 2	Image 3	Image 4
horizontal	h1=0.5*(x-y)*√3	h2=0.5*(-x-y)*√3	h3=0.5*(-x+y)*√3	h4=0.5*(x+y)*√3
vertical	v1=0.5*(-x-y)+z	v2=0.5*(-x+y)+z	v3=0.5*(x+y)+z	v4=0.5*(x-y)+z

In our GEB-object there were various solutions to create a minimal art object, determined by the rules of a Latin square plus some connectivity requirements. In this Beatle-object we did not yet discover a recipe to create a minimal art result. Until now we found only 2 solutions. We expect that there are no further solutions. Our first solution has one centre point, and from that point 6 pointed slabs (triangles) start in perpendicular directions. This solution fits in a icosahedron [see Figure 3a]. Each opposite pair of triangles fits in a golden rectangle. Seen from the right 4 viewing directions these triangles fill the plane without overlaps!



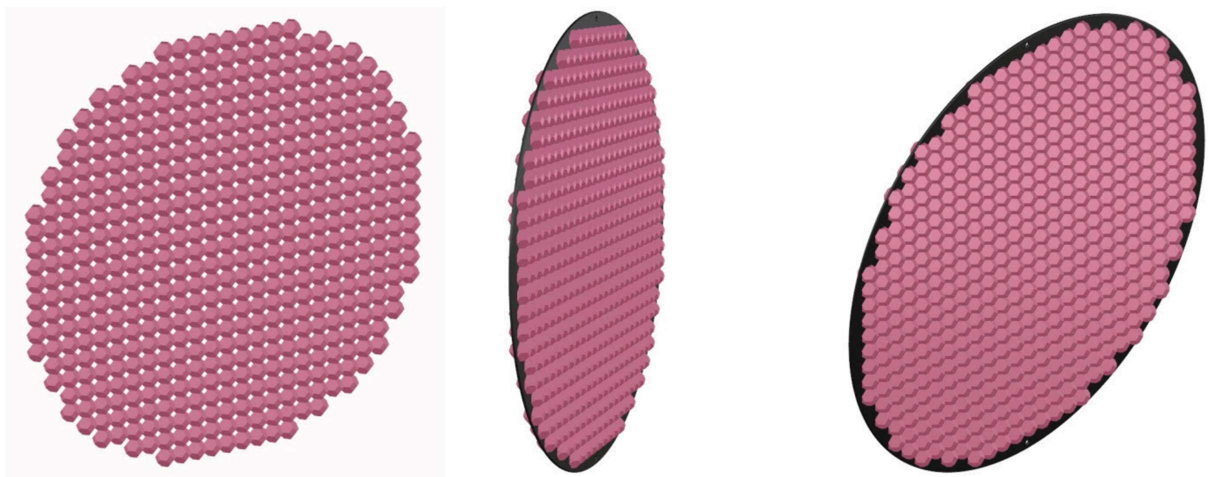
(a) (b) (c) (d)

Figure 3: *First solution: (a) 6 pointed slabs, fitting in a icosahedron, (b) 6 pointed slabs built up with unit cells, (c) 6 pointed slabs with reinforcing square frames, (d) 6 pointed slabs with perfectly fitting edges*

The slabs consist of connected unit cells [see Figure 3b]. Because the slope of the angle (of the pointed slab) is an irrational number (φ:1) there is no regular pattern in the shape of the slab borders which form the central point. Yet the borders fit perfectly to each other without holes and without double cells. The remaining closing border of the pointed slab depends on the edges of the image we want to show. In our case we decided to show the faces of the Beatles framed in hexagons. This leads to pointed slabs in the shape of isosceles triangles. This object [see Figure 3b] is not strong enough to build. The central unit cell must carry all the weight forces of the pointed slabs and will be overloaded. That is why we added three interconnected square frames to reinforce the structure. The reinforcing frame coincides partly with the borders of the image hexagon [see Figure 3c]. The extended parts project to a star shape [see Figure 3d and 9].

This first solution arose more or less intuitively. Starting with a central truncated octahedron we connected unit cells on all six squares. The result was that in all four viewing directions the central cell was surrounded by six other cells. The following step was to extend this process layer by layer. The resulting contour is a hexagon with triangular extensions (see Figure 3d). We decided to use a 12 layered hexagon to create our images.

The second solution we found is a straight forward flat slab filled with unit cells with freely chosen borders [see Figure 4]. Although this second solution is far more simple than our first, it is derived from it. Observing this first solution, we realised that by projecting all cells along one viewing direction to one single plane, it would also fulfil the requirements. We tried to find other solutions "in between", but that did not succeed.

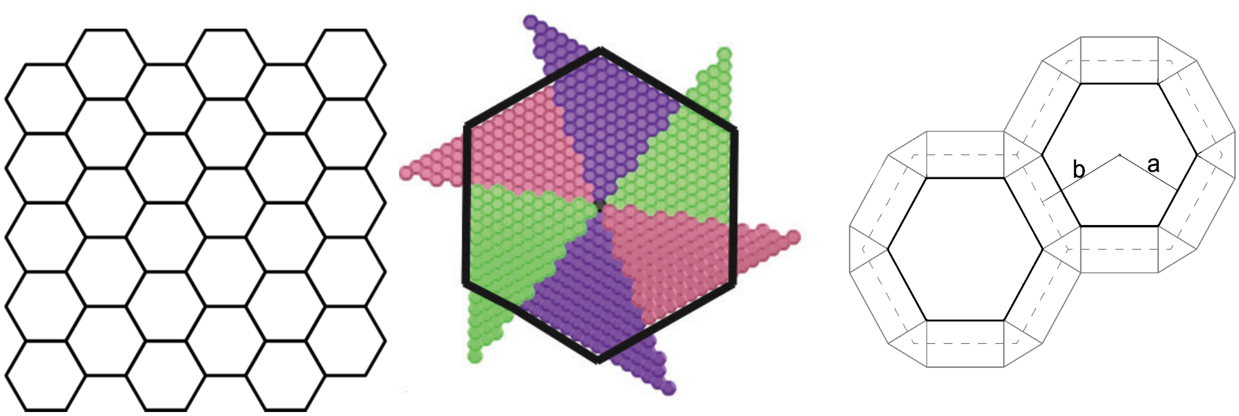


(a) (b) (c)

Figure 4: *Second solution: (a) circular slab built up with 517 unit cells, (b) unit cells reinforced with a circular disk frame, (c) one of the 4 views with an oval border*

Optical Minimal Art Graphics

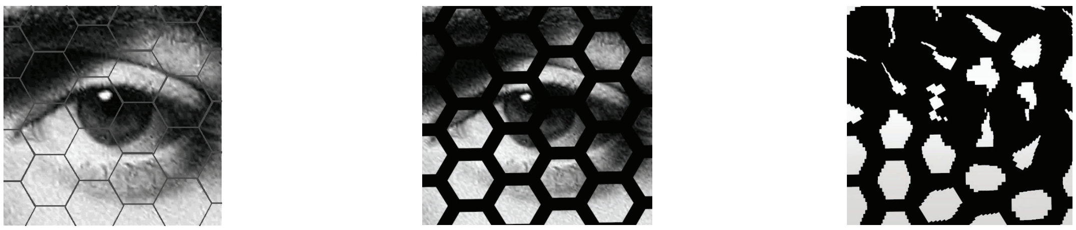
We have to transform a high resolution image with various grey colours into a low resolution black and white image, using tiles in a grid. In the GEB-object we used a grid built up with squares. In the Beatles object we have to use hexagons. The grid is the result of the projection of the connected truncated octahedrons in the viewing directions [see Figure 5].



(a) (b) (c)

Figure 5: *Grid of hexagons: (a) Tiling with hexagons, (b) Tiling in the object; 469 hexagon tiles inside the hexagonal contour and 108 beyond, (c) Detail of two unit cells projected to the viewing plane; notice the overlaps; a:b=3:4 so a^2:b^2=9:16*

We will perforate the truncated octahedron perpendicular to each pair of opposite hexagons to simulate the white areas in the image. The black areas are simulated by the remaining non perforated areas of the hexagons and the (overlapping) projections of the 6 remaining hexagons and the 6 squares. The area of the hexagon available for perforation is 9/16 of the hexagon in the grid [see Figure 5c]. This property limits the maximum brightness of the perceived images. That is why the low res image is darker than the high res image.

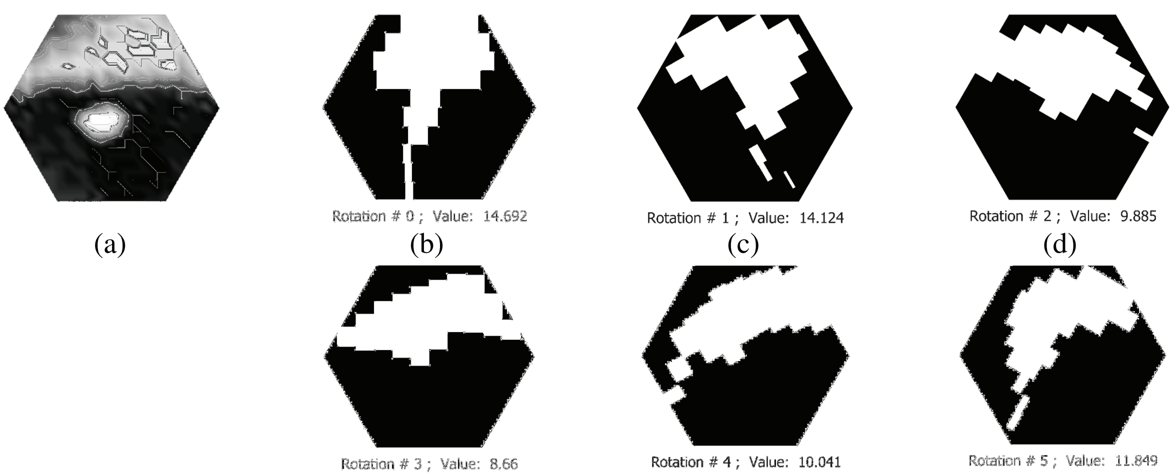


(a) (b) (c)

Figure 6: *From high resolution to low resolution: (a) division of the hi res image in hexagon tiles; (b) downscaled hi res hexagon tiles; (c) result: white areas in low res*

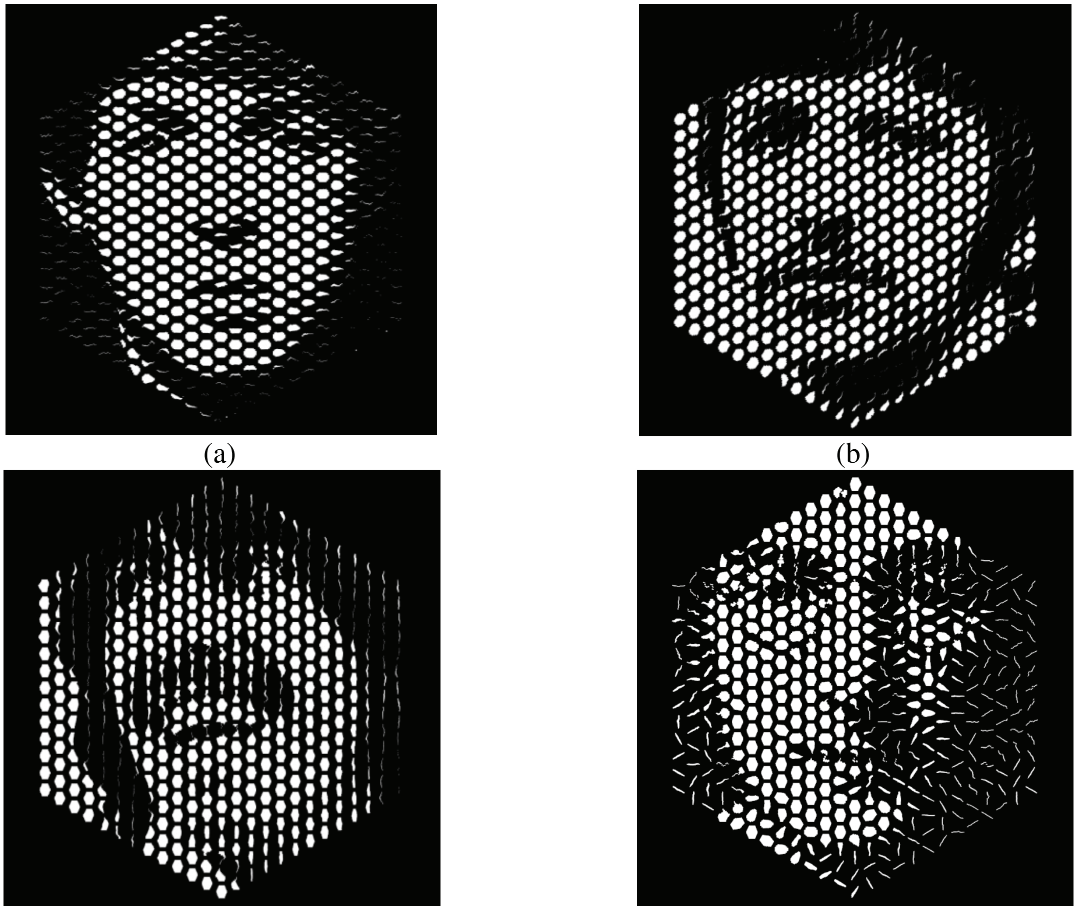
We "downscaled" (by 75%) each tile of the image into the inner hexagon and after that we analysed the shape of the hole in that hexagon [see Figure 6]. We had to accept that there is an overlap of the projections outside of these hexagons, which is always black. The white could only be concentrated within the inner hexagons. We chose to generate a low resolution image with 12 layers around the central cell and the contour of a hexagon. This led to 469 perforated tiles within the hexagon border [see Figure 5b]. Then we had to make holes in the unit cells. To determine the shape of these holes we used a new optical minimal art technique. Until now we had used techniques applied on different shaped strips (straight, circular, spiral, superformula-based) and squares (see [1] and [3]). For our new object we had to apply the optical minimal art technique on hexagons.

We used the technique of the strips, but in such a way that the magnitude of the width of pixel row within the strip varies. We distinguished six different directions within each hexagon [see Figure 7]. We made our calculations with strips with the directions 0, 30, 60, 90,120 and 150 degrees. So we calculated six different shaped holes for each hexagon tile. Then we assumed that the hole with the largest polar moment of inertia [7] was the most expressive. That hole was selected to be part of the final picture. Figure 8 shows the results of that process.



(e) (f) (g)

Figure 7: *Selection of the six different holes. (a) input image; (b-g) output images; (b) selected output image*



(a) (b) (c) (d)

Figure 8: *Using different directions (a) Paul in 0 degrees, (b) Ringo in 60 degrees, (c) George in 90 degrees, (d) John in all six directions*

Different Ways to Display the Objects

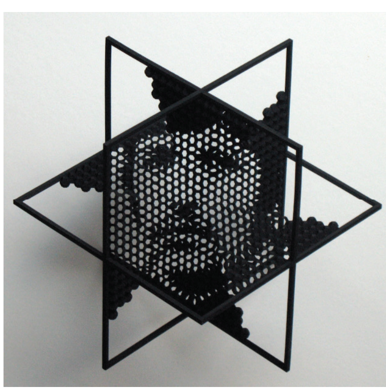


Figure 9: *Ringo in direct view*

There are several ways to display the Beatles object.

One can view the black object against a white background from 4 different viewing points [see Figure 9].

Another way is to look at its shadow in four different orientations of the object [see Figure 10].

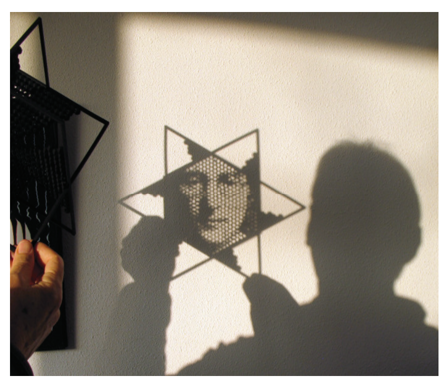


Figure 10: *shadow of John*

But the most spectacular way is to see the black object in one view with its 3 reflections with the help of a configuration of 2 mirrors and a white lighted background [see Figure 11]. Our second solution, the disk version is shown in Figure 12.

(a) (b) (c)

Figure 11: *One object, one lighted background, two mirrors: (a) lighted background switched off, (b) lighted background switched on, (c) 1 direct view and 3 reflections*

(a) (b) (c)

Figure 12: *Second solution (virtual): (a) disk with 4 hidden faces, (b) disk seen in different perspective, (c) Paul in direct view*

Another way to show our art is a 2D parallel projection of the direct view and its 3 reflections [see Figure 13].

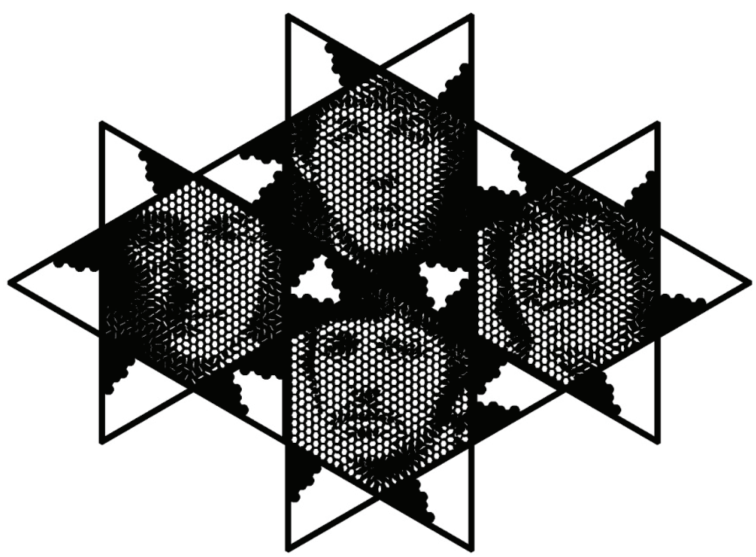


Figure 13: *The Beatles in 2D*

More Famous Quartets

Of course there are more famous quartets. What to think of the four presidents of Mount Rushmore [see Figure 14]?

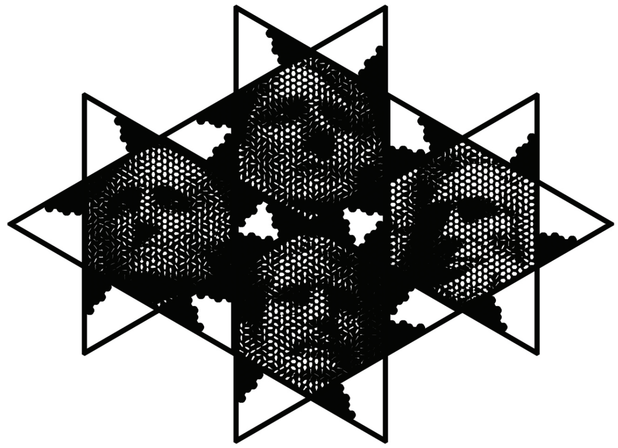


Figure 14: *Mount Rushmore Presidents in 2D*

Figure 15: **ABBA** *in 2D*

Or another group of four famous persons: the Swedish musicians of ABBA [see Figure 15]?

References

[1] H. Kuiper and W. van Ballegooijen. "Gödel, Escher Bach: Just Another Braid." *Bridges Conference Proceedings*, Jyväskylä, Finland, 2016 pp. 403-406. http://archive.bridgesmathart.org/2016/bridges2016-403.pdf.

[2] H. Kuiper. "A SUDOKU Puzzle Generates a Minimal Art Object." *Bridges Conference Proceedings*, Leeuwarden, The Netherlands, 2008, pp. 475-476. http://archive.bridgesmathart.org/2008/bridges2008-475.pdf.

[3] H. Kuiper. "Optical Minimal Art." *Bridges Conference Proceedings*, Towson, USA, 2012, pp. 405-408. http://archive.bridgesmat0068art.org/2012/bridges2012-405.pdf.

[4] H. Kuiper and W. van Ballegooijen. "3D SUDOKO Puzzle with 81 Connected Cubes." *Bridges Conference Proceedings*, Enschede, The Netherlands, 2013, pp. 407-410. http://archive.bridgesmathart.org/2013/bridges2013-407.pdf.

[5] Rhinoceros. Modeling Tools for Designers. https://www.rhino3d.com/.

[6] Shapeways. 3D printing. https://www.shapeways.com/shops/kuiper-van-ballegooijen-math-art.

[7] Polar Moment of Inertia on Wikipedia. https://en.wikipedia.org/wiki/Polar_moment_of_inertia.